Collectivity in small systems from the small-x perspective

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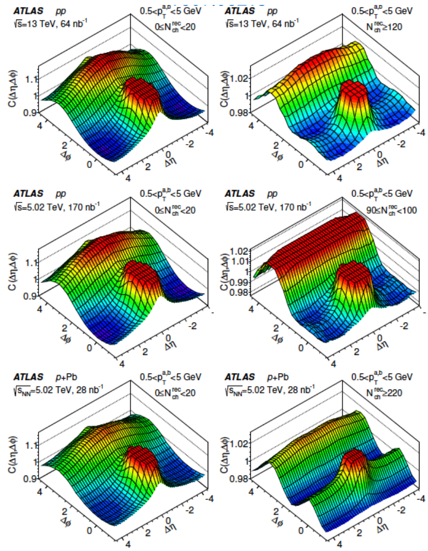
Two particle correlations

Motivation: Ridge structure

[ATLAS Collaboration - arXiv:1609.06213]



- *correlations between particles over large intervals of rapidity peaking at zero and π relative azimuthal angle.*
- observed first at RHIC in Au-Au collisions.
- observed at LHC for high multiplicity pp and pA collisions.

Correlations within the CGC framework

Ridge in HICs ↔ collective flow due to strong final state interactions

(good description of the data in the framework of relativistic viscous hydrodynamics)

Ridge in small size systems: similar reasoning looks tenuous but hydro describes the data very well.

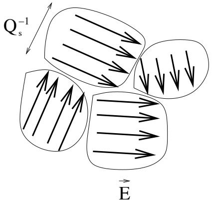
Can it be initial state effect?

idea: *final state particles carry the imprint of the partonic correlations that exist in the initial state.*

Several mechanisms have been suggested to explain the ridge correlations in the CGC framework:

(i) Local anisotropy of the target fields → rotational symmetry is broken.

[Kovner, Lublinsky - arXiv:1012.3398 / arXiv:1109.0347 / arXiv:1211.1928]



particles correlated in the incoming w.f.

transverse separation $\ll 1/Q_s$

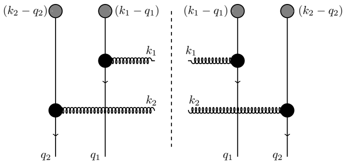
scatter through the same domain.

initial state correlations → final state correlations

Numerical studies based on local anisotropy of the target:

[Dumitru, Skokov - arXiv:1411.6030] / [Dumitru, McLerran, Skokov - arXiv:1410.4844] *(ii) Glasma graph approach to two gluon production:*

[Dumitru, Gelis, McLerran, Venugopalan - arXiv:0804.3858]
[Dumitru, Dusling, Gelis, Jalilian-Marian, Lappi, Venugopalan - arXiv:1009.5295]



What is the physics behind the glasma graph approximation?

⋆ Glasma graph calculation contains two physical effects:

- Bose enhancement of the gluons in projectile/target wave function

  [TA, Armesto, Beuf, Kovner, Lublinsky - arXiv:1503.07126]

  $$\sigma|_{BE,P} \propto \left\{ \delta^{(2)}\left[(k_1 - q_1) - (k_2 - q_2)\right] + \delta^{(2)}\left[(k_1 - q_1) + (k_2 - q_2)\right] \right\}$$
  $$\sigma|_{BE,T} \propto \left\{ \delta^{(2)}(q_1 - q_2) + \delta^{(2)}(q_1 + q_2) \right\}$$

- Hanbury-Brown-Twiss (HBT) correlations between gluons far separated in rapidity.

  $$\sigma|_{HBT} \propto \left\{ \delta^{(2)}(k_1 - k_2) + \delta^{(2)}(k_1 + k_2) \right\}$$

  [TA, Armesto, Beuf, Kovner, Lublinsky - arXiv:1509.03223]

Correlations within the CGC framework -III

Two particle correlations beyond the glasma graph approach: 2 gluon production in pA collisions

[TA, Armesto, Wertepny - arXiv:1804.02910] $\rightarrow$ $k_\perp$-factorized approach

[TA, Armesto, Kovner, Lublinsky - arXiv:1805.07739] $\rightarrow$ Glasma graph approach .

scattering on a dense target $\rightarrow$ *dipole and quadrupole operators.*

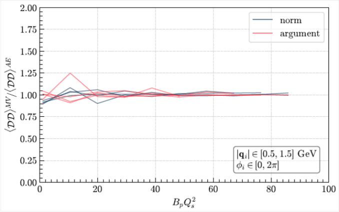
Factorization assumption (Area enhancement (AE) model):

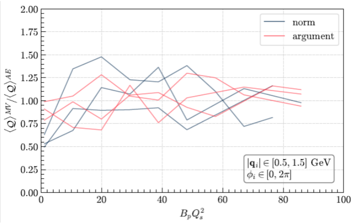
$$\langle Q(x,y,z,v)\rangle_T \;\rightarrow\; d(x,y)d(z,v)+d(x,v)d(z,y)+\frac{1}{N_c^2-1}d(x,z)d(y,v)$$

$$\langle D(x,y)D(z,v)\rangle_T \;\rightarrow\; d(x,y)d(z,v)+\frac{1}{(N_c^2-1)^2}\left[d(x,v)d(y,z)+d(x,z)d(v,y)\right]$$

[Agostini, TA, Armesto - arXiv:2103.08485]

Comparison of the AE model and MV model for fundamental operators:





with B_p being the transverse area of the projectile.

Correlations within the CGC framework -IV

[TA, Armesto, Kovner, Lublinsky - arXiv:1805.07739]

double inclusive X-section within the AE model:

$$\frac{d\sigma}{d^3k_1 d^3k_2} \propto \int_{q_1 q_2} \left\{ d(q_1)d(q_2)\left[I_0 + \frac{1}{N_c^2-1} I_1 + \frac{1}{(N_c^2-1)^2} I_2 \right] + (k_2 \to -k_2) \right\} + O\left(\frac{1}{Q_s S_\perp}\right)$$

symmetry under $(k_2 \to -k_2)$: **"accidental symmetry of the CGC"**

$I_0 \propto \delta^{(2)}(0) \to$ uncorrelated contribution.

$$I_1 \propto \Big\{ \underbrace{f_1 \delta^{(2)}\big[(k_1 - q_1) - (k_2 - q_2)\big]}_{\text{BE. proj.}} + \underbrace{f_2 \delta^{(2)}(k_1 - k_2)}_{\text{HBT}} \Big\}$$

$$I_2 \propto \Big\{ \underbrace{g_1 \delta^{(2)}(q_1 - q_2)}_{\text{BE. target}} + \underbrace{g_2 \delta^{(2)}\big[(k_1 - q_1) - (k_2 - q_2)\big]}_{\text{BE. proj.}} \Big\}$$

Convenient way to study the two particle correlations: Fourier decomposition into harmonics in $\Delta\phi$

$\Delta\phi \equiv$ azimuthal angle between the produced gluons with transverse momenta $\mathbf{k}_1$ and $\mathbf{k}_2$

[T. Lappi, B. Schenke, S. Schlichting, R. Venugopalan - arXiv: 1509.03499]

$$2V_{n\Delta}(k_1, k_2) = \frac{a_n(k_1,k_2)}{a_0(k_1,k_2)} = 2\,\frac{\int_0^\pi N(k_1,k_2,\Delta\phi)\,\cos(n\Delta\phi)\,d\Delta\phi}{\int_0^\pi N(k_1,k_2,\Delta\phi)\,d\Delta\phi}$$

- set $k_1 = p_T^{ref}$ and $k_2 = p_T$. Then, *the azimuthal harmonics are defined as*

$$v_n(p_T) = \frac{V_{n\Delta}(p_T, p_T^{ref})}{\sqrt{V_{n\Delta}(p_T^{ref}, p_T^{ref})}}$$

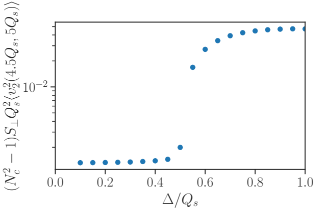
Correlated 3 gluon production: v_2 and N correlations

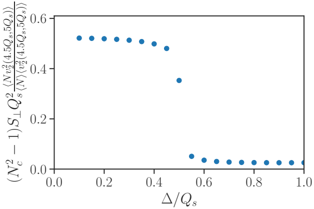
[TA, Armesto, Kovner, Lublinsky, Skokov - arXiv: 2012.01810]

$$v_2^2(k,k',\Delta) = \frac{\int_{k-\Delta/2}^{k+\Delta/2} k_2 dk_2 \int_{k'-\Delta/2}^{k'+\Delta/2} k_3 dk_3 \int d\phi_2 d\phi_3 e^{i2(\phi_2-\phi_3)} \frac{d^2N^{(2)}}{d^2k_2 d^2k_3}}{\int_{k-\Delta/2}^{k+\Delta/2} k_2 dk_2 \int_{k'-\Delta/2}^{k'+\Delta/2} k_3 dk_3 \int d\phi_2 d\phi_3 \frac{d^2N^{(2)}}{d^2k_2 d^2k_3}}$$

In momentum space: width of the BE correlations $\sim Q_s$ & width of the HBT correlations $\ll Q_s$

non-overlapping bins				overlapping bins
$\Delta < \lvert k-k'\rvert$	$\rightarrow$	$\Delta \approx \lvert k-k'\rvert$	$\rightarrow$	$\Delta > \lvert k-k'\rvert$
only BE contribution		HBT starts to contribute		BE+HBT contribution





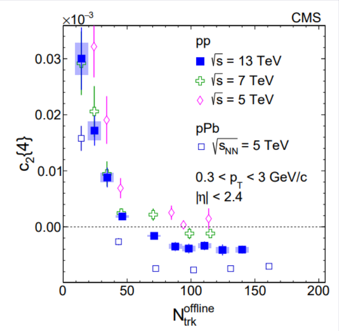
non-overlapping bins: $\Delta/Q_s < 0.5$ – v_2 is small & sizable correlation between N and v_2 (No HBT contribution)

non-overlapping bins: $\Delta/Q_s > 0.5$ – v_2 is large & negligable correlation between N and v_2 (HBT start contributing)

4 gluon production and correlations

- negative 4-particle cumulant, $c_2\{4\}$, at high multiplicity
- positive 4-particle cumulant, $c_2\{4\}$, at low multiplicity

[CMS - arXiv:1606.06198]
[ALICE - arXiv:1406.2474]



previous CGC calculations to study 4-particle correlations:

- positive 4-particle cumulant in the dilute-dilute regime
  [Dumitru, McLerran , Skokov - arXiv:1410.4844]
- negative 4-particle cumulant in the dilute-dense regime (quarks only)
  [Dusling, Mace, Venugopalan - arXiv:1706.06260]

⋆ cumulants:

$$c_n\{2\} = \left\langle e^{in(\phi_1-\phi_2)} \right\rangle$$

$$c_n\{4\} = \left\langle e^{in(\phi_1+\phi_2-\phi_3-\phi_4)} \right\rangle - 2\left\langle e^{in(\phi_1-\phi_2)} \right\rangle^2$$

⋆ azimuthal harmonics:

$$v_n\{2\} = (c_n\{2\})^{1/2}$$

$$v_n\{4\} = (-c_n\{4\})^{1/4}$$

⋆ event average:

$$\left\langle e^{in(\phi_1+\cdots+\phi_{m/2}-\phi_{m/2+1}-\cdots-\phi_m)} \right\rangle = \frac{\int \left(\prod_{i=1}^m \frac{d^2\mathbf{k}_i}{(2\pi)^2}\right) \frac{d^m N}{\prod_{i=1}^m d^2\mathbf{k}_i} e^{in(\phi_1+\cdots+\phi_{m/2}-\phi_{m/2+1}-\cdots-\phi_m)}}{\int \left(\prod_{i=1}^m \frac{d^2\mathbf{k}_i}{(2\pi)^2}\right) \frac{d^m N}{\prod_{i=1}^m d^2\mathbf{k}_i}}$$

Multi-particle production: technical aspects (i)

[Agostini, TA, Armesto - arXiv:2103.08485]

multi-gluon spectra in dilute-dense regime at LO:

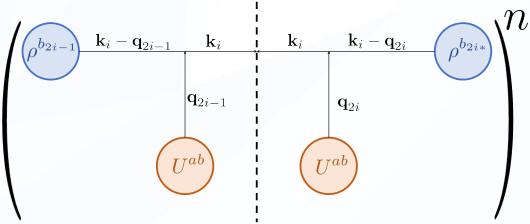
$$\frac{d^n N}{\prod_{i=1}^n d^2\mathbf{k}_i} = \int \left(\prod_{i=1}^n \frac{d^2\mathbf{q}_{2i-1}}{(2\pi)^2}\frac{d^2\mathbf{q}_{2i}}{(2\pi)^2}\right) \underbrace{\left\langle \left(\prod_{i=1}^n g^2 \rho^{b_{2i-1}}(\mathbf{k}_i-\mathbf{q}_{2i-1})\rho^{b_{2i}*}(\mathbf{k}_i-\mathbf{q}_{2i})\right)\right\rangle_p}_{\text{Contribution from the projectile sources}} \underbrace{\left\langle \left(\prod_{i=1}^n \overline{\mathcal{M}}^{a_i b_{2i-1}}_{\lambda_i}(\mathbf{k}_i,\mathbf{q}_{2i-1})\overline{\mathcal{M}}^{\dagger a_i b_{2i}}_{\lambda_i}(\mathbf{k}_i,\mathbf{q}_{2i})\right)\right\rangle_T}_{\text{Contribution from the strong field of the target}}$$

$$\overline{\mathcal{M}}^{ab}_{\lambda}(\mathbf{k},\mathbf{q}) = 2i\epsilon^{i*}_{\lambda}(\mathbf{k})L^i(\mathbf{k},\mathbf{q})\int_y e^{-i\mathbf{q}\cdot\mathbf{y}}U^{ab}(\mathbf{y})$$

Wilson line

$$L^i(\mathbf{k},\mathbf{q}) = \frac{\mathbf{k}^i}{\mathbf{k}^2} - \frac{(\mathbf{k}-\mathbf{q})^i}{(\mathbf{k}-\mathbf{q})^2}$$

Lipatov vertex



Multi-particle production: technical aspects (ii)

[Agostini, TA, Armesto - arXiv:2103.08485]

Models for the projectile and target averaging, and for the Lipatov vertex:

- Gaussian (MV-like) model for an extended projectile (with area B_p):

$$g^2\Big\langle \rho^{b_i}(\mathbf{k}_i-\mathbf{q}_i)\rho^{b_j}(\mathbf{k}_j-\mathbf{q}_j)\Big\rangle_p = \frac{\delta^{b_ib_j}}{N_c^2-1}\mu^2\left[\mathbf{k}_i-\mathbf{q}_i,\mathbf{k}_j-\mathbf{q}_j\right] \qquad \mu^2(\mathbf{k},\mathbf{q}) = e^{-\frac{(\mathbf{k}+\mathbf{q})^2}{4B_p^{-1}}}$$

$$\Big\langle \rho^{b_1}(\mathbf{k}_1-\mathbf{q}_1)\rho^{b_2\dagger}(\mathbf{k}_1-\mathbf{q}_2)\cdots\rho^{b_{2n-1}}(\mathbf{k}_n-\mathbf{q}_{2n-1})\rho^{b_{2n}\dagger}(\mathbf{k}_n-\mathbf{q}_{2n})\Big\rangle_p = \sum_{\omega\in\Pi(\chi)}\prod_{\{i,j\}\in\omega}\Big\langle \rho^{b_i}(\mathbf{k}_i-\mathbf{q}_i)\rho^{b_j}(\mathbf{k}_j-\mathbf{q}_j)\Big\rangle_p$$

- AE model for target averaging and GBW model for the dipole operators:

$$\Big\langle U(\mathbf{y}_1)^{a_1b_1}U(\mathbf{y}_2)^{a_2b_2}\cdots U(\mathbf{y}_{2n})^{a_{2n}b_{2n}}\Big\rangle_T = \sum_{\sigma\in\Pi(\chi)}\prod_{\{\alpha,\beta\}\in\sigma}\Big\langle U(\mathbf{y}_\alpha)^{a_\alpha b_\alpha}U(\mathbf{y}_\beta)^{a_\beta b_\beta}\Big\rangle_T$$

$$\Big\langle U(\mathbf{x})^{ab}U^\dagger(\mathbf{y})^{dc}\Big\rangle_T = \frac{\delta^{ac}\delta^{bd}}{(N_c^2-1)^2}\Big\langle \mathrm{Tr}\left(U(\mathbf{x})U^\dagger(\mathbf{y})\right)\Big\rangle_T \qquad \frac{1}{N_c^2-1}\Big\langle Tr\left[U(\mathbf{y}_1)U^\dagger(\mathbf{y}_2)\right]\Big\rangle_T = e^{-\frac{Q_s^2}{4}(\mathbf{y}_1-\mathbf{y}_2)^2}$$

- Lipatov vertex contains IR divergences: use a Gaussian instead.

$$L^i(\mathbf{k},\mathbf{q}_1)L^i(\mathbf{k},\mathbf{q}_2) \to \frac{(2\pi)^2}{\xi^2}\exp\left\{-\frac{[\mathbf{k}-(\mathbf{q}_1+\mathbf{q}_2)/2]^2}{\xi^2}\right\}$$

it connects with the Wigner function approach but includes correlations:

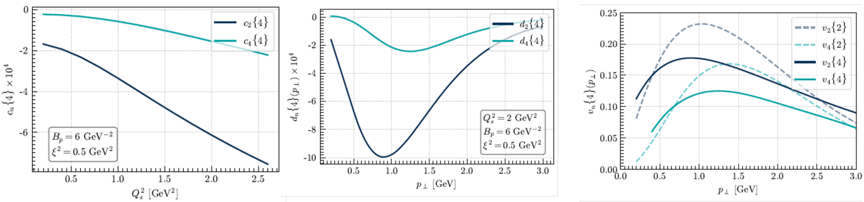
$$W^{b_1b_2b_3b_4}(\mathbf{b}_1,\mathbf{p}_1,\mathbf{b}_2,\mathbf{p}_2) = \frac{1}{(N_c^2-1)^2}\frac{1}{\pi^4\xi^4B_p^2}e^{-(\mathbf{p}_1^2+\mathbf{p}_2^2)/\xi^2}e^{-(\mathbf{b}_1^2+\mathbf{b}_2^2)/B_p}\Big[\delta^{b_1b_2}\delta^{b_3b_4}$$

$$+\,\delta^{b_1b_3}\delta^{b_2b_4}2\pi B_p\delta^{(2)}(\mathbf{b}_1-\mathbf{b}_2)e^{-(\mathbf{p}_1+\mathbf{p}_2)^2/(2B_p^{-1})} + \delta^{b_1b_4}\delta^{b_2b_3}2\pi B_p\delta^{(2)}(\mathbf{b}_1-\mathbf{b}_2)e^{-(\mathbf{p}_1-\mathbf{p}_2)^2/(2B_p^{-1})}\Big]$$

4 gluon production: numerical results

- Large number of terms for 4 gluon production ($\sim$11000), reduced by using the symmetries

$n = 4$



⋆ negative 4-particle cumulants ⇒ well-defined azimuthal harmonics
⋆ numerical values lie in the bulk of the experimental data
⋆ if we do not include the correlations in the projectile the cumulants are positive

Accidental symmetry in the CGC

"accidental symmetry in CGC" $\Rightarrow$ vanishing odd harmonics

• *breaking the accidental symmetry with nonlinear Gaussian approximation for dipole-dipole correlator:*

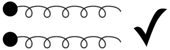
[Lappi, Schenke, Schlichting, Venugopalan - arXiv:1509.03499]

$$\langle D(x,y)D(u,v)\rangle = d_1 + \frac{1}{N_c^2}\left[\frac{\ln(d_3/d_2)}{\ln(d_1/d_2)}\right]^2 \left\{ d_1 + d_2\left[\ln(d_1/d_2) - 1\right]\right\}$$

$$d_1 \equiv D(x-y)D(u-v)$$
$$d_2 \equiv D(x-v)D(u-y)$$
$$d_3 \equiv D(x-u)D(y-v)$$

• *breaking the accidental symmetry with the density corrections to the projectile:*

[Kovner, Lublinsky, Skokov - arXiv:1612.07790] / [Kovchegov, Skokov - arXiv:1802.08166]
[Kendi, Marquet, Vila] (in prepration)



Subeikonal corrections in the CGC

Eikonal approximation amounts dropping the energy suppressed terms!

For realistic values of energy one should go beyond eikonal approximation.

• dense target is defined by $\mathcal{A}^{\mu}(x)$ and eikonal approximation amounts to:

1. $\mathcal{A}^{\mu}_a(x) \simeq \delta^{\mu -}\mathcal{A}^-_a(x)$
2. $\mathcal{A}^{\mu}_a(x) \simeq \mathcal{A}^{\mu}_a(x^+, \mathbf{x})$
3. $\mathcal{A}^{\mu}_a(x) \propto \delta(x^+)$

1. other components of the target background field $\mathcal{A}^{\mu}_a(x)$
2. dynamics of the target : x^- dependence of $\mathcal{A}^{\mu}_a(x)$
3. Finite width L^+ of the target along x^+

(1) Other components of the background field (quark production):

[TA, Beuf, Czajka, Tymowska - arXiv:2012.03886]

(2) Dynamics of the target (scalar and quark propogators):

[TA, Beuf - arXiv:2109.01620]

(3) Finite width corrections in single inclusive gluon production:

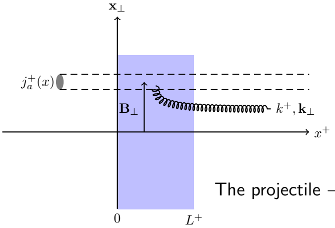
[TA, Armesto, Beuf, Martínez, Salgado - arXiv:1404.2219]
[TA, Armesto, Beuf, Moscoso - arXiv:1505.01400]

$$\mathcal{A}^{\mu} = \delta^{\mu -}\delta(x^+)\mathcal{A}^-(\mathbf{x}) \to \mathcal{A}^{\mu} = \delta^{\mu -}\mathcal{A}^-(x^+, \mathbf{x})$$

direct relation with jet quenching (BDMPS-Z formulation)!

Subeikonal corrections in the CGC - II

[TA, Armesto, Beuf, Moscoso - arXiv:1505.01400]



The target $\rightarrow \mathcal{A}^\mu(x) \equiv \delta^{\mu -} \mathcal{A}_a^-(x^+, \mathbf{x})$

The projectile $\rightarrow j_a^\mu(x) \propto \delta^{\mu +} \delta(x^-)\, \rho^b(\mathbf{x} - \mathbf{B})$

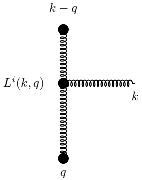
Prod. Amp. $\mathcal{M} \propto$ scalar background propagator $\rightarrow$ *eikonal expansion (in powers of L^+/k^+)*

eikonal order: standard Wilson line / higher orders: new operators (decorated Wilson lines)

[TA, Dumitru - arXiv:1512.00279] $\rightarrow$ *corrections to the Lipatov vertex.*

from pA to pp: expand the standard & decorated Wilson lines to first order in the background field.

$$\mathcal{M} \propto \left[\frac{(k-q)^i}{(k-q)^2} - \frac{k^i}{k^2} \right] \left\{ 1 + i \frac{k^2}{2k^+} x^+ - \frac{1}{2} \left(\frac{k^2}{2k^+} x^+ \right)^2 \right\}$$



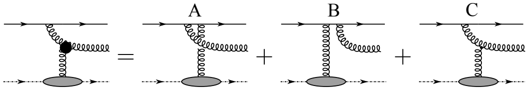
$O(1)$ term eikonal Lipatov vertex.

the form of the corrections suggests exponentiation.

Subeikonal corrections in the CGC - III

[Agostini, TA, Armesto - arXiv:1902.04483]

- calculate the diagrams by keeping the phase $e^{ik^- x^+}$ which is taken to be 1 in the eikonal limit.



$$L^i_{\rm NE}(\underline{k}, q; x^+) = \left[\frac{(k-q)^i}{(k-q)^2} - \frac{k^i}{k^2}\right] e^{ik^- x^+}$$

$$\underline{k} \equiv (k^+, k)$$
$$k^- = k^2/2k^+$$

Double inclusive cross section with Non-Eik Lipatov vertex

$$\frac{d\sigma}{d^2k_1 d\eta_1 d^2k_2 \eta_2}\Bigg|^{\rm NE}_{\rm dilute} \propto \int_{q_1 q_2} \left\{ \left[f(k_1, q_1, k_2, q_2) + \mathcal{G}_2^{\rm NE}(k_1^-, k_2^-; L^+)\, g(k_1, q_1, k_2, q_2) \right] + (\underline{k}_2 \to -\underline{k}_2) \right\}$$

all non-eikonal effects are encoded in

$$\mathcal{G}_2^{\rm NE}(k_1^-, k_2^-; L^+) = \left\{ \frac{2}{(k_1^- - k_2^-)L^+} \sin\left[\frac{(k_1^- - k_2^-)}{2} L^+\right] \right\}^2$$

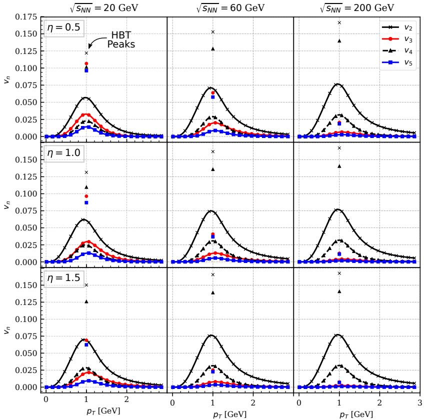
$\mathcal{G}_2^{\rm NE}(k_1^-, k_2^-; L^+)$ is not symmetric under $(\underline{k}_2 \to -\underline{k}_2)$

⇒non-eikonal corrections seem to be breaking the accidental symmetry!!

odd-harmonics from the non-eikonal corrections in *pp*?

[Agostini, TA, Armesto - arXiv:1907.03668]

Non-eikonal corrections do generate odd harmonics.



$$V_{n\Delta}(k_1, k_2) = \frac{\int_0^{\pi} N(k_1, k_2, \Delta\phi)\cos(n\Delta\phi)\, d\Delta\phi}{\int_0^{\pi} N(k_1, k_2, \Delta\phi)\, d\Delta\phi}$$

$$v_n(p_T) = \frac{V_{n\Delta}(p_T, p_T^{ref})}{\sqrt{V_{n\Delta}(p_T^{ref}, p_T^{ref})}}$$

- $L^+ = 6$ fm in the rest frame and we scale it with the γ factor for different energies.
- $\mu_T = 0.4$ GeV and $\mu_P = 0.2$ GeV (these are the values that maximize v_3).
- $\eta_1 = \eta_2$ & $p_t^{ref} = 1$ GeV.

Non-eikonal effects alone can not explain the odd-harmonics HOWEVER there is a contribution originating from these effects for certain kinematic region.

[Agostini, TA, Armesto, Dominguez, Milhano - arXiv:2207.10472]

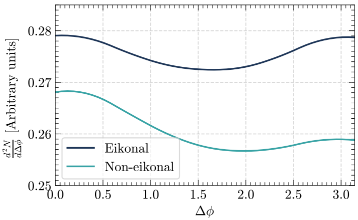
- *NonEik. double inclusive spectrum*

(all order finite width effects) with operators like:

$$\frac{1}{N_c^2-1}\Big\langle \mathrm{Tr}\left[\mathcal{G}_{k^+}(x^+,\mathbf{x};y^+,\mathbf{y})U^\dagger_{\mathbf{y}}(x^+,y^+)\right]\Big\rangle_T$$

$$\frac{1}{N_c^2-1}\Big\langle \mathrm{Tr}\left[\mathcal{G}_{k_1^+}(x^+,\mathbf{x};y^+,\mathbf{y})\mathcal{G}^\dagger_{k_2^+}(x^+,\bar{\mathbf{x}};y^+,\bar{\mathbf{y}})\right]\Big\rangle_T$$

- $\sqrt{s_{\mathrm{NN}}} = 100$ GeV, $\eta_1 = \eta_2 = 0.2$, $|k_1| = |k_2| = 1$ GeV
- near-side %4 and away-side %8



[Agostini, TA, Armesto - arXiv:2212.XXXXX]

- *Harmonics from NNEik spectrum:* non-eikonal parameter $\epsilon_i = Q_s^2 L^+/(2k_i^+)$

$k_i^+ = |k_i|e^{\eta_i}/\sqrt{2}$
$L^+ = 2r_A/(\gamma\sqrt{2})$
with
nuclear-radius $r_A \sim 5A^{1/3}$ GeV^{-1}
Lorentz factor $\gamma = \sqrt{s}/(2m_N)$
nucleon mass, $m_N \sim 1$ GeV

In the plot:
$\eta_1 = 0.1$, $\eta_2 = 0.5$
$Q_s = 1$ GeV, $A^{1/3} = 6$