Quark mass effects in the splitting part of double parton distributions.

a consistent treatment

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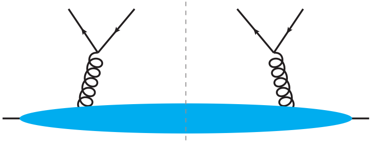
Part I

Small distance DPDs and quark mass effects.

Perturbative splitting in DPDs.

In the limit of small distance $\boldsymbol{y}$ the leading contribution to a DPD is due to the perturbative splitting of one parton into two:



$$F_{a_1 a_2}(x_1, x_2, y; \mu) = \frac{1}{\pi y^2} \, V_{a_1 a_2, a_0}(y; \mu) \underset{12}{\otimes} f_{a_0}(\mu) \,,$$

where

$$\Big[V \underset{12}{\otimes} f\Big](x_1, x_2) = \int_x^1 \frac{dz}{z^2} \, V\left(\frac{x_1}{z}, \frac{x_2}{z}\right) f(z)$$

Note that in applications the splitting DPDs are multiplied with a Gaussian damping factor:

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formally from OPE for $y \to 0$

At which scale μ_{split} should the splitting be evaluated?

- Natural scale is $\mu_{\text{split}}(y) \sim \frac{1}{y}$.
- Define regulated function $\mu_{\text{split}}(y)$ to avoid evaluation at non-perturbative scales for large y:

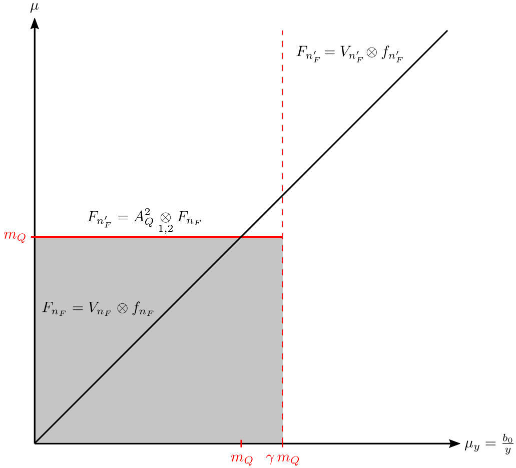
$$\mu_{\text{split}}(y) = \frac{b_0}{y^*(y)}\,, \qquad y^*(y) = \frac{y}{\sqrt[4]{1+y^4/y_{\text{max}}^4}}\,, \qquad y_{\text{max}} = \frac{b_0}{\mu_{\text{min}}}$$

where y^* is adapted from b^* in TMD studies.

What happens if μ_{split} is similar to a heavy quark mass m_Q?

Purely massless quarks.

The simplest scheme to handle massive quarks is to treat them as absent below a certain scale and as massless above a certain scale.



- Below $\mu_y = \gamma\, m_Q$ the DPD is initialized for n_F massless flavours with a n_F flavour PDF.

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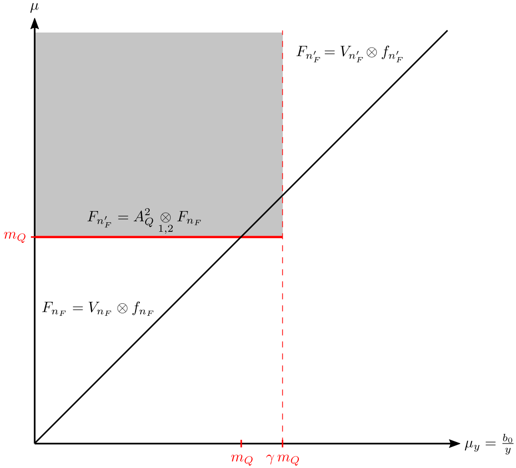

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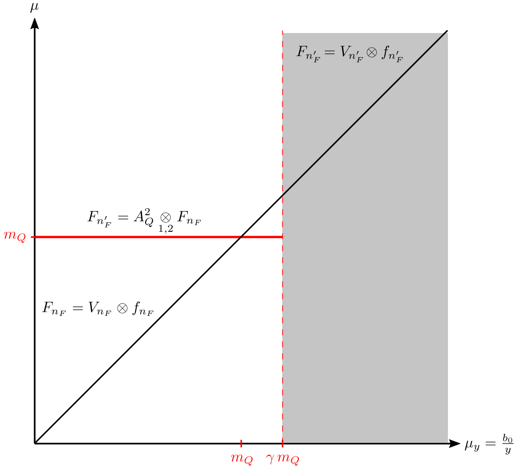
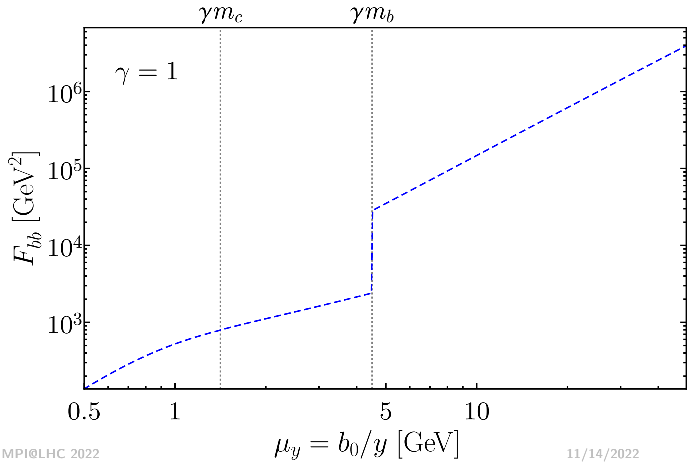


- Above $\mu_y = \gamma\, m_Q$ the DPD is initialized for $n_F + 1$ massless flavours with a $n_F + 1$ flavour PDF.

Purely massless quarks.

Consider $n_F = 5$ LO splitting DPDs at $\mu_1 = \mu_2 = m_{\text{dijet}} = 25\,\text{GeV}$ initialized with the scheme shown in the previous slide:

$$F_{b\bar{b}}(x_1 = x_2 = m_{\text{dijet}}/\sqrt{s},\ y,\ m_{\text{dijet}})$$

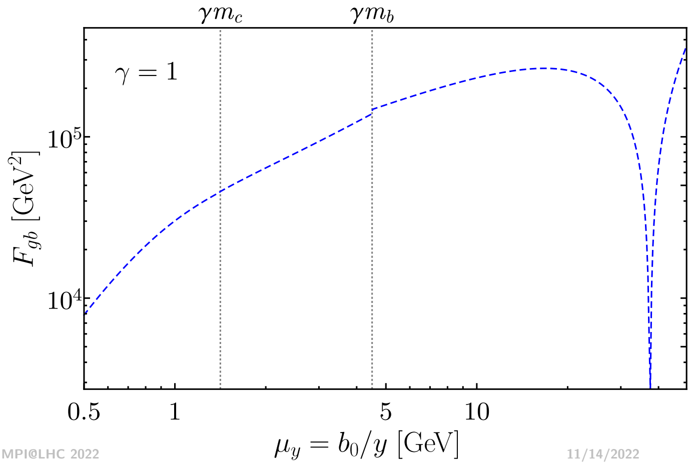


- Below $\mu_y = m_b$ the $b\bar{b}$ DPD is produced only by flavour matching and evolution.
- Above $\mu_y = m_b$ the $b\bar{b}$ DPD is produced by a direct (massless) $g \to q\bar{q}$ splitting.

Purely massless quarks.

Consider $n_F = 5$ LO splitting DPDs at $\mu_1 = \mu_2 = m_{\text{dijet}} = 25\,\text{GeV}$ initialized with the scheme shown in the previous slide:

$$F_{gb}(x_1 = x_2 = m_{\text{dijet}}/\sqrt{s},\ y,\ m_{\text{dijet}})$$

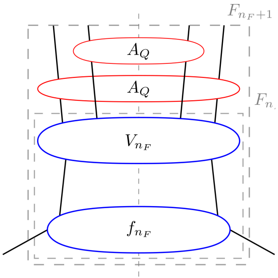


- At LO the gb DPD is produced by a direct splitting only for $\mu_y > \gamma m_b$.
- Heavy quark effects in the splitting seem to be unimportant.

A more realistic treatment of quark mass effects.

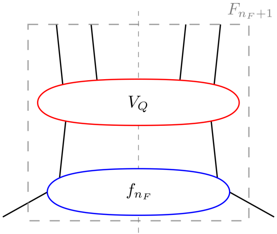
In the splitting DPDs one can distinguish three regions of μ_{split}:

$\mu_{\text{split}} \ll m_Q$:



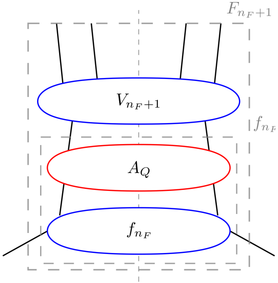
- In the splitting the heavy quarks decouple.
- $n_F + 1$ DPDs obtained by flavour matching.

$\mu_{\text{split}} \sim m_Q$:



- Heavy quarks treated as massive in the splitting kernel V_Q.

$\mu_{\text{split}} \gg m_Q$:



- Heavy quarks can be treated as massless in the splitting.

Massive DPD splitting kernels.

Just like the massless V_{n_F} kernels the massive V_Q kernels can be computed in perturbation theory!

At leading order the only splitting with massive quarks is $g \to Q\bar{Q}$, where the kernel reads:

$$V^{(1)}_{Q\bar{Q},g}(z_1, z_2, m_Q, y) = T_f \, (m_Q \, y)^2 \left[(z_1^2 + z_2^2) \, K_1^2(m_Q \, y) + K_0^2(m_Q \, y) \right] \, \delta(1 - z_1 - z_2)$$

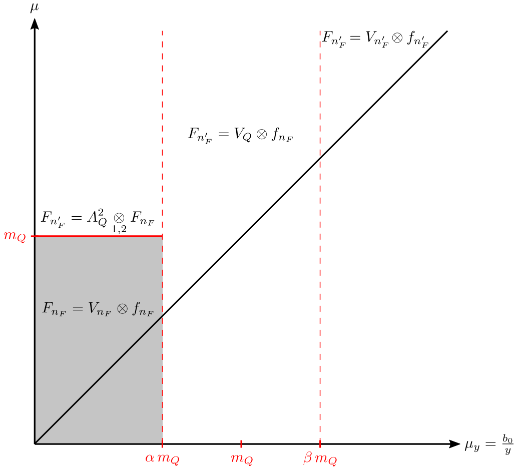
with the following limiting behaviour for small and large μ_{split} (corresponding to large and small $m_Q \, y$, respectively):

$$\begin{aligned} \mu_{\text{split}} \ll m_Q : &\qquad V^{(1)}_{Q\bar{Q},g}(z, m_Q, y) \longrightarrow 0 \\ \mu_{\text{split}} \gg m_Q : &\qquad V^{(1)}_{Q\bar{Q},g}(z_1, z_2, m_Q, y) \longrightarrow T_f (z_1^2 + z_2^2) \, \delta(1 - z_1 - z_2) = V^{(1)}_{q\bar{q},g}(z_1, z_2) \end{aligned}$$

$\longrightarrow$ The massive kernel interpolates between the regions where the heavy quark decouples and where it can be treated as massless!

One heavy flavour.

Consider now the initialization of a splitting DPD with one heavy flavour (where $\alpha \ll 1$ and $\beta \gg 1$):



- Below $\mu_y = \alpha\, m_Q$ the DPD is initialized for n_F massless flavours with a n_F flavour PDF.

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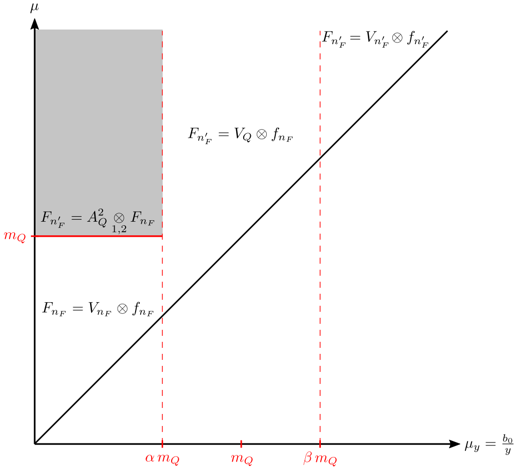

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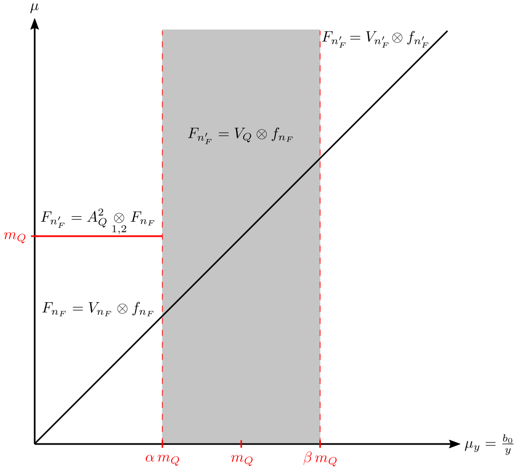

- For $\alpha\, m_Q < \mu_y < \beta\, m_Q$ the DPD is initialized for n_F massless and one massive flavours with a n_F flavour PDF.

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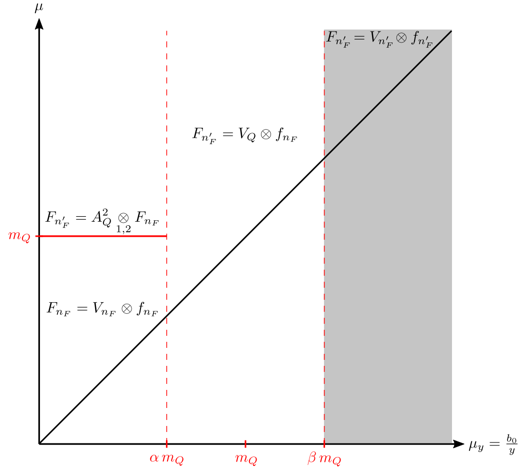

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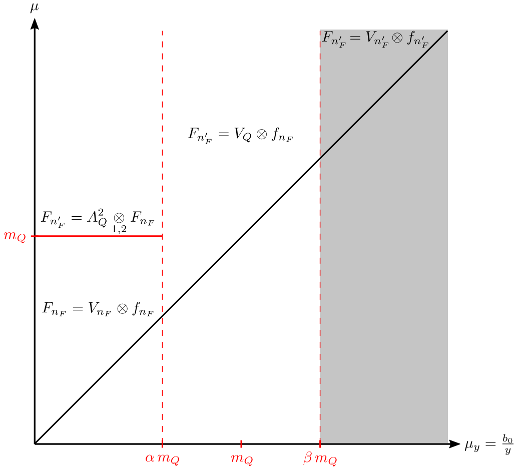
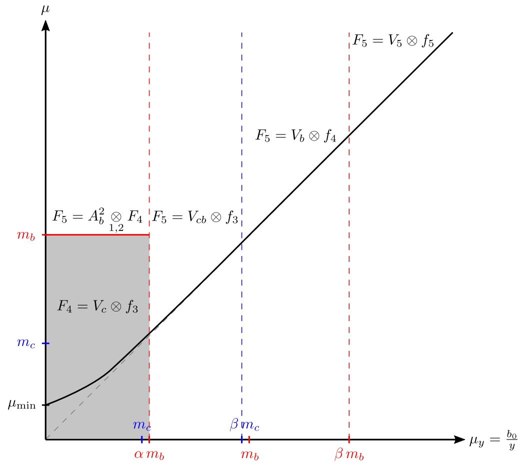


- Above $\mu_y = \beta\, m_Q$ the DPD is initialized for $n_F + 1$ massless flavours with a $n_F + 1$ flavour PDF.

What happens for charm and bottom which have to be treated as massive simultaneously?

Two heavy flavours: charm and bottom.

Consider now the initialization of a splitting DPD with massive c and b quarks:



- Below $\mu_y = \alpha\, m_b$ the DPD is initialized for 3 massless and one heavy flavours with a 3 flavour PDF.

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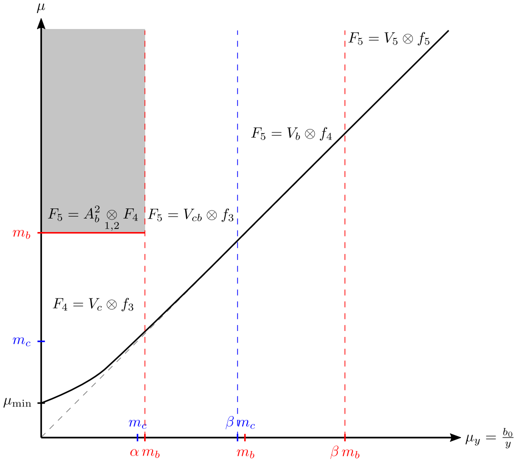

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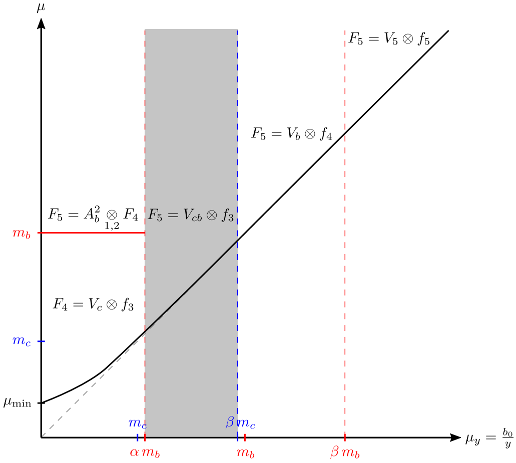

- For $\alpha\, m_b < \mu_y < \beta\, m_c$ the DPD is initialized for 3 massless and two massive flavours with a 3 flavour PDF.

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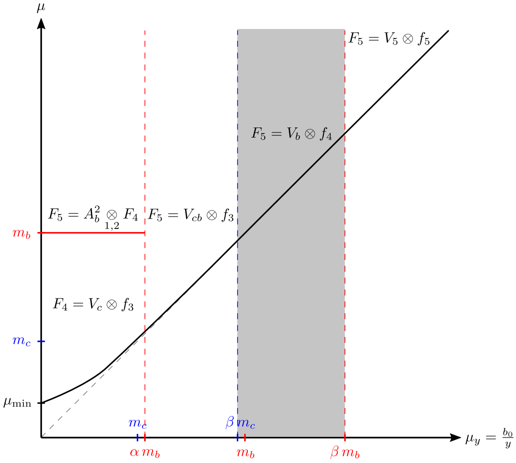

- For $\beta\, m_c < \mu_y < \beta\, m_b$ the DPD is initialized for 4 massless and one massive flavours with a 4 flavour PDF.

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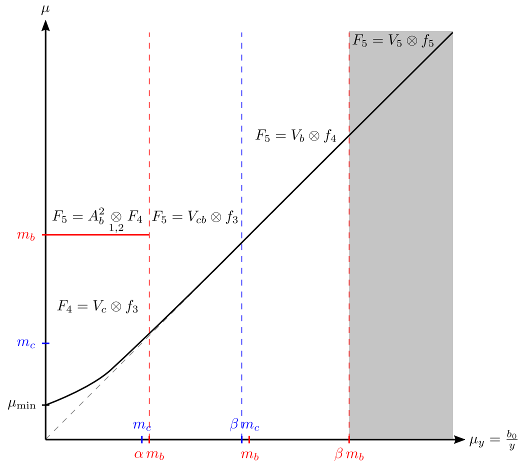

- Above $\mu_y = \beta\, m_b$ the DPD is initialized for 5 massless flavours with a 5 flavour PDF.

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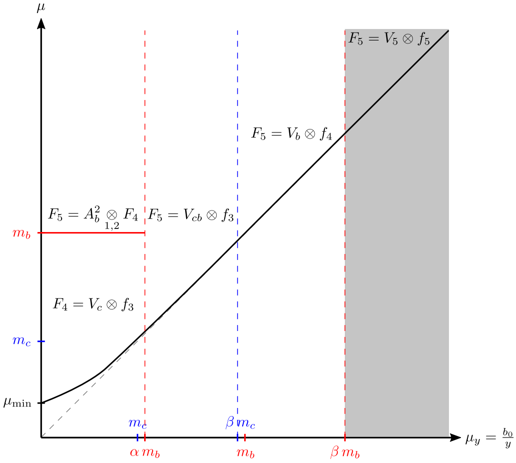

- Above $\mu_y = \beta\, m_b$ the DPD is initialized for 5 massless flavours with a 5 flavour PDF.

Let's see how the DPDs look like in this scheme!

Part II

Results.

Splitting DPDs for dijet production.

Consider now $n_F = 5$ LO splitting DPDs at $\mu_1 = \mu_2 = m_{\text{dijet}} = 25\,\text{GeV}$ for dijet production, initialized with the scheme shown in the previous slide (for different α and β):

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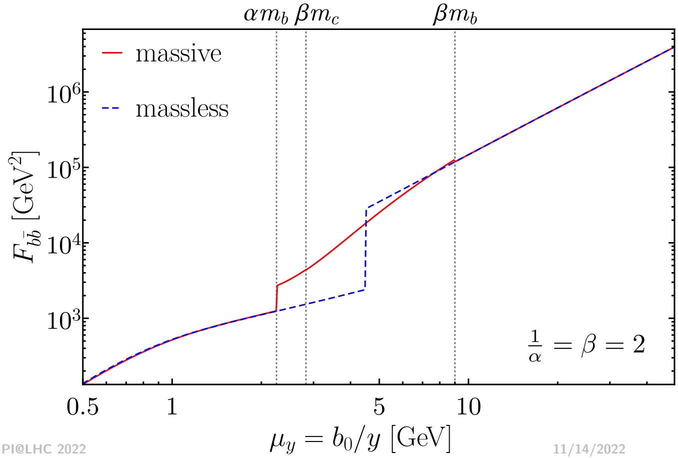

- DPDs still discontinuous, but greatly improved compared to the massless scheme!

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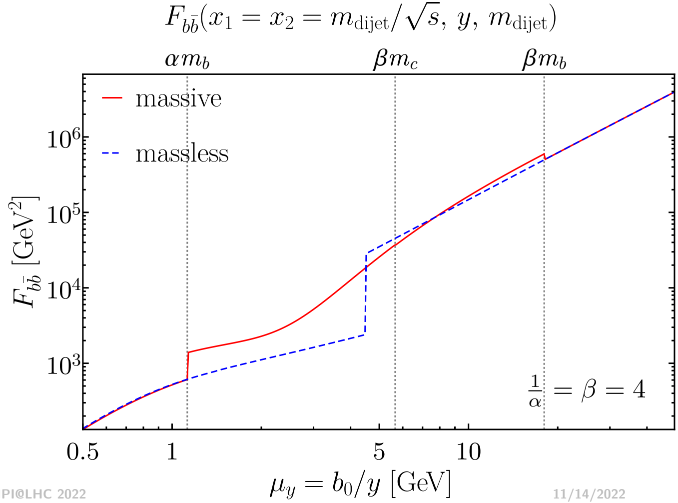

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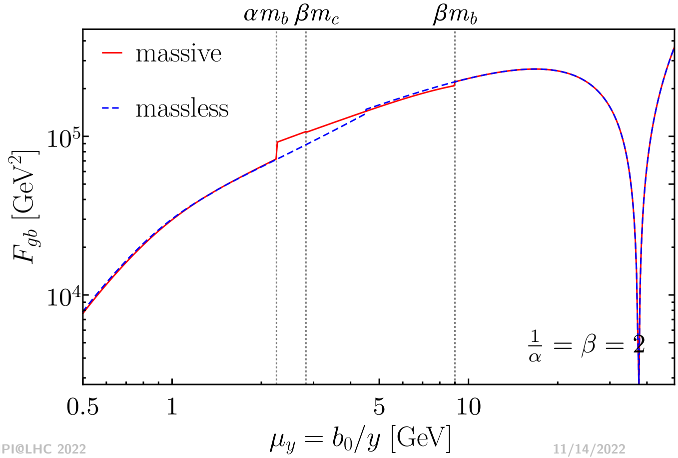

- Increased discontinuity for gb at $\mu_y = \alpha m_b$ due to direct production of $\bar{b}b$ DPD!
- Increased discontinuity for gb at $\mu_y = \beta m_b$ due to more production modes in the massless case!

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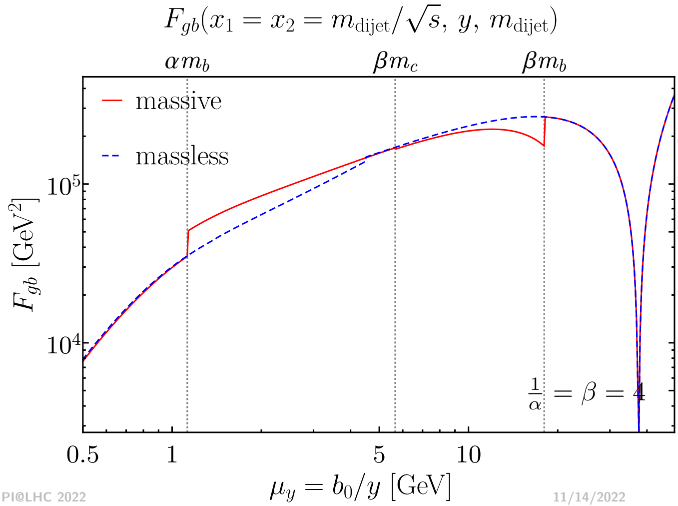

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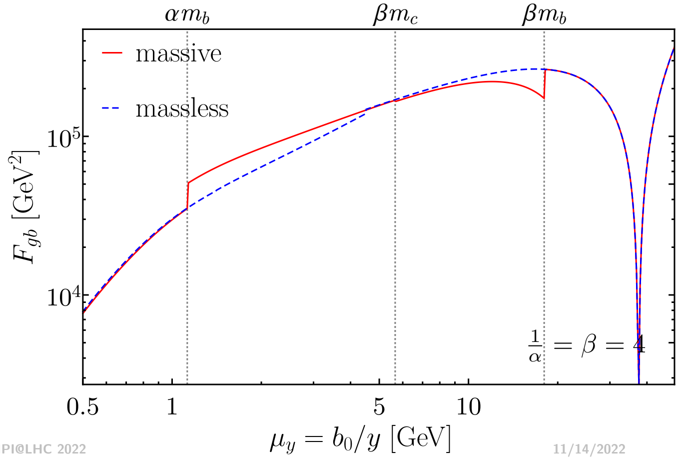

- Smallest discontinuities for $\beta = 2$ and $\alpha = \frac{1}{4}$!
- Seen also in other DPDs and at different scales, cf. backup slides.

DPD luminosities.

In order to study the effect of heavy quarks on DPS cross sections, consider DPD luminosities, i.e. products of DPDs integrated over y:

$$\mathcal{L}_{a_1 a_2 b_1 b_2}(x_{1a}, x_{2a}, x_{1b}, x_{2b}; \nu, \mu_1, \mu_2) = \int\limits_{b_0/\nu} \mathrm{d}^2\boldsymbol{y}\, F_{a_1 a_2}(x_{1a}, x_{2a}, y; \mu_1, \mu_2) F_{b_1 b_2}(x_{1b}, x_{2b}, y; \mu_1, \mu_2)$$

where the lower cut-off regulates the y^{-2} splitting singularity (very small y are related to loop corrections to SPS [Diehl, Gaunt and Schönwald, 2017]).

Here we include also "intrinsic" non-splitting contributions to the DPDs, modelled as:

$$F^{\mathrm{int}}_{a_1 a_2}(x_1, x_2, y; \mu_1, \mu_2) = \frac{(1 - x_1 - x_2)^2}{(1 - x_1)^2 (1 - x_2)^2} \frac{\exp\left(-\frac{y^2}{4h_{a_1 a_2}}\right)}{4\pi h_{a_1 a_2}} f_{a_1}(x_1, \mu_1)\, f_{a_2}(x_2, \mu_2)$$

In the following all possible combinations containing splitting DPDs are considered:

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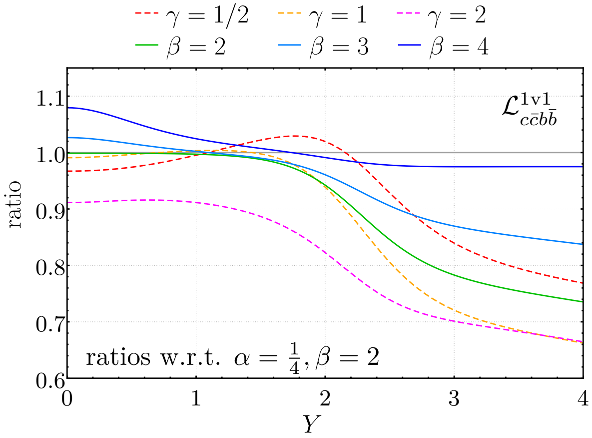
split x split (1v1), split x int (1v2), int x split (2v1).

DPD luminosities for dijet production.

Consider now ratios of LO DPD luminosities for dijet production with different scheme parameters:



Jets at rapidities Y and $-Y$:

$$x_{1a} = \frac{m_{\text{dijet}}}{\sqrt{s}} \exp(Y)$$

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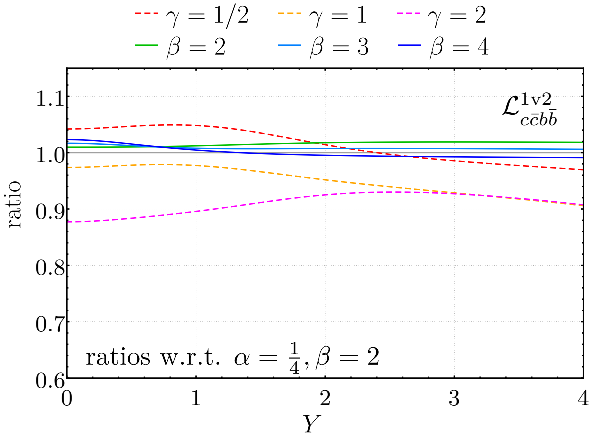

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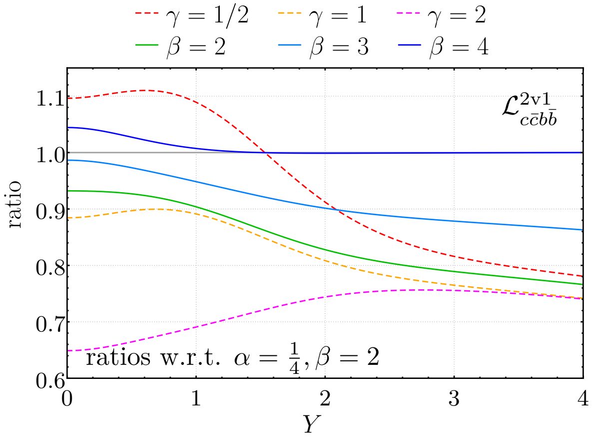

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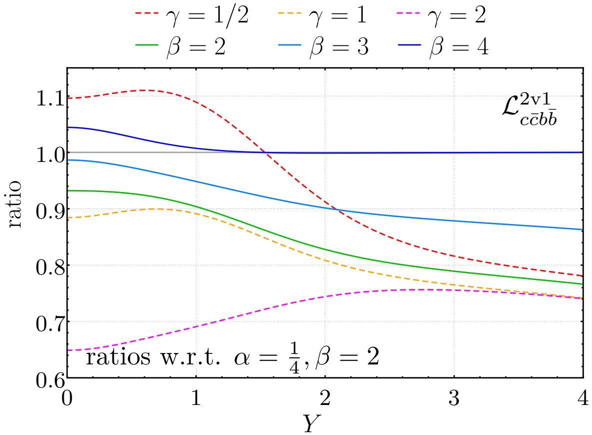

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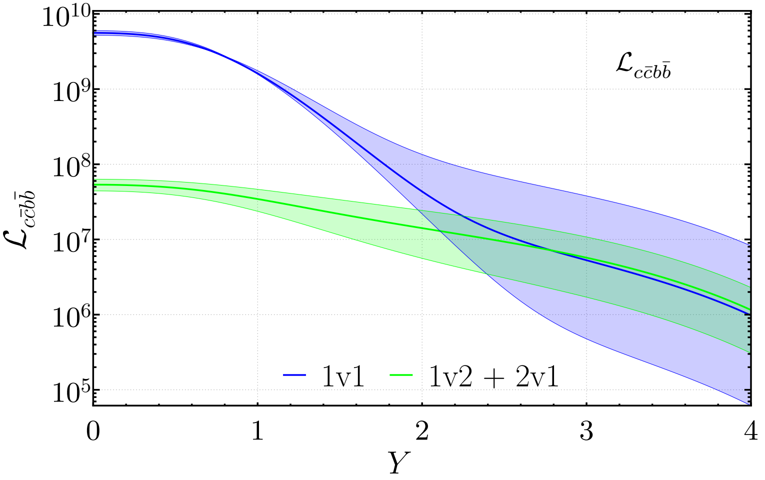
$\longrightarrow$ Smaller dependence of luminosities on α and β compared to γ!

Scale dependence of splitting DPDs.

Finally consider the dependence of DPD luminosities involving LO splitting DPDs on the scale μ_{split} (varied by a factor of 2):



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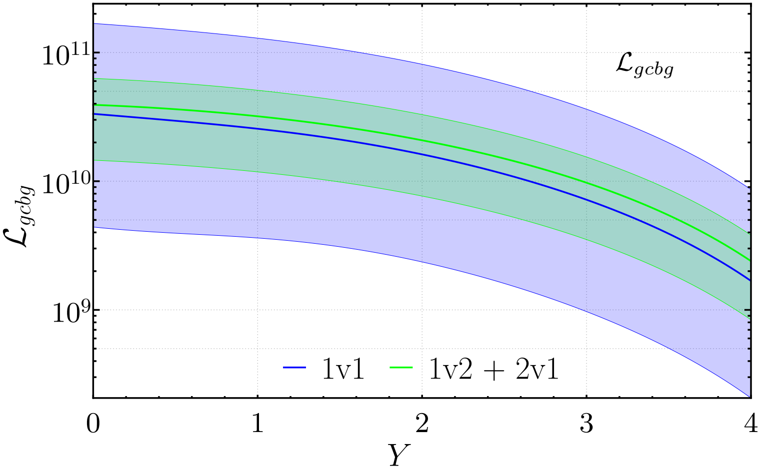

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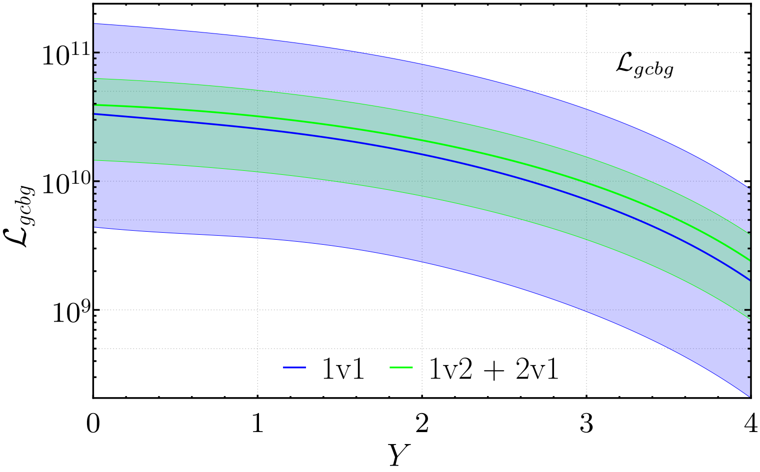

- Large scale uncertainties hint at importance of higher order splitting!

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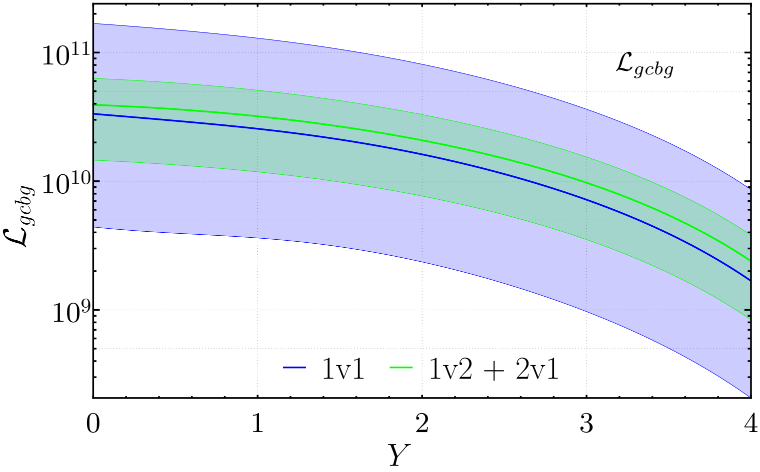

- Massless NLO kernels already calculated!

  [Diehl, Gaunt, PP, Schäfer, 2019; Diehl, Gaunt, PP, 2021]

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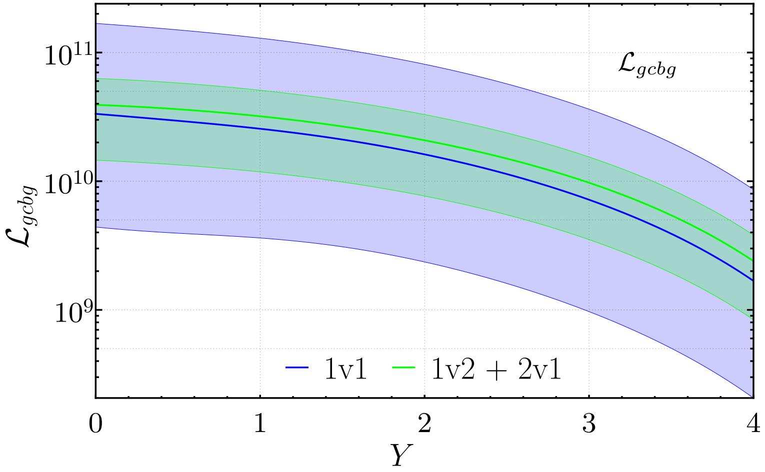

- Massive NLO kernels still unknown!

Constraints for the massive kernels.

For now a full calculation of the massive NLO kernels is out of reach for us (involves massive two-loop diagrams).

$\longrightarrow$ construct approximate solutions!

To this end make use of the following constraints:

- RGE dependence of the massive kernels.
- Small and large distance limits of the massive kernels.
- DPD number and momentum sum rules.

The limiting behaviour and RGE dependence are uniquely fixed by these constraints, while the DPD sum rules constrain also intermediate inter parton distances!

Part III

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At small interparton distances y DPDs can be matched onto PDFs with perturbative $1 \to 2$ splitting kernels:

- Splitting evaluated at $\mu_{\text{split}} \sim 1/y$.
- For $\mu_{\text{split}} \sim m_Q$ quark mass effects have to be taken into account!

Consistent treatment of quark mass effects:

- Heavy quark decouples for $\mu_{\text{split}} \ll m_Q$.
- Heavy quark treated as massive for $\mu_{\text{split}} \sim m_Q$.
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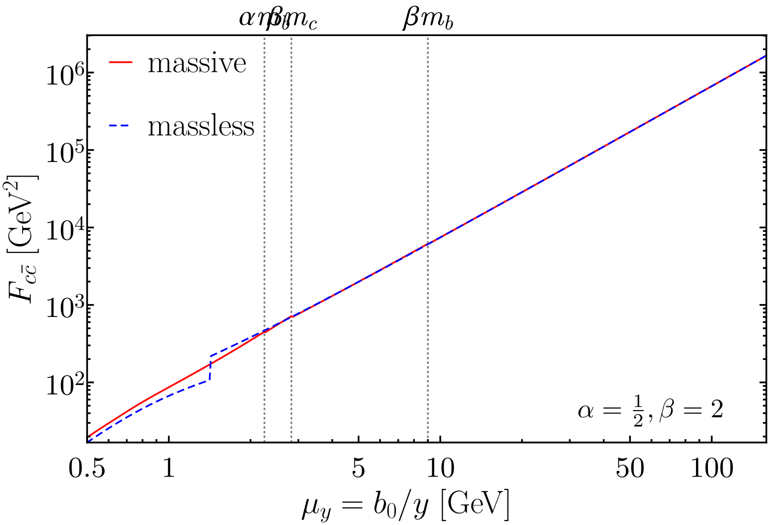
Thank you for your attention!

Part IV

Backup.

Splitting DPDs at $\mu_1 = \mu_2 = m_W$.

$n_F = 5$ LO splitting DPDs for WW production:



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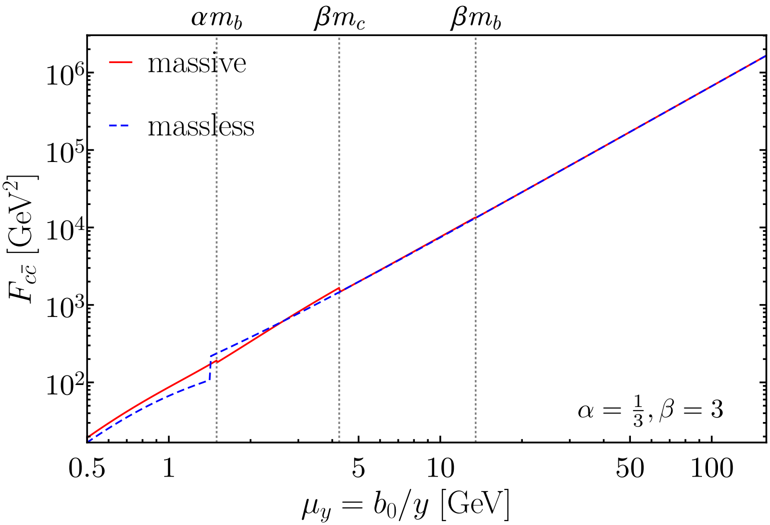

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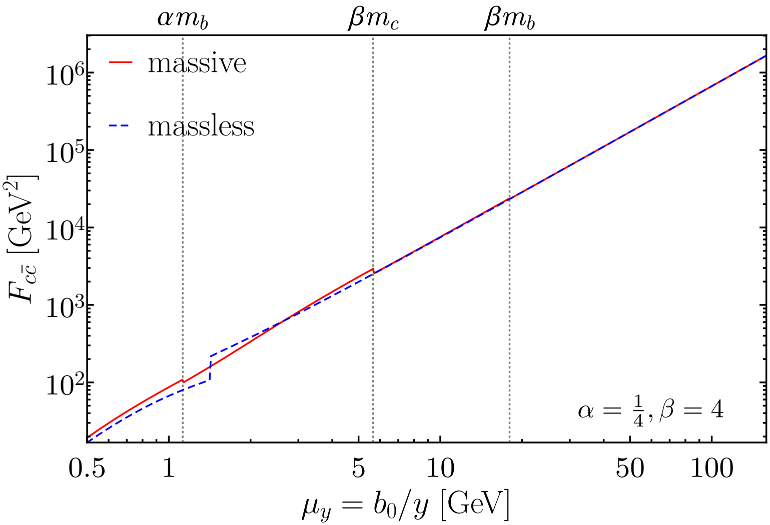

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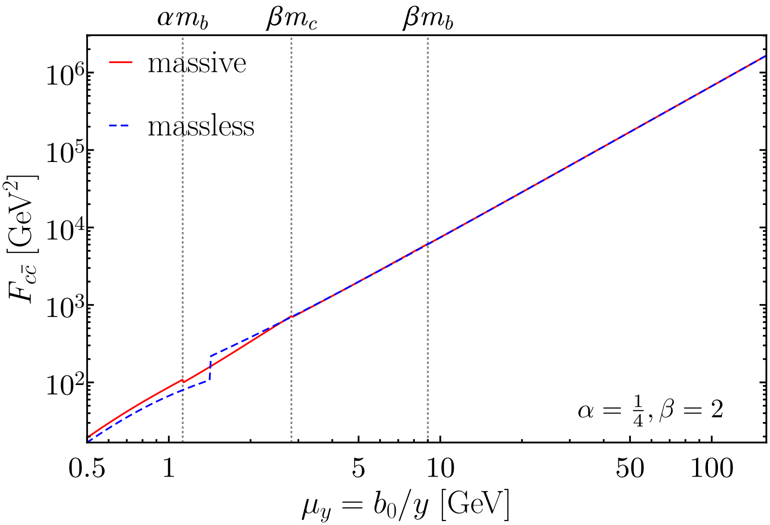

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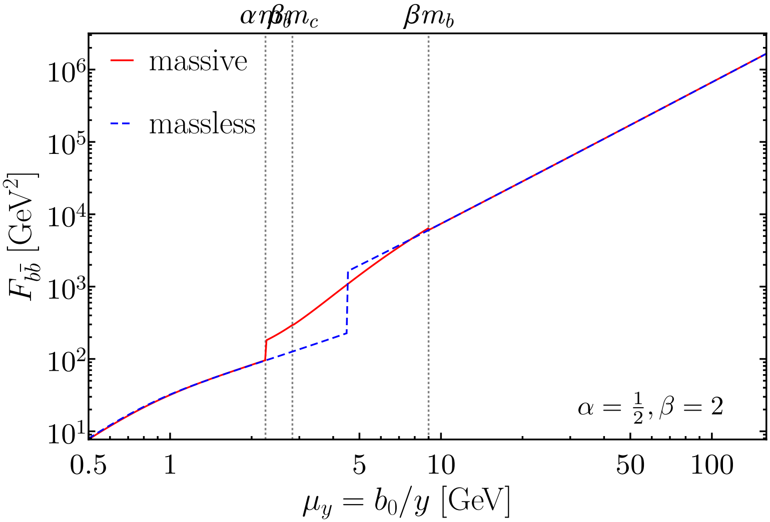

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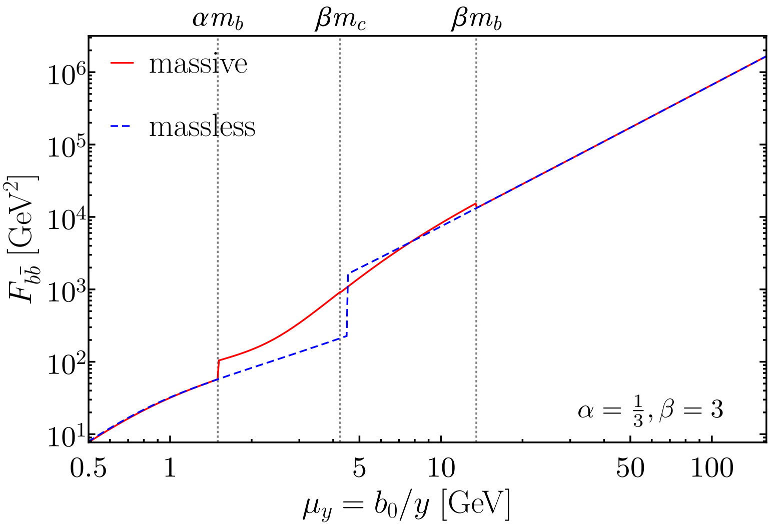

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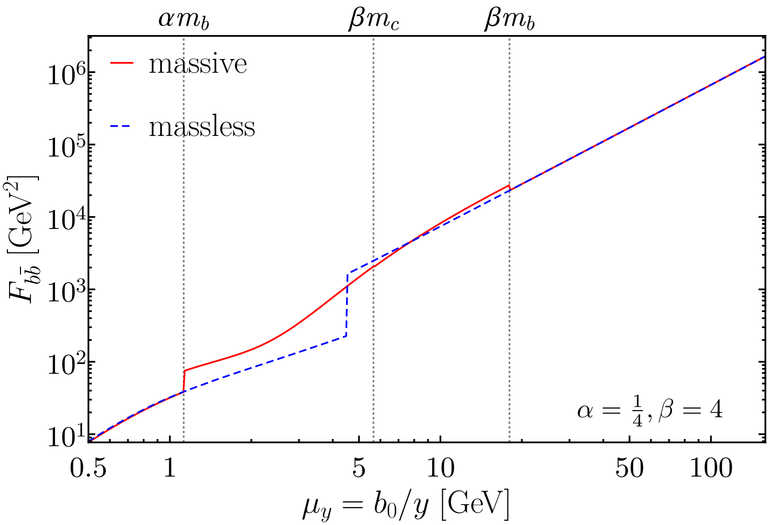

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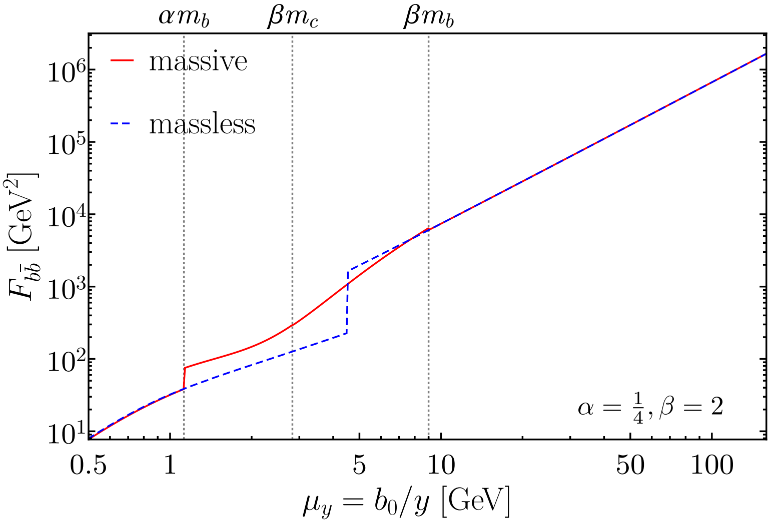

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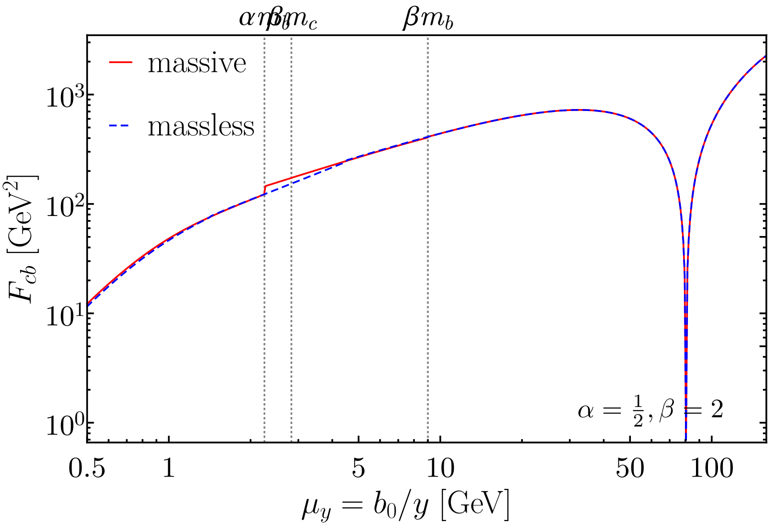

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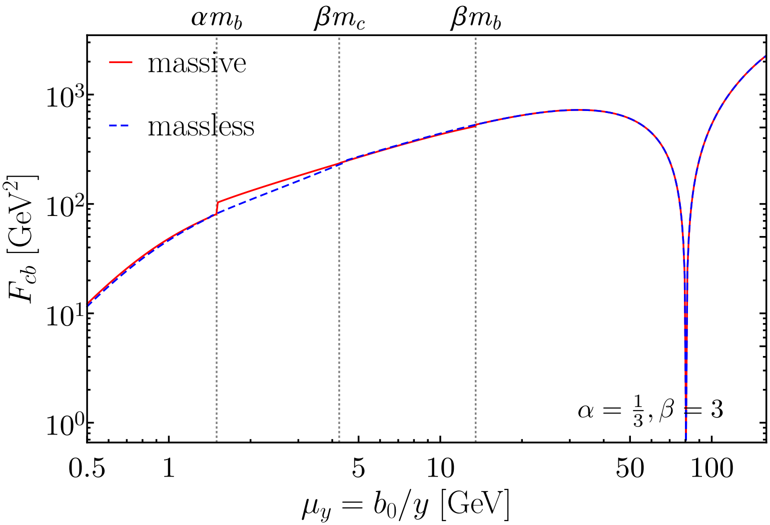

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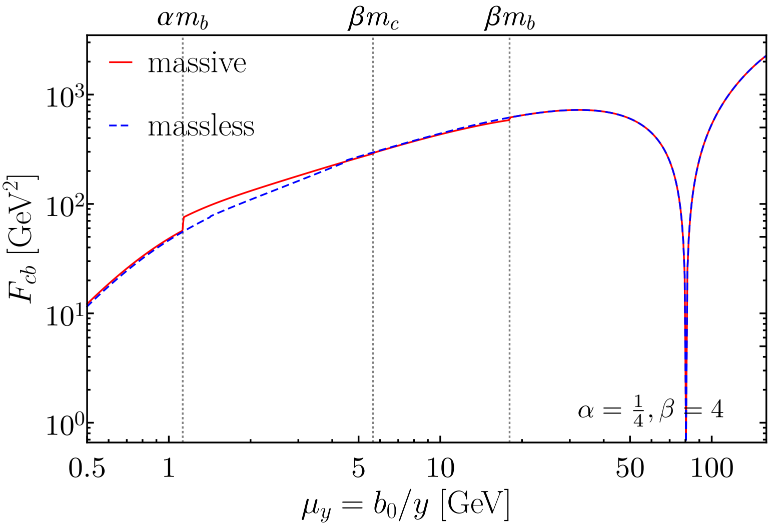

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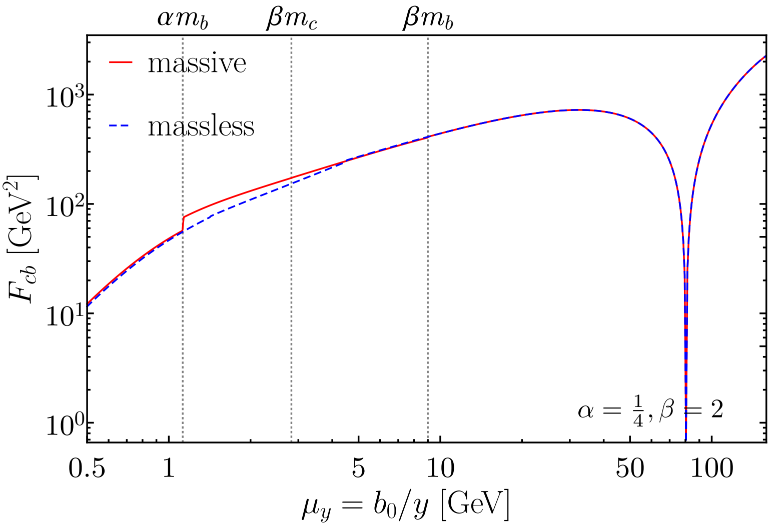

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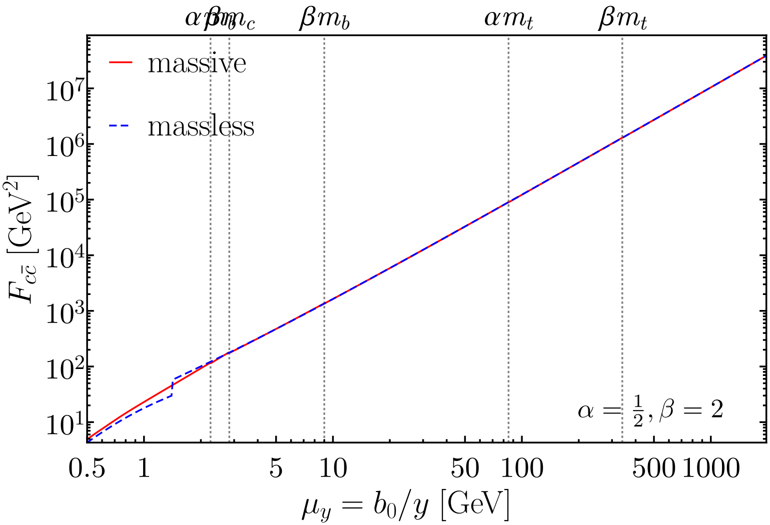

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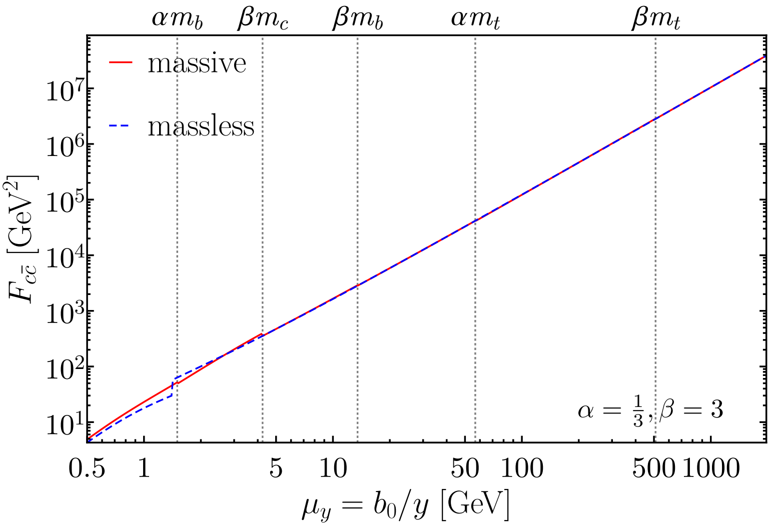

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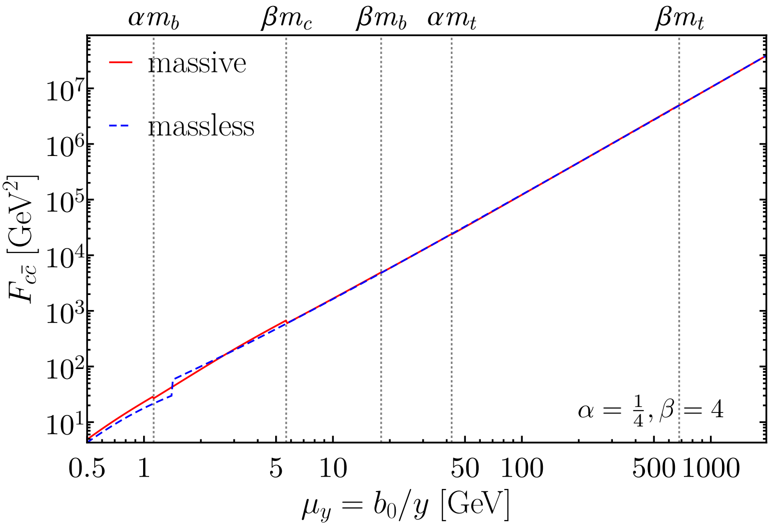

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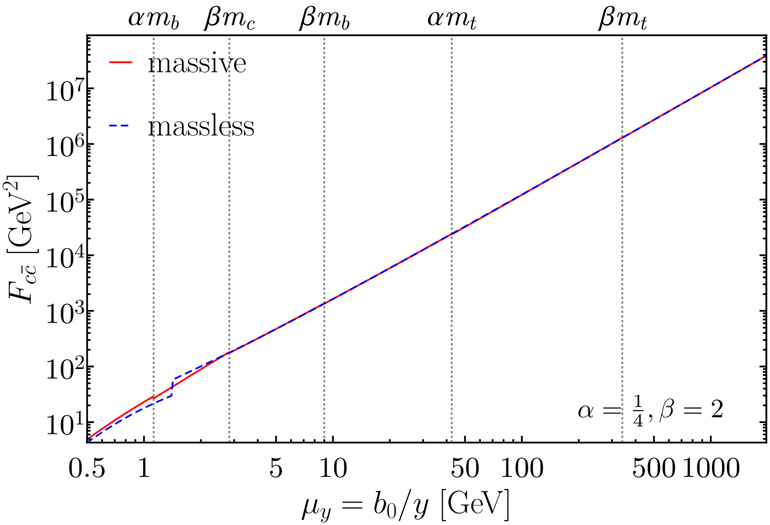

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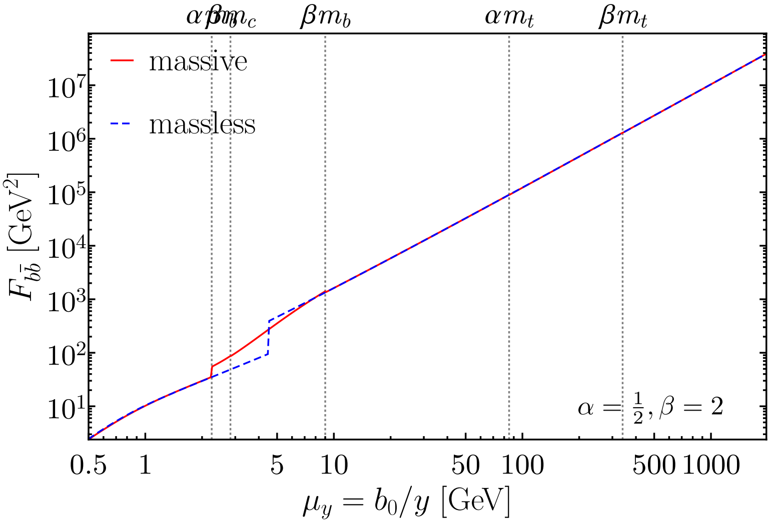

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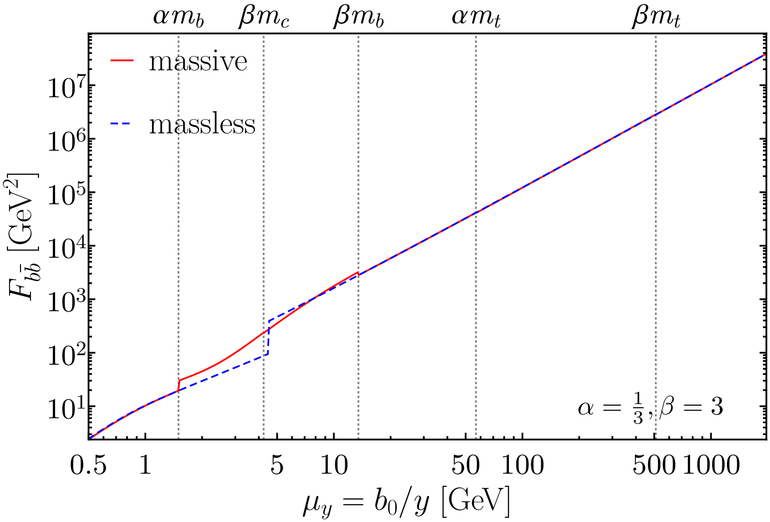

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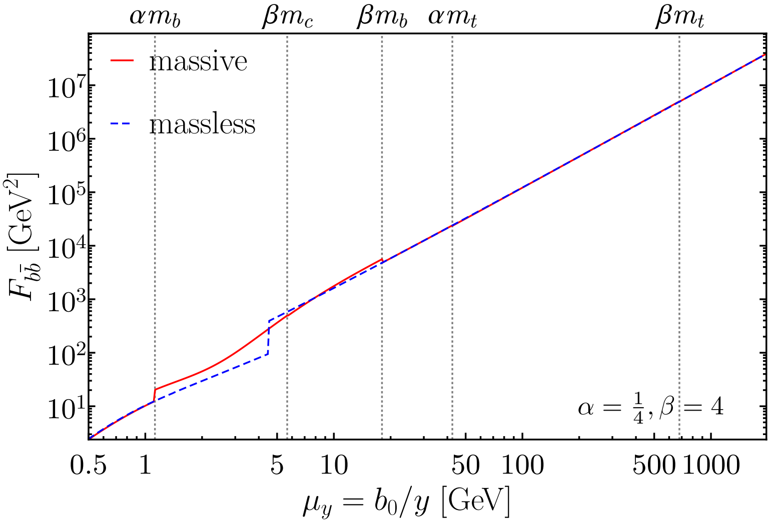

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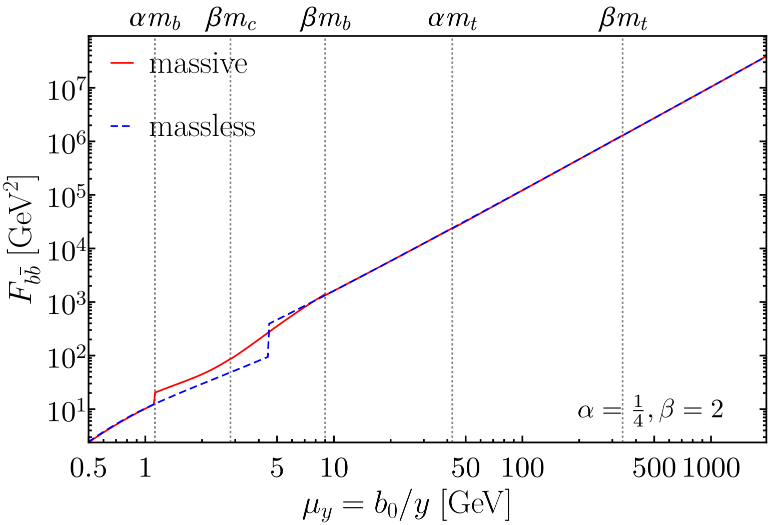

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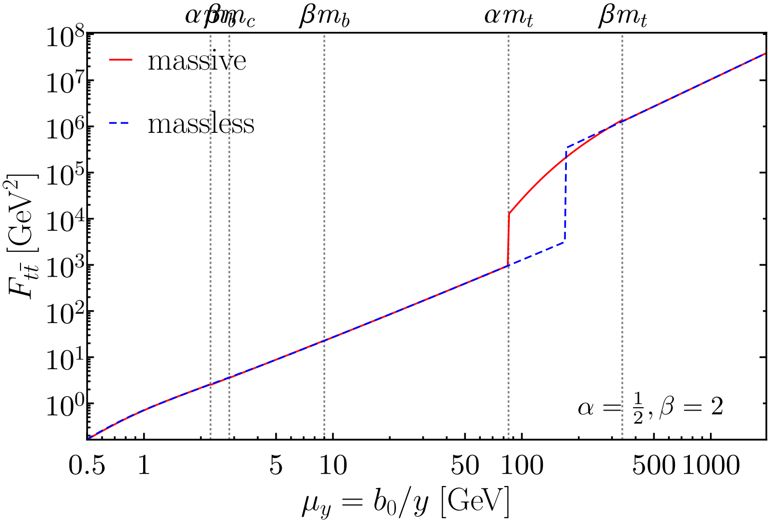

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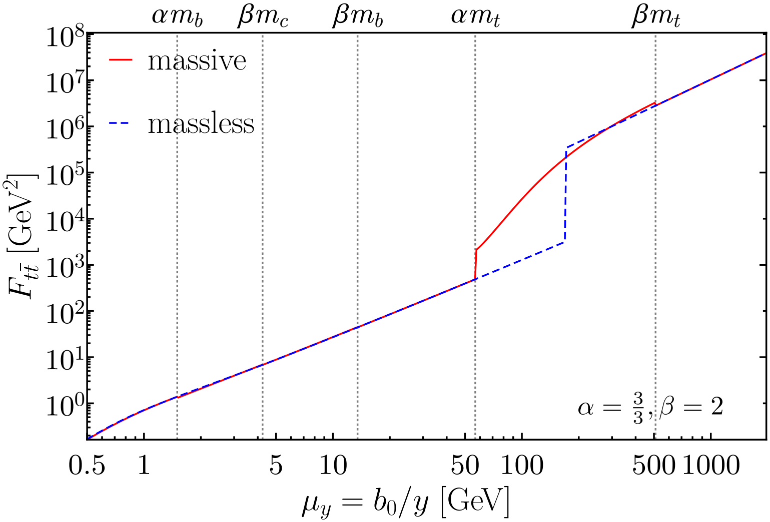

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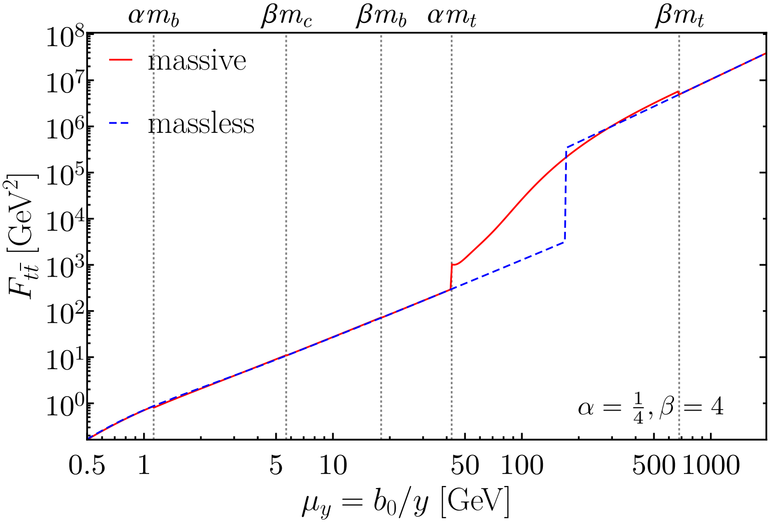

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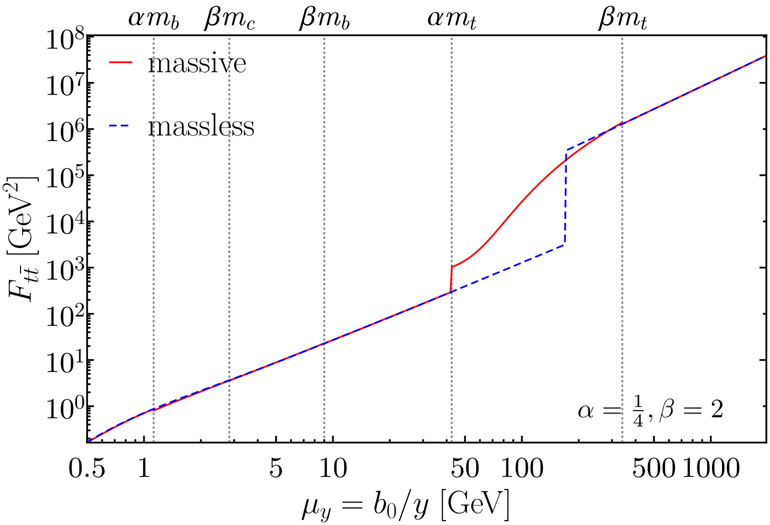

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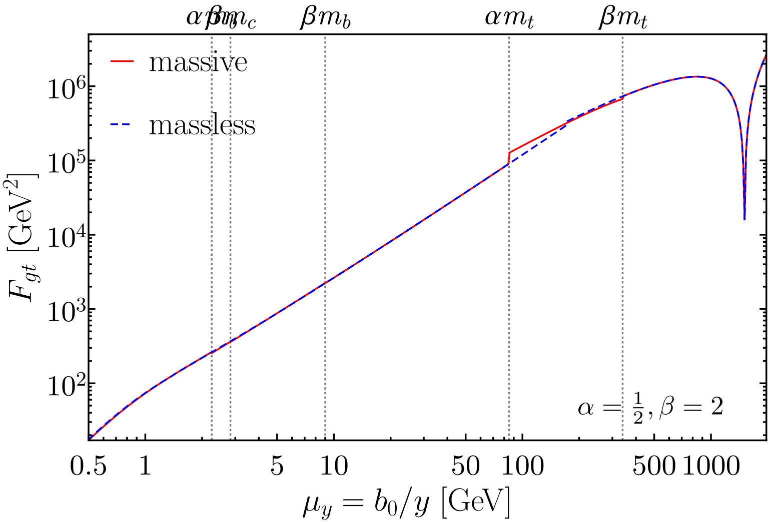

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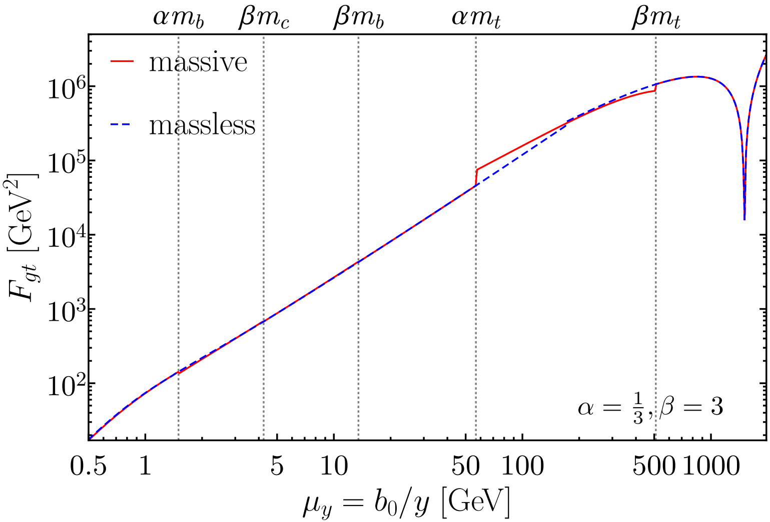

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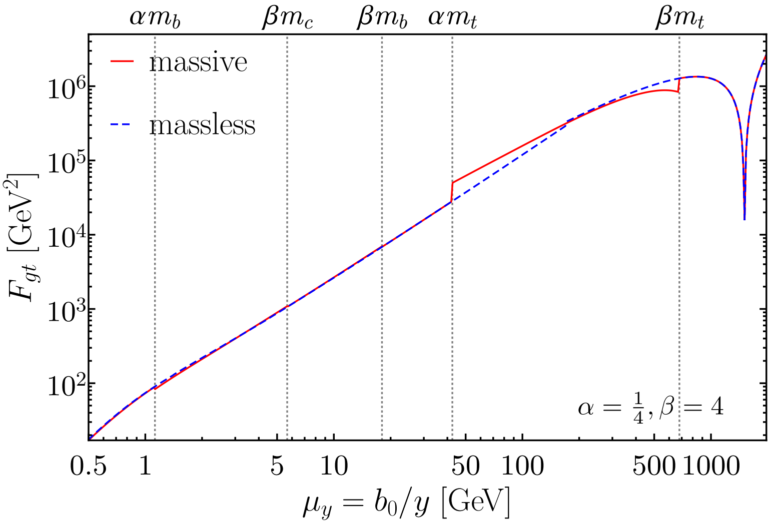

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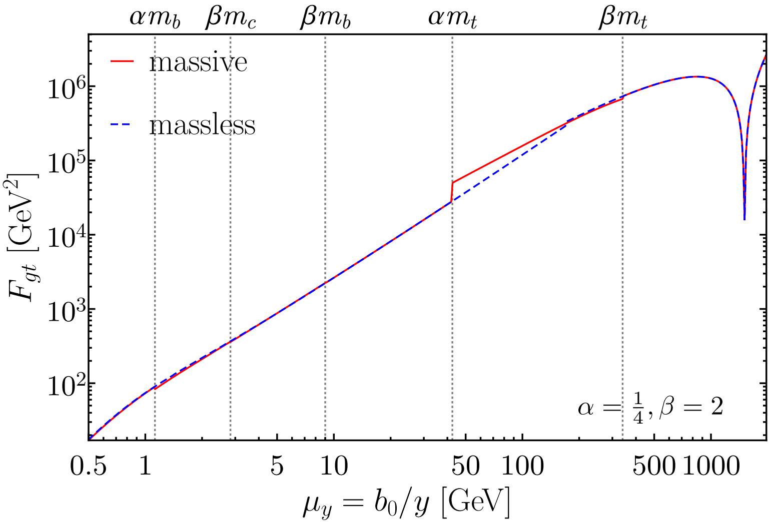

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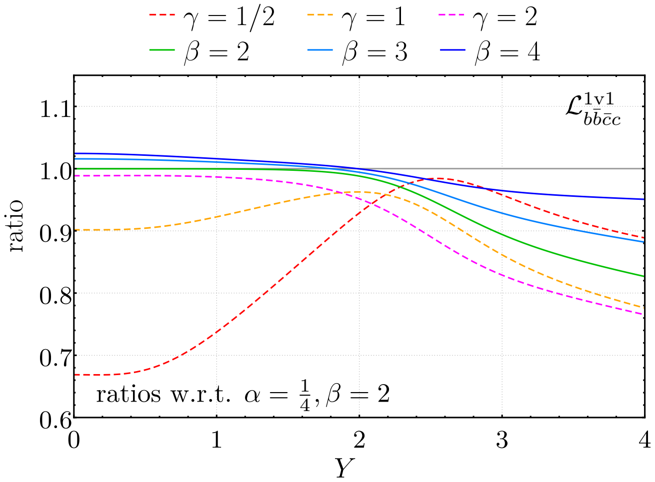

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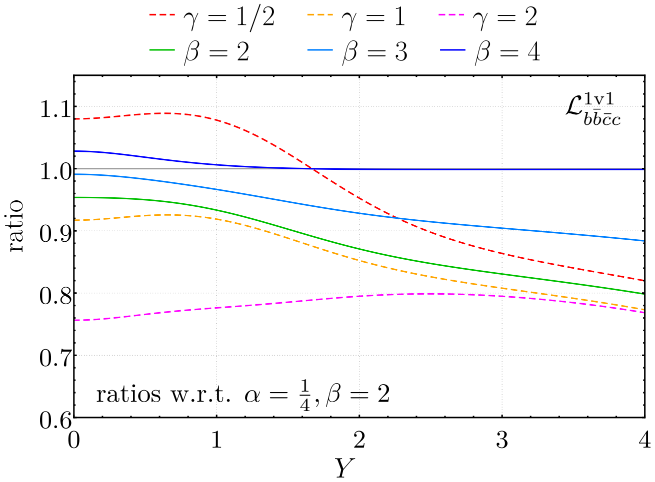

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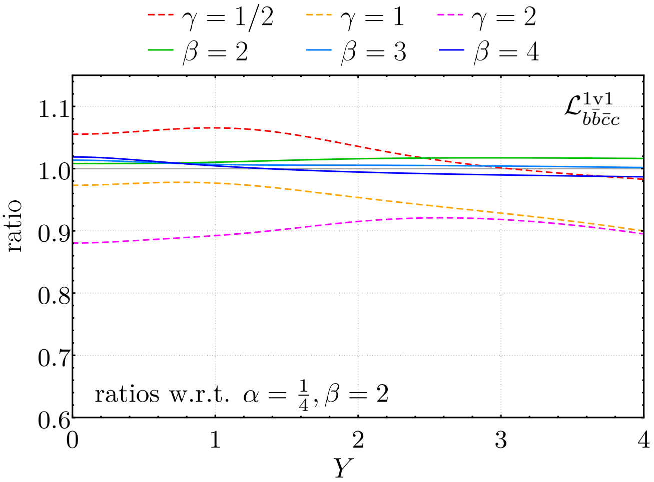

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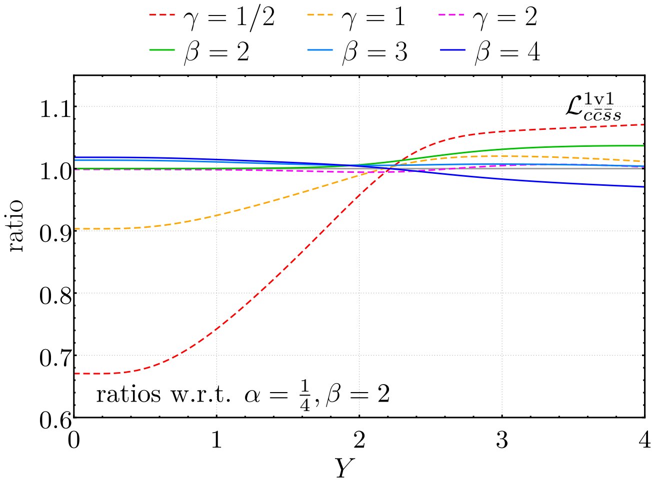

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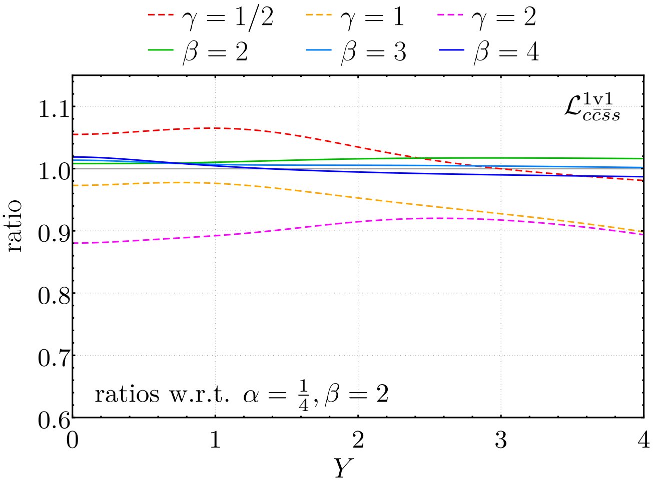

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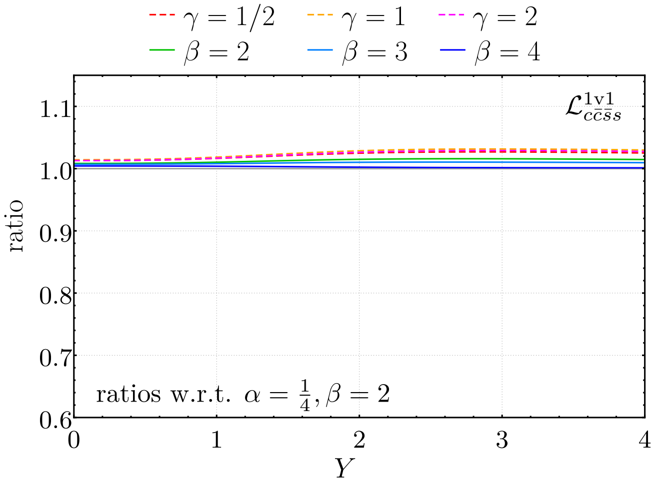

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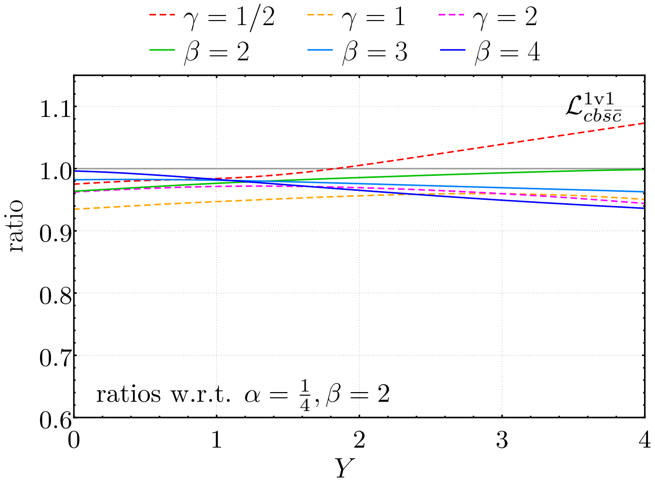

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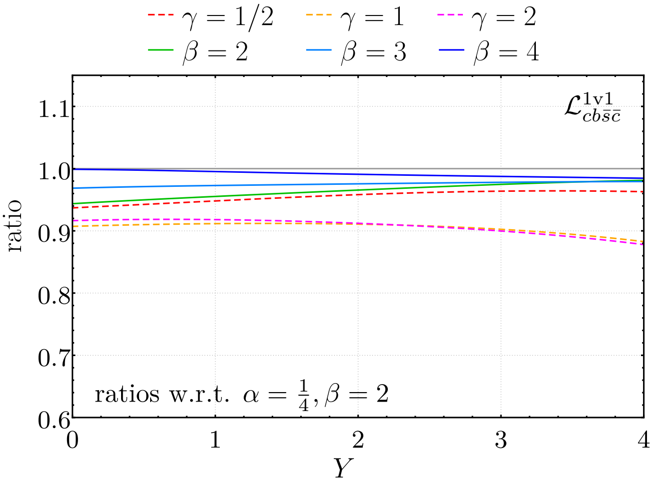

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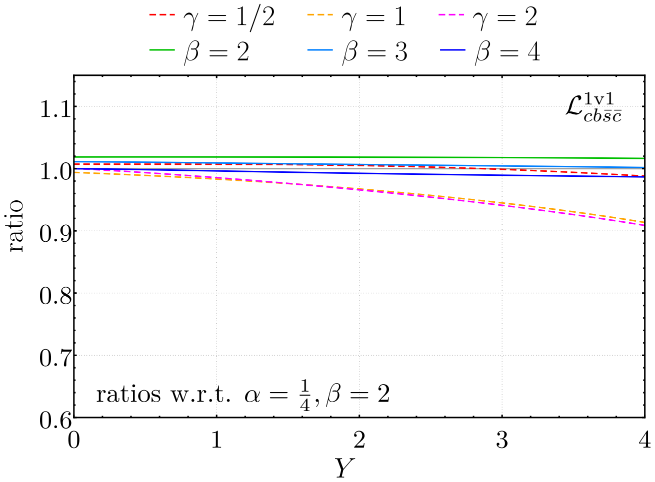

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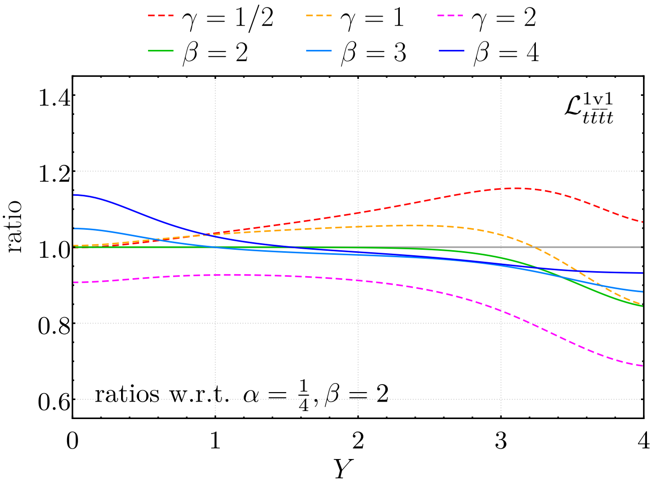

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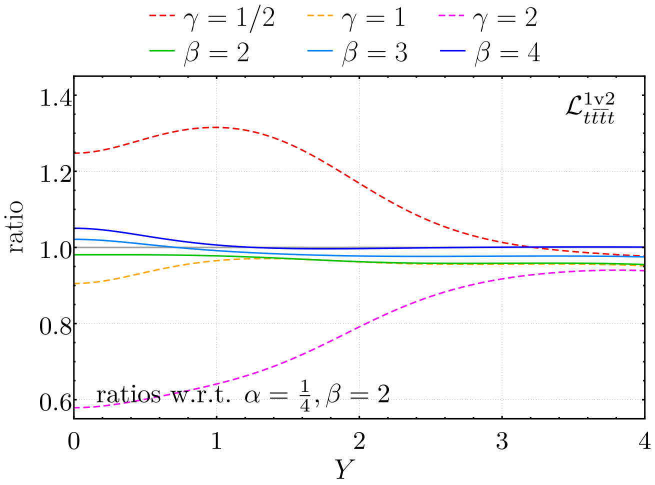

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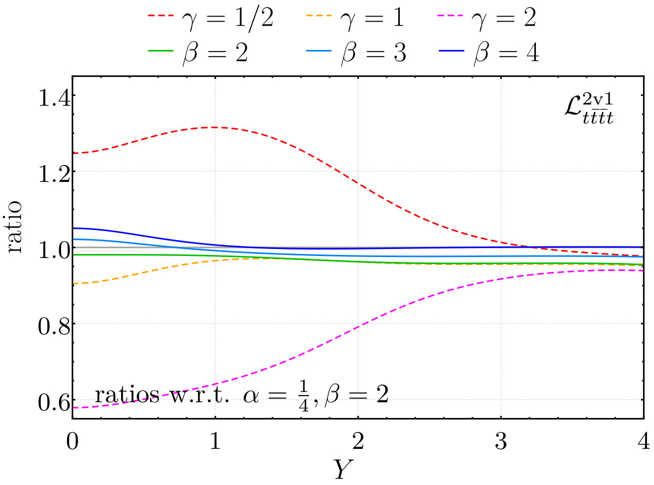
DPD luminosities at $\mu_1 = \mu_2 = 1000\,\mathrm{GeV}$.

$n_F = 6$ LO DPD luminosities for $t\bar{t}$ production:



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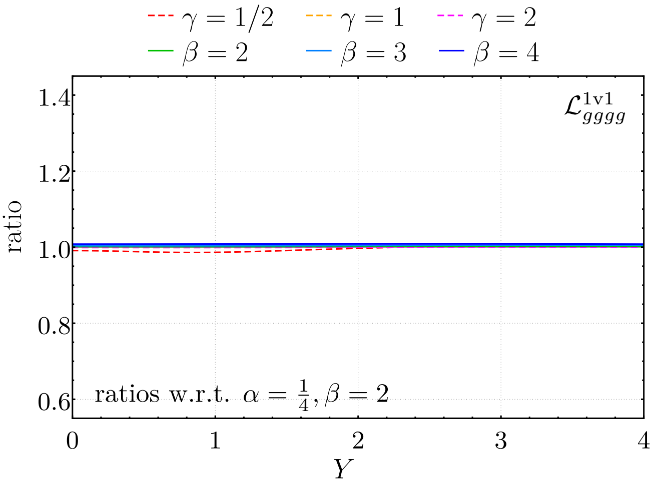

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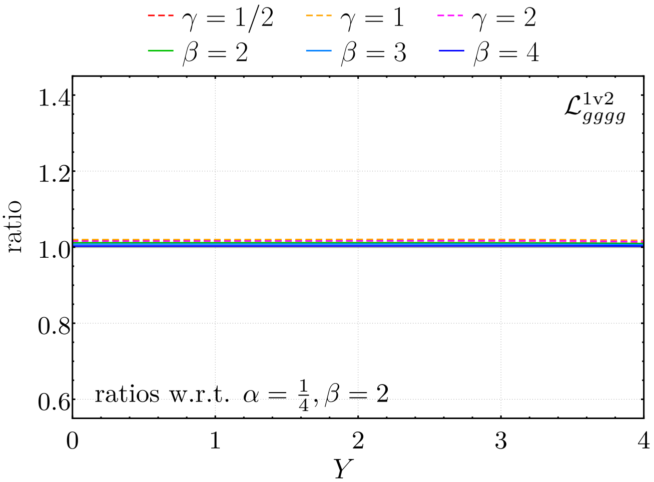

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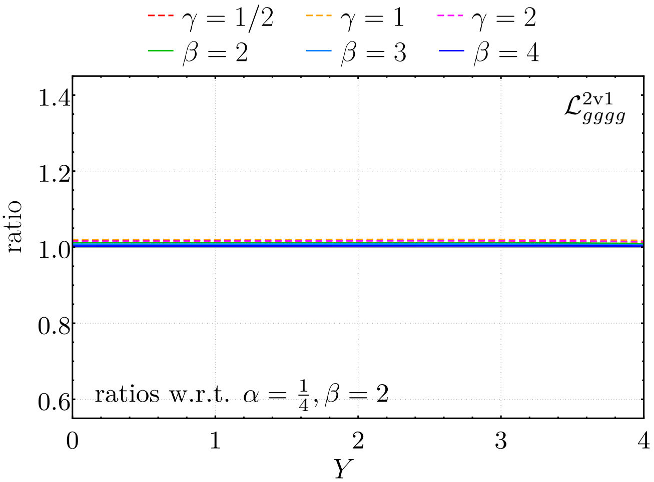

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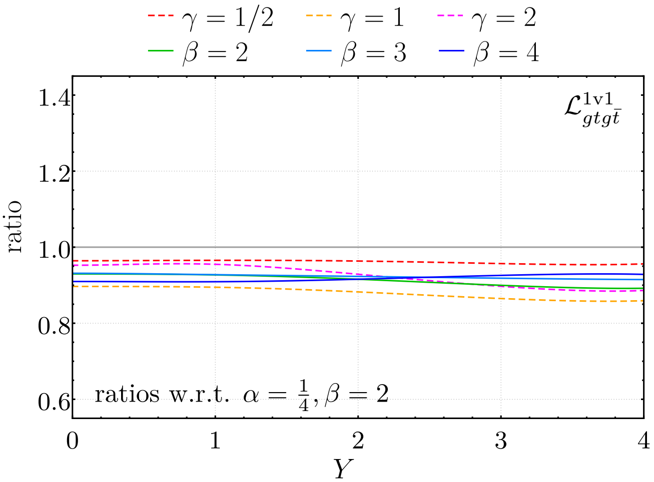

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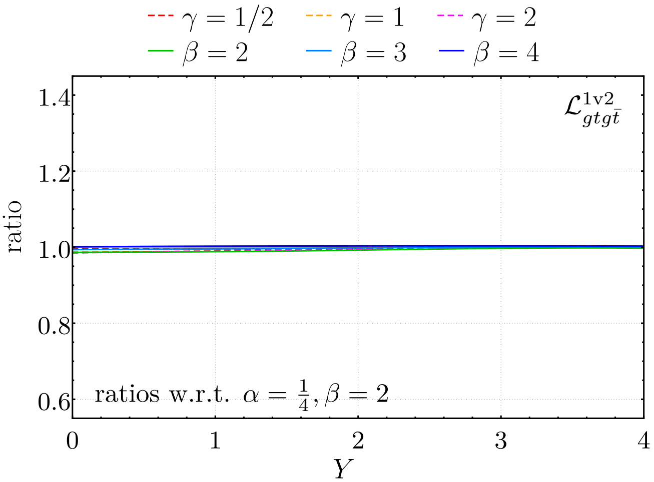

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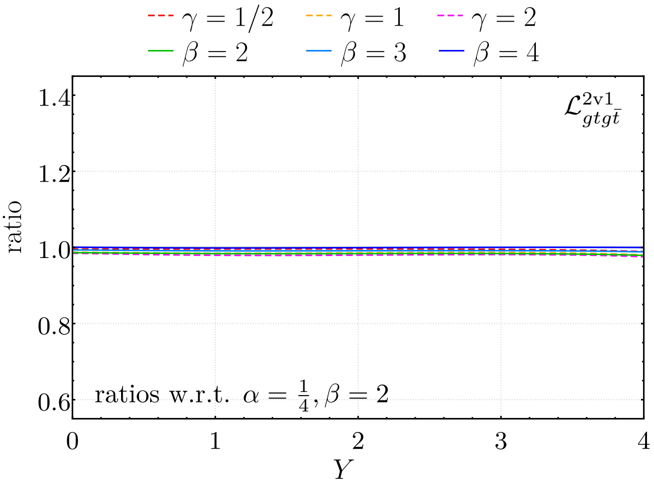

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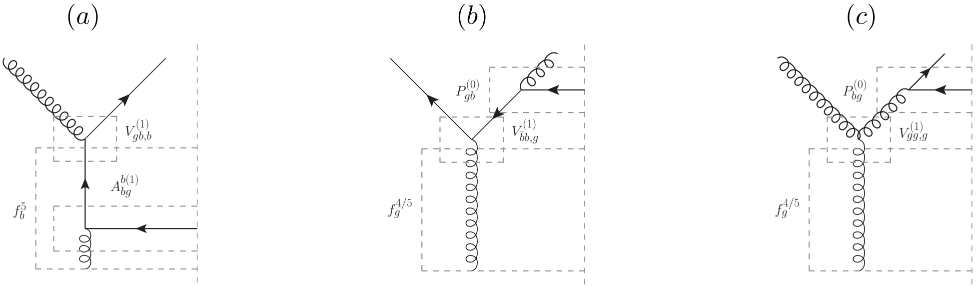
F_{gb}: massless vs. massive scheme



(a)
- Only contributes in the massless scheme.
- DPD produced by direct splitting, no evolution necessary.

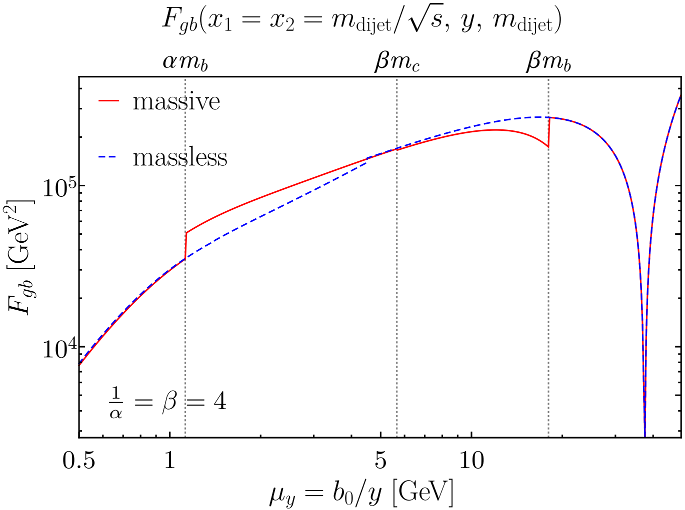
(b)
- Contributes in the massive and massless schemes.
- DPD only produced by evolution.

(c)
- Contributes in the massive and massless schemes.
- DPD only produced by evolution.

- Contributions (b) and (c) vanish when the splitting scale is identical to the target scale!

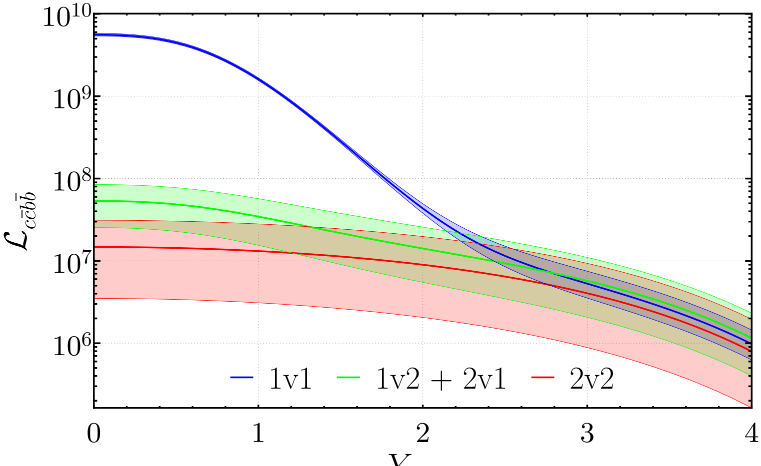
DPD luminosities.

F_{gb}: massless vs. massive scheme



Matching scale dependence of splitting DPDs.

Finally consider the dependence of LO DPD luminosities for dijet production on the flavour matching scales (at LO, varied by a factor of 2):



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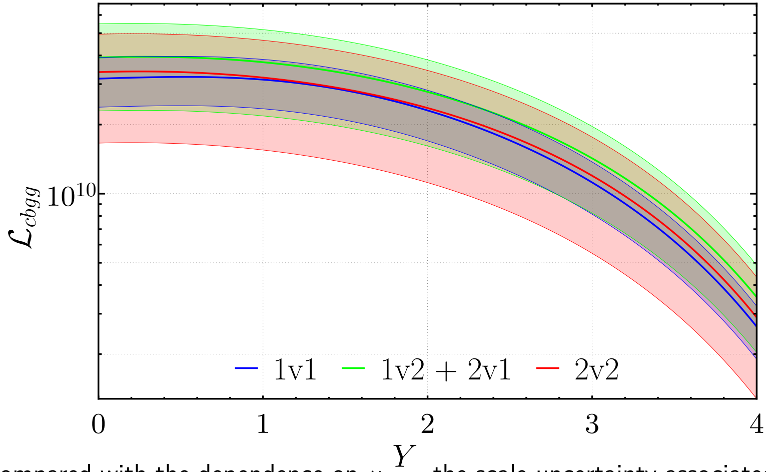

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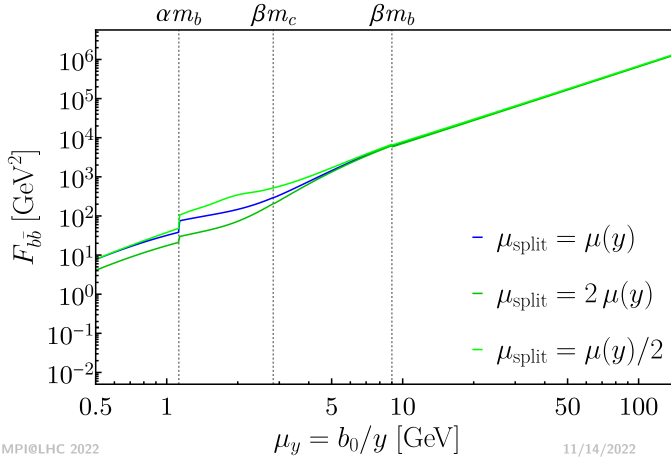
Scale dependence of splitting DPDs: in depth.

In order to understand the μ_{split} dependence of LO DPD luminosities involving $q\bar{q}$ DPDs consider the scale variation of the involved DPDs ($x_1 = \frac{m_W}{\sqrt{s}} \exp Y$, $x_2 = \frac{m_W}{\sqrt{s}} \exp -Y$):

Central rapidity ($Y = 0$):



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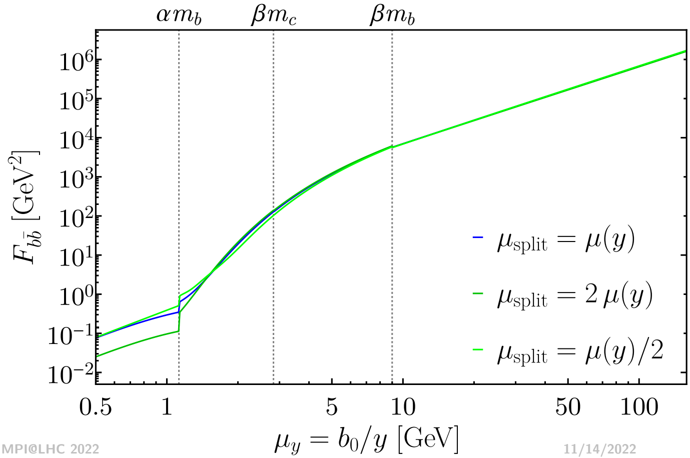
Central rapidity ($Y = 0$), only $g \to q\bar{q}$ splitting:

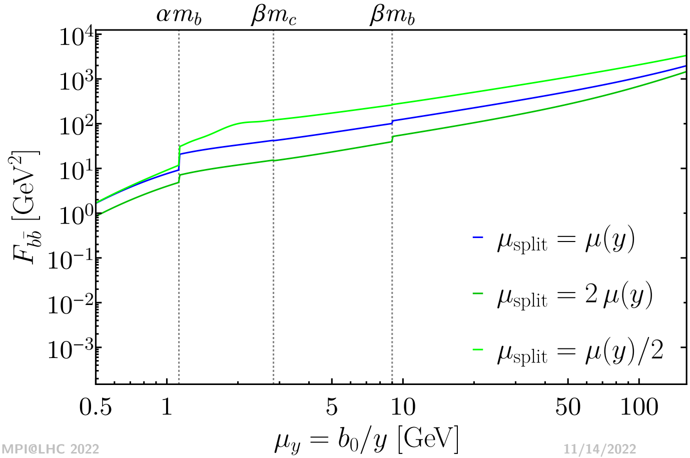


- Contribution from $g \to gg$ and $q \to qg, gq$ splitting and evolution negligible for central rapidity ($x_1 = x_2$).
- Only scale variation from initial gluon PDF.

Scale dependence of splitting DPDs: in depth.

In order to understand the μ_{split} dependence of LO DPD luminosities involving $q\bar{q}$ DPDs consider the scale variation of the involved DPDs ($x_1 = \frac{m_W}{\sqrt{s}} \exp Y$, $x_2 = \frac{m_W}{\sqrt{s}} \exp -Y$):

Non-central rapidity ($Y = 3$):



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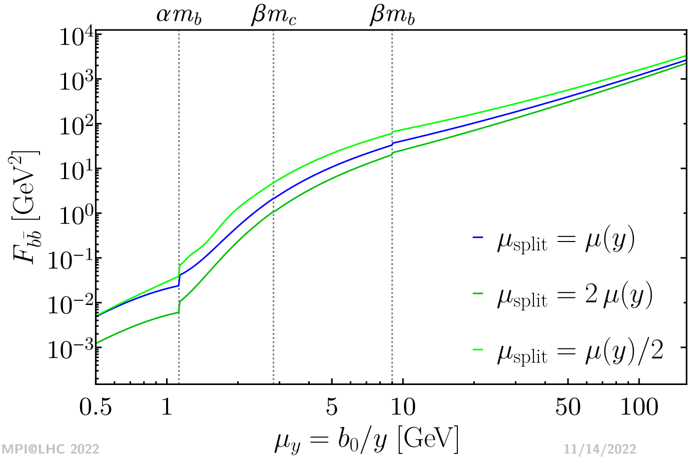
Non-central rapidity ($Y = 3$), only $g \to q\bar{q}$ splitting:



- Sizeable contribution from $g \to gg$ and $q \to qg, gq$ splitting and evolution for non-central rapidity ($x_1 \ll x_2$).
- In addition to scale variation from initial gluon PDF also uncertainties from evolution.

RGE dependence of the massive kernels.

The RGE dependence of the massive NLO kernels is completely fixed by LO perturbative ingredients:

Scale dependence of the massive NLO kernels:

$$
\begin{aligned}
\frac{\mathrm{d}}{\mathrm{d}\log\mu^2} V^{Q,n_F(2)}_{a_1a_2,a_0} &= \sum_{b_1} P^{n_F+1(0)}_{a_1b_1} \underset{1}{\otimes} V^{Q(1)}_{b_1a_2,a_0} + \sum_{b_2} P^{n_F+1(0)}_{a_2b_2} \underset{2}{\otimes} V^{Q(1)}_{a_1b_2,a_0} \\
&\quad - \sum_{b_0} V^{Q(1)}_{a_1a_2,b_0} \underset{12}{\otimes} P^{n_F(0)}_{b_0a_0} + \frac{\beta_0^{n_F+1}}{2} V^{Q(1)}_{a_1a_2,a_0} \\
&= v^{n_F,\,\mathrm{RGE}}_{a_1a_2,a_0}
\end{aligned}
$$

where the $V^{Q(1)}$ are the massive LO kernels and the $P^{n_F(0)})_{ab}$ are the LO DGLAP kernels.

Limiting behaviour of the massive kernels.

For small and large interparton distances the massive kernels can be expressed in terms of convolutions of massless kernels and flavour matching kernels:

Small distance limit.

$$V^{Q,n_F(2)}_{a_1 a_2, a_0} \xrightarrow{y \to 0} \delta^{n_F}_{a_0 l}\, V^{n_F+1(2)}_{a_1 a_2, a_0} + \sum_{b_0} V^{n_F+1(1)}_{a_1 a_2, b_0} \underset{12}{\otimes} A^{Q(1)}_{b_0 a_0} \,,$$

Large distance limit.

$$V^{Q,n_F(2)}_{a_1 a_2, a_0} \xrightarrow{y \to \infty} V^{n_F(2)}_{a_1 a_2, a_0} + \sum_{b_1} A^{Q(1)}_{a_1 b_1} \underset{1}{\otimes} V^{(1)}_{b_1 a_2, a_0} + \sum_{b_2} A^{Q(1)}_{a_2 b_2} \underset{2}{\otimes} V^{(1)}_{a_1 b_2, a_0} + A^{Q(1)}_{\alpha}\, V^{(1)}_{a_1 a_2, a_0} \,.$$

Sum rules for the massive kernels.

The Gaunt-Stirling DPD sum rules can be used to derive sum rules for the massive kernels:

Number sum rule.

$$\int_2 \int_{y_\beta}^{y_\alpha} \mathrm{d}^2 y \frac{1}{\pi y^2} V^{Q,n_F}_{a_1 a_{2_v}, a_0} = \left(\delta_{a_1 \bar{a}_2} - \delta_{a_1 a_2} - \delta_{a_2 \bar{a}_0} + \delta_{a_2 a_0}\right) A^{Q,n_F}_{a_1 a_0}$$
$$+ \sum_{b_1} A^{Q,n_F}_{a_1 b_1} \otimes \left(\int_2 U^{n_F}_{b_1 a_{2_v}, a_0}(r_\alpha) \right) - \sum_{b_2} \left(\int_2 U^{n_F+1}_{a_1 a_{2_v}, b_0}(r_\beta) \right) \otimes A^{Q,n_F}_{b_0 a_0} \,,$$

Momentum sum rule.

$$\sum_{a_2} \int_2 X_2 \int_{y_\beta}^{y_\alpha} \mathrm{d}^2 y \frac{1}{\pi y^2} V^{Q,n_F}_{a_1 a_2, a_0} = (1 - X)\, A^{Q,n_F}_{a_1 a_0}$$
$$+ \sum_{b_1, a_2} A^{Q,n_F}_{a_1 b_1} \otimes \left(\int_2 X_2\, U^{n_F}_{b_1 a_2, a_0}(r_\alpha) \right) - \sum_{a_2, b_0} \left(\int_2 X_2\, U^{n_F+1}_{a_1 a_2, b_0}(r_\beta) \right) \otimes \left(X A^{Q,n_F}_{b_0 a_0} \right) .$$

Ansatz for the massive kernels.

The following ansatz fulfils the RGE and limiting behaviour constraints:

$$V^{Q,n_F(2)}_{a_1a_2,a_0} = V^{n_F[2,0]}_{a_1a_2,a_0} + V^{n_F[2,1]}_{a_1a_2,a_0} \log\frac{m_Q^2}{\mu_y^2} + k_{00}(y\, m_Q)\, v^{n_F,I}_{a_1a_2,a_0}(z_1,z_2)$$
$$+ k_{11}(y m_Q)\left(V^{n_F+1[2,0]}_{a_1a_2,a_0} - V^{n_F[2,0]}_{a_1a_2,a_0}\right) - k_{02}(y m_Q)\left(V^{n_F+1[2,1]}_{a_1a_2,a_0} - V^{n_F[2,1]}_{a_1a_2,a_0}\right)$$
$$+ \log\frac{\mu^2}{m_Q^2}\, v^{n_F,\mathrm{RGE}}_{a_1a_2,a_0}(z_1,z_2)\,,$$

where

$$k_{ij}(w) = w^2 K_i(w) K_j(w)\,.$$

⟶ Sum rules can be used to constrain $v^{n_F,I}_{a_1a_2,a_0}$!