Photon initiated double parton scattering: a new light on the proton structure

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in collaboration with

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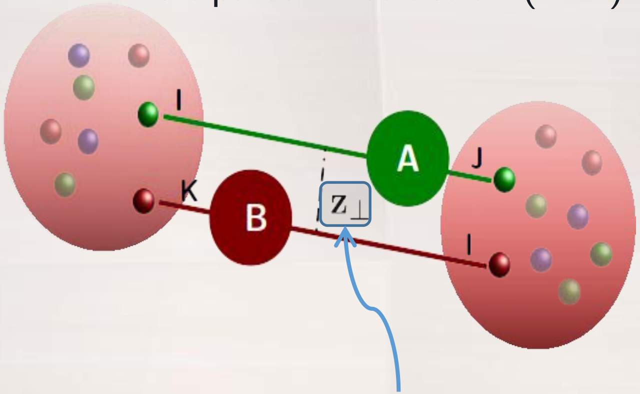
Roadmap

1. INTRODUCTION
2. 3D PROTON STRUCTURE VIA p-p DPS
3. INTERPRETATION OF LHC DATA
4. DPS VIA PHOTON-PROTON INTERACTIONS
5. DPS AND DOUBLE J/Ψ PRODUCTION AT THE FUTURE EIC
6. CONCLUSIONS

Double Parton Scattering @LHC

*previous talks

Multiparton interaction (MPI) can contribute to the, pp and pA, cross section @ the LHC:



Transverse distance between partons

The cross section for a double parton scattering (DPS) event can be written in the following way:

N. Paver, D. Treleani, Nuovo Cimento 70A, 215 (1982)

double PDF (dPDF)

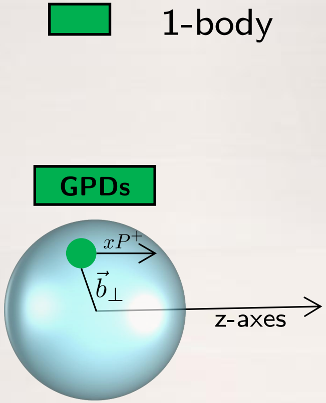
$$d\sigma \propto \int d^2 z_\perp \, F_{ik}(x_1, x_2, \vec{z}_\perp; \mu_A, \mu_B) \cdot F_{jl}(x_3, x_4, \vec{z}_\perp; \mu_A, \mu_B)$$

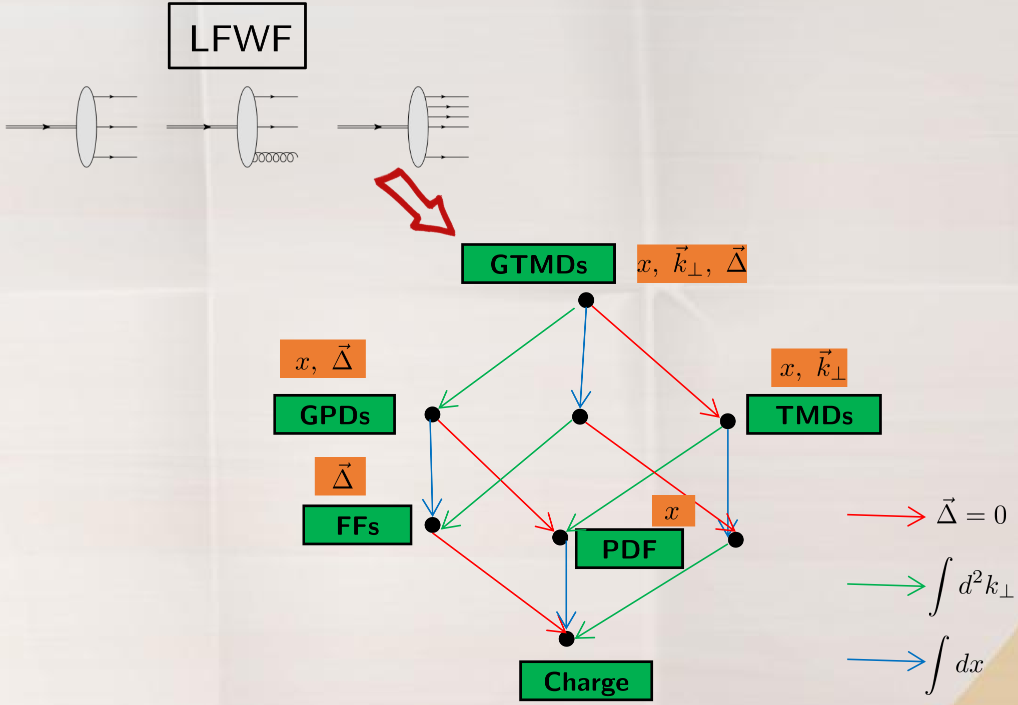
Momentum fractions carried by the parton inside the proton

Momentum scales

DPS processes are important for fundamental studies, e.g. the background for the research of new physics and to grasp information on the **3D PARTONIC STRUCTURE OF THE PROTON**

Multidimensional Pictures of Hadron





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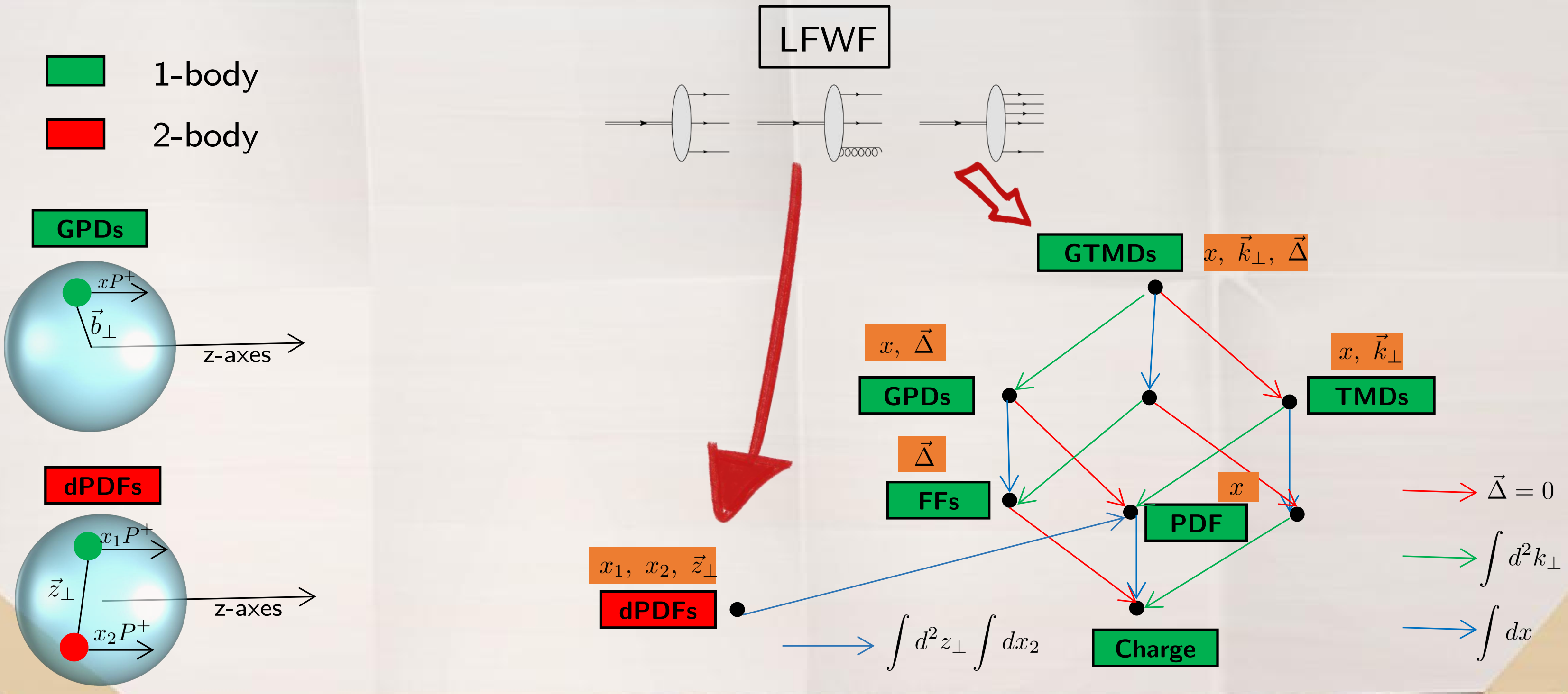
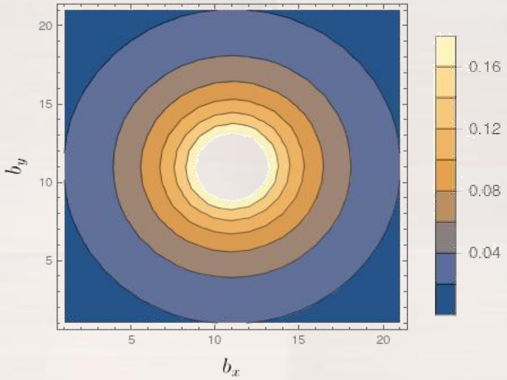


Information from quark models

$z_\perp = b_\perp$



M.R. and F. A. Ceccopieri, JHEP 09 (2019) 097

1) e.g. the distance distribution of **two gluons** in the proton

$$\langle z_\perp^2 \rangle^{ij}_{x_1,x_2} = \frac{\int d^2 z_\perp \; z_\perp^2 F_{ij}(x_1, x_2, z_\perp)}{\int d^2 z_\perp \; F_{ij}(x_1, x_2, z_\perp)}$$

Information from quark models

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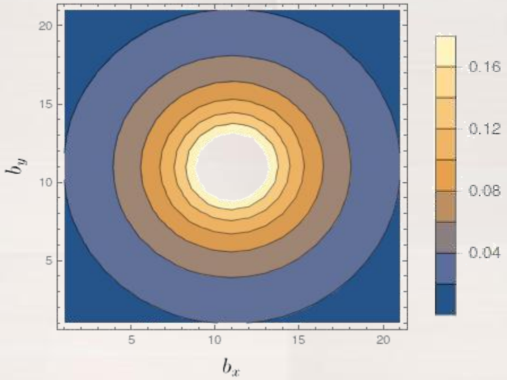
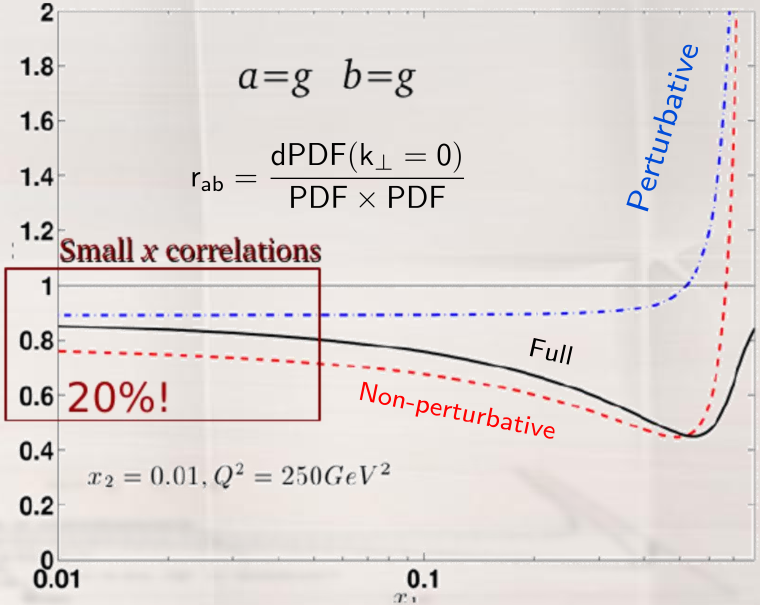


M.R. and F. A. Ceccopieri, JHEP 09 (2019) 097



2) Correlations are important

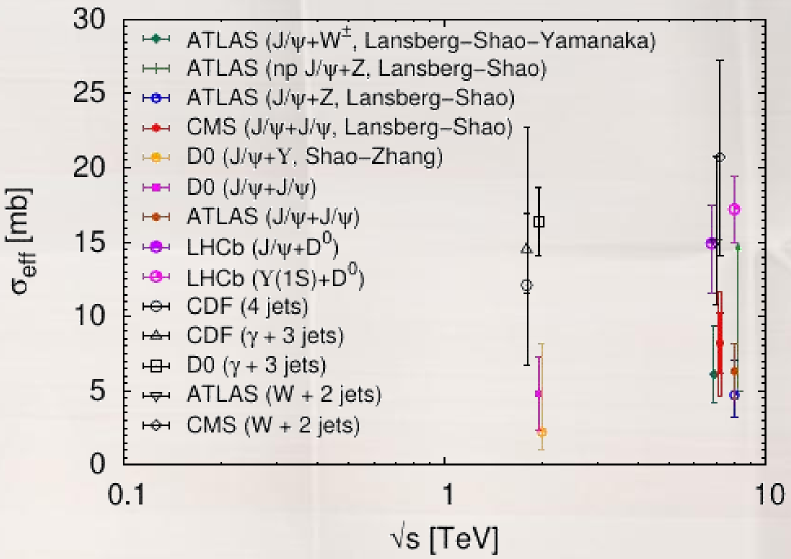
M.R., S. Scopetta et al, JHEP 10 (2016) 063

M.R. and F. A. Ceccopieri PRD 95 (2017) 034040

Data and Effective Cross Section

J.P. Lansberg's slide
MPI-2019 workshop

$$\sigma_{\text{eff}}^{pp} = \frac{m}{2}\frac{\sigma_A^{pp}\sigma_B^{pp}}{\sigma_{\text{DPS}}^{pp}}$$



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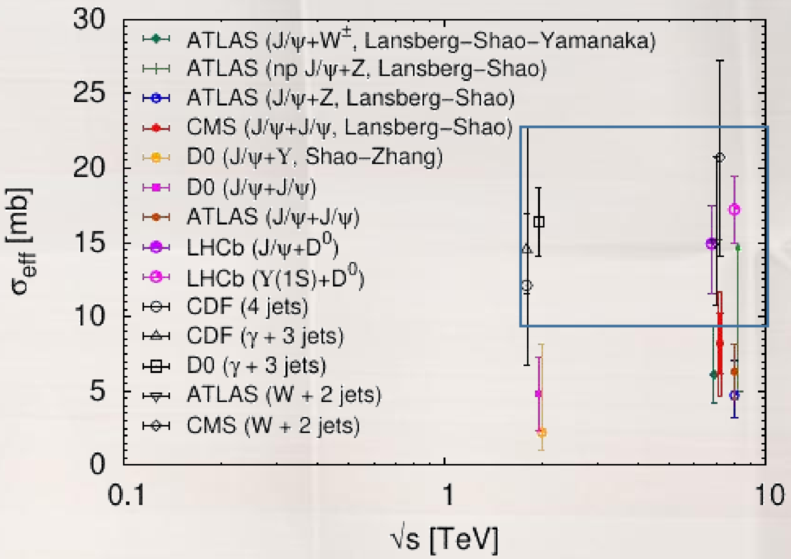

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MPI-2019 workshop

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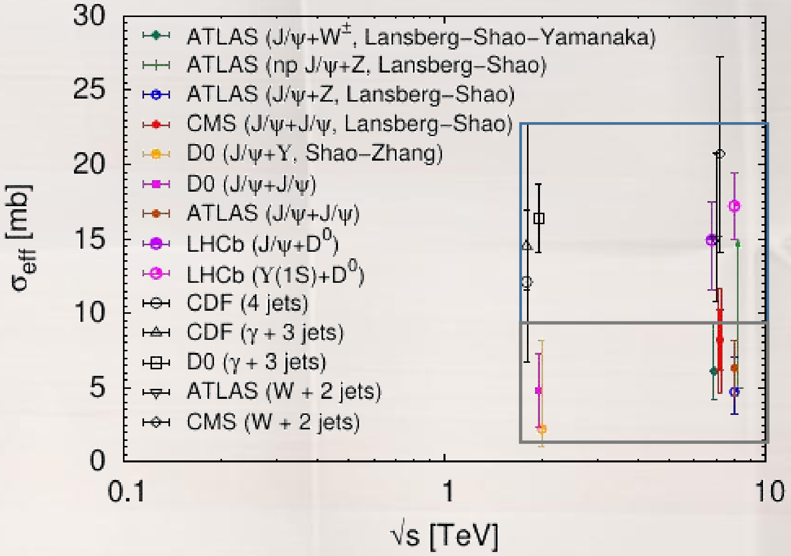
- SENSITIVE TO CORRELATIONS
- PROCESS DEPENDENT?
- SENSITIVE TO INFORMATION ON THE PROTON STRUCTURE?
  As predicted by quark models

M.R. et al PLB 752,40 (2016)
M. Traini, M. R. et al, PLB 768, 270 (2017)
M. R. et al, Phys.Rev. D95 (2017) no.11, 114030



M. R. and F. A. Ceccopieri, PRD 97, no. 7, 071501 (2018)

Clues from data?

If dPDFs factorize in terms of PDFs then $\sigma_{\text{eff}}^{-1} = \int \frac{d^2k_\perp}{(2\pi)^2} \boxed{T(k_\perp)}^2$ → Effective form factor (EFF)

EFF can be formally defined as **FIRST MOMENT** of dPDF in momentum space

$$T(k_\perp) \propto \int dx_1 dx_2 \, \tilde{F}(x_1, x_2, k_\perp)$$

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$$T(k_\perp) \propto \int dx_1 dx_2 \, \tilde{F}(x_1, x_2, k_\perp)$$

$k_\perp$ is the conjugate variable to $z_\perp$. In analogy with the charge form factor:

$$\langle z_\perp^2 \rangle \propto \frac{d}{k_\perp dk_\perp} T(k_\perp) \Big|_{k_\perp = 0}$$

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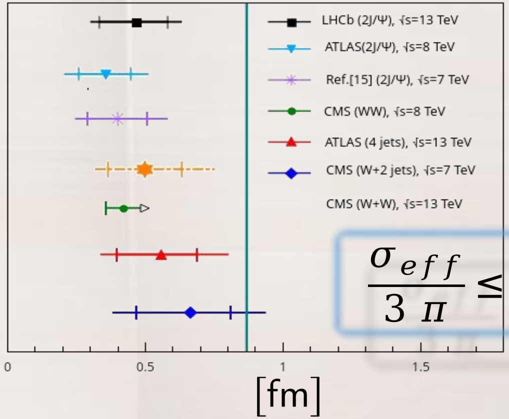
DPS processes:

The vertical line stands for the transverse proton radius



$$\frac{\sigma_{eff}}{3\pi} \leq \langle z^2 \rangle \leq \frac{\sigma_{eff}}{\pi}$$

M. R. and F. A. Ceccopieri, PRD 97, no. 7, 071501 (2018)

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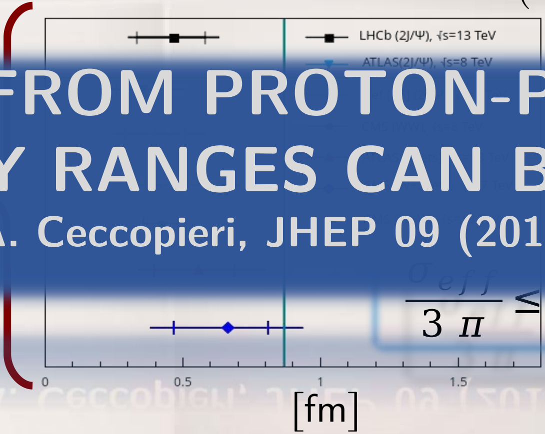
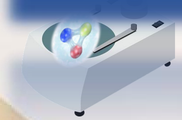
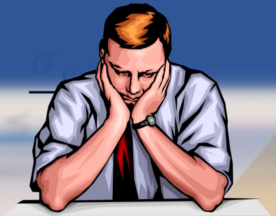


$\frac{\sigma_{eff}}{3\pi} \leq \langle z^2 \rangle \leq$

HOWEVER FROM PROTON-PROTON COLLISIONS ONLY RANGES CAN BE ACCESSED

M.R. and F. A. Ceccopieri, JHEP 09 (2019) 097





The effective cross section: a key for the proton structure

The effective cross section can be also written in terms of Fourier Transform of the EFF, i.e. the probability distribution of finding a parton pair at distance:

$$\left[\sigma_{eff}^{pp}\right]^{-1} = \int d^2z_\perp \boxed{F_2^p(z_\perp)^2}$$

UNKNOWN!

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$$\left[\sigma_{\text{eff}}^{\text{pp}}\right]^{-1} = \int d^2 z_\perp \boxed{F_2^p(z_\perp)^2}$$

UNKNOWN!

but for DPS involving two different hadrons (A and p) we would have:

$$\left[\sigma_{\text{eff}}^{\text{Ap}}\right]^{-1} = \int d^2 z_\perp \; F_2^p(z_\perp) F_2^A(z_\perp)$$

M. R. and F. A. Ceccopieri, PRD 105 (2022) 1, L011501

The effective cross section: a key for the proton structure

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$$\left[\sigma_{\text{eff}}^{Ap}\right]^{-1} = \int d^2 z_\perp \; F_2^p(z_\perp) F_2^A(z_\perp)$$

if we expand the distribution for the A hadron

$$F_2^A(z_\perp) = \sum_n C_n^A z_\perp^n$$

we get:

$$\left[\sigma_{\text{eff}}^{Ap}\right]^{-1} = \sum_n C_n^A \boxed{\langle z_\perp^n \rangle_p}$$

We could access for the first time the <u>mean transverse distance between partons in the proton</u>

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The effective cross section: a key for the proton structure

The effective cross section can be also written in terms of Fourier Transform of the EFF, i.e. the probability distribution of finding a parton pair at distance:

Is it possible to:
1) find an hadron with known Cn?
2) measure the effective x-section?

We could access for the first time the mean transverse distance between partons in the proton

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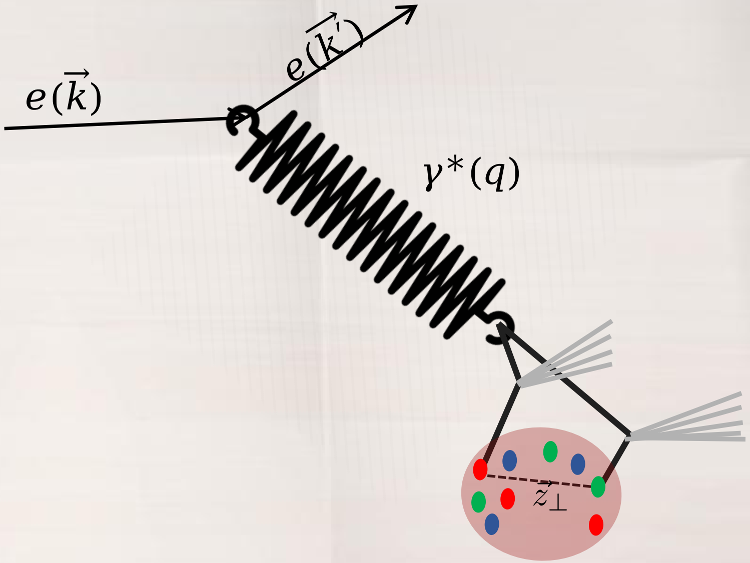
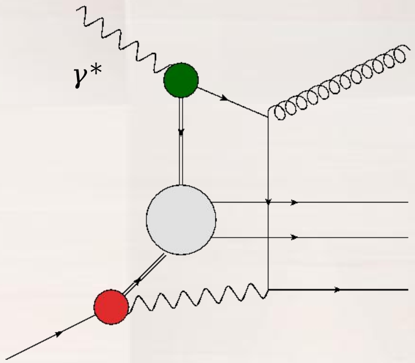
New Idea: DPS via γ-p interaction

We consider the possibility offered by a DPS process involving a photon FLACTUATING in a quark-antiquark pair interacting with a proton:



New Idea: DPS via γ-p interaction

In order to study the impact of the DPS contribution to a process initiated via photon-proton interactions we evaluated the 4-JET photoproduction at HERA **(S. Checkanov et al. (ZEUS), Nucl. Phys B792, 1 (2008))**



In

1) **G. Abbiend et al, Phys. Commun 67, 465 (1992)**
2) **J.R. Forshaw et al, Z. Phys. C 72, 637 (1992)**

It has been shown that the agreement with data improves if MPI are included in the Monte Carlo

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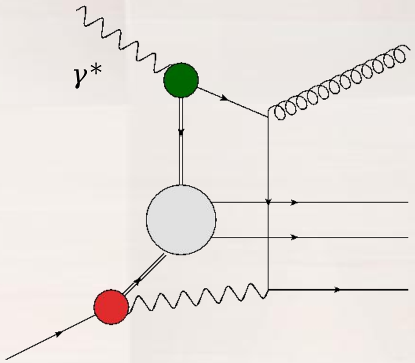

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⇓

WE EVALUATE THE DPS CONTRIBUTION TO THIS PROCESS

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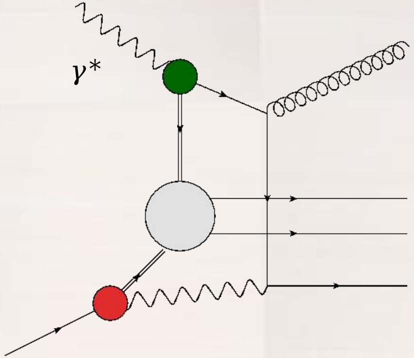

For this first investigation, we make use of the POCKET FORMULA:

$$d\sigma^{4j}_{DPS} = \frac{1}{2}\sum_{ab,cd}\int dy\, dQ^2 \frac{f_{\gamma/e}(y,Q^2)}{\sigma^{\gamma p}_{eff}(Q^2)} \times$$

$$\left.\begin{array}{l} \times \int dx_{p_a} dx_{\gamma_b} f_{a/p}(x_{p_a}) f_{b/\gamma}(x_{\gamma_b}) d\hat{\sigma}^{2j}_{ab}(x_{p_a}, x_{\gamma_b}) \\ \times \int dx_{p_c} dx_{\gamma_d} \underbrace{f_{c/p}(x_{p_c})}_{\text{p-PDF}} \underbrace{f_{d/\gamma}(x_{\gamma_d})}_{\gamma\text{-PDF}} d\hat{\sigma}^{2j}_{cd}(x_{p_c}, x_{\gamma_d}) \end{array}\right\}$$

SPS* × SPS

Flux Factor ($f_{\gamma/e}(y,Q^2)$)
P. Nason et al, PLB319 339 (1993)

p-PDF **(J. Pumplin et al. JHEP 07, 012 (2002))**

γ-PDF **(M. Gluck et al. PRD46, 1973 (1992))**

*Single Parton Scattering (SPS)

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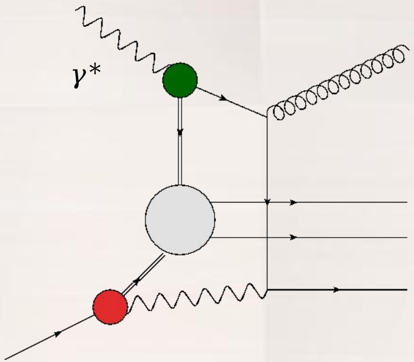

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$$\times \ldots) d\hat{\sigma}^{2j}_{ab}(x_{p_a}, x_{\gamma_b}) \quad \text{SPS*}$$

$$\times$$

$$\times \ldots) d\hat{\sigma}^{2j}_{cd}(x_{p_c}, x_{\gamma_d}) \quad \text{SPS}$$

Flux Factor
P. Nason et al,PLB319 339 (1993)

γ-PDF **(M. Gluck et al. PRD46, 1973 (1992))**

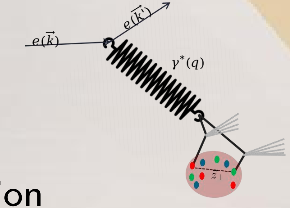
2002))

The main quantity we have to evaluate is:

$$\sigma^{\gamma p}_{eff}(Q^2)$$

*Single Parton Scattering (SPS)

The γ-p effective cross section



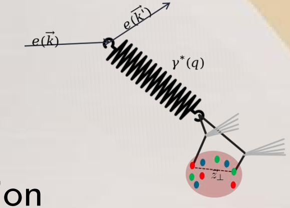
M. R. and F. A. Ceccopieri, PRD 105 (2022) 1, L011501

The expression of this quantity is very similar to the proton-proton collision case and can be formally derived by comparing the product of SPS cross sections and the DPS one obtained in **Gaunt, JHEP 01, 042 (2013)** and describing a DPS from a vector bosons splitting with given Q^2 virtuality

Proton EFF

$$\left[\sigma_{\rm eff}^{\gamma p}(Q^2)\right]^{-1} = \int \frac{d^2k_\perp}{(2\pi^2)} \boxed{T_p(k_\perp)} T_\gamma(k_\perp; Q^2)$$

The γ-p effective cross section



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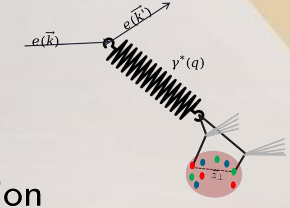

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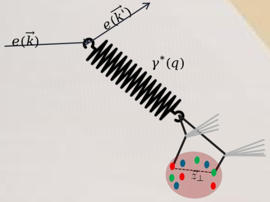
This quantity is similar to an EFF

The latter can be formally described within a Light-Front (LF) approach in terms of LF wave functions.

The γ-p effective cross section



1 $\left[\sigma_{\text{eff}}^{\gamma p}(Q^2)\right]^{-1} = \int \frac{d^2 k_\perp}{(2\pi^2)} T_p(k_\perp) T_\gamma(k_\perp; Q^2)$

2 $T_p(k_\perp)$ proton EFF

3 ψ_γ Photon WF

For the proton EFF use has been made of three choices:

1) G1: $e^{-\alpha_1 k_\perp^2}$, $\alpha_1 = 1.53\ \text{GeV}^{-2} \Longrightarrow \sigma_{\text{eff}}^{pp} = 15\ \text{mb}$

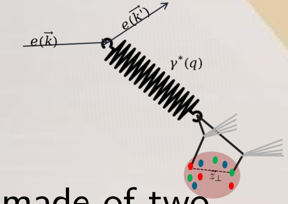
2) G2: $e^{-\alpha_2 k_\perp^2}$, $\alpha_2 = 2.56\ \text{GeV}^{-2} \Longrightarrow \sigma_{\text{eff}}^{pp} = 25\ \text{mb}$

3) S: $\left(1 + \frac{k_\perp^2}{m_g^2}\right)^{-4}$, $m_g^2 = 1.1\ \text{GeV}^2 \Longrightarrow \sigma_{\text{eff}}^{pp} = 30\ \text{mb}$

B. Blok et al, EPJC74, 2926 (2014)

M. R. and F. A. Ceccopieri, PRD 105 (2022) 1, L011501

The γ-p effective cross section



1 $\left[\sigma_{eff}^{\gamma p}(Q^2)\right]^{-1} = \int \frac{d^2k_\perp}{(2\pi^2)} T_p(k_\perp) T_\gamma(k_\perp; Q^2)$

$T_p(k_\perp)$ proton EFF

2

3 ψ^γ Photon WF

For the photon W.F. use has been made of two choices representing two extreme cases:

1) QED at LO (S.J. Brodsky et al. PRD50, 3134 (1994)):

$$\psi_{q,\bar{q}}^{\lambda=\pm}(x, k_{1\perp}; Q^2) = -e_f \frac{\bar{u}_q(k)\ \gamma \cdot \varepsilon^\lambda\ v_{\bar{q}}(q-k)}{\sqrt{x(1-x)}\left[Q^2 + \frac{k_{1\perp}^2 + m^2}{x(1-x)}\right]}$$

2) Non-Pertubative (NP) effects (E.R.Arriola et al, PRD74,054023 (2006))

$$\psi_A^\gamma(x, k_{\perp 1}; Q^2) = \frac{6(1+Q^2/m_\rho^2)}{m_\rho^2\left(1 + 4\frac{k_{\perp 1}^2 + Q^2 x(1-x)}{m_\rho^2}\right)^{5/2}}$$

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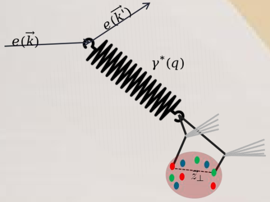
The γ-p effective cross section



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2 $T_p(k_\perp)$ proton EFF

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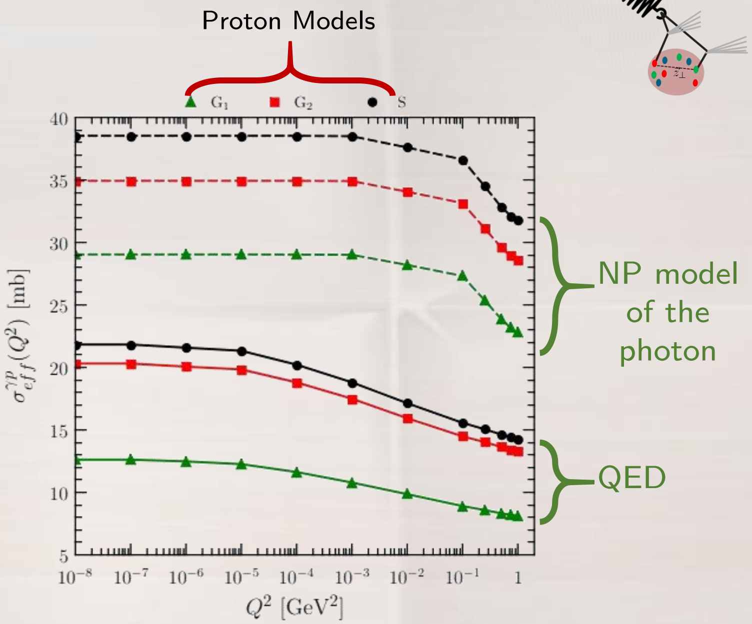


M. R. and F. A. Ceccopieri, PRD 105 (2022) 1, L011501

The 4 jet DPS cross section

KINEMATICS:

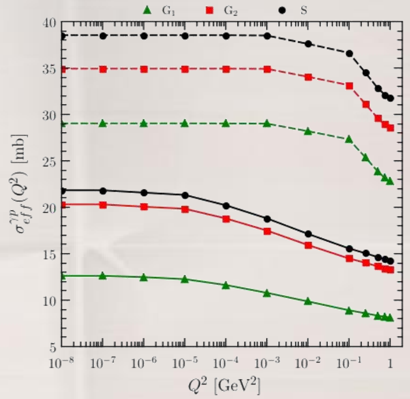
$E_T^{jet} > 6$ GeV

$|\eta_{jet}| < 2.4$

$Q^2 < 1$ GeV^2

$0.2 \leqslant y \leqslant 0.85$

$$d\sigma^{4j}_{DPS} = \frac{1}{2}\sum_{ab,cd}\int dy\, dQ^2\, \frac{f_{\gamma/e}(y,Q^2)}{\sigma^{\gamma p}_{eff}(Q^2)} \times \int dx_{p_a} dx_{\gamma_b} f_{a/p}(x_{p_a}) f_{b/\gamma}(x_{\gamma_b}) d\hat{\sigma}^{2j}_{ab}(x_{p_a}, x_{\gamma_b}) \times \int dx_{p_c} dx_{\gamma_d} f_{c/p}(x_{p_c}) f_{d/\gamma}(x_{\gamma_d}) d\hat{\sigma}^{2j}_{cd}(x_{p_c}, x_{\gamma_d})$$



The ZEUS collaboration quoted an integrated total 4-jet cross section of 136 pb

S. Checkanov et al. (ZEUS), Nucl. Phys B792, 1 (2008)

M. R. and F. A. Ceccopieri, PRD 105 (2022) 1, L011501

The 4 jet DPS cross section

KINEMATIC

$E_T^{jet} > 6$ Ge

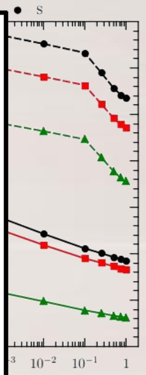
$|\eta_{jet}| < 2.4$

$Q^2 < 1$ GeV

$0.2 \leqslant y \leqslant 0$

photon		$Q^2 \leq 10^{-2}$ [GeV^2]	$10^{-2} \leq Q^2 \leq 1$ [GeV^2]	$Q^2 \leq 1$ [GeV^2]	$\frac{\sigma_{DPS}}{\sigma_{tot}}$ [%]
		σ_{DPS} [pb]			
NP model	G_1	35.1	18.6	53.7	40
	G_2	29.1	15.2	44.3	33
	S	26.4	13.7	40.1	30
QED	G_1	87.8	54.3	142.1	101
	G_2	54.3	33.4	87.7	65
	S	50.5	31.1	81.6	60

proton



The effective cross section: a key for the proton structure

The effective cross section can be also written in terms of Fourier Transform of the EFF:

$$\tilde{F}_2^{\gamma}(z_\perp; Q^2) = \sum_n C_n(Q^2) z_\perp^n$$

$$\left[\sigma_{eff}^{\gamma p}(Q^2)\right]^{-1} = \int d^2 z_\perp \; \tilde{F}_2^{p}(z_\perp) \tilde{F}_2^{\gamma}(z_\perp; Q^2)$$

$$= \sum_n C_n(Q^2) \langle (z_\perp)^n \rangle_p$$

If we can measure the dependence of the effective-cross section on the photon VIRTUALITY

We could access for the first time the mean transverse distance between partons in the proton

This coefficient can be determined from the structure of the photon described in a given approach

M. R. and F. A. Ceccopieri, PRD 105 (2022) 1, L011501

The effective cross section: a key for the proton structure

The effective cross section can be also written in terms of Fourier Transform of the EFF:

$\left[\sigma_{\text{eff}}^{\gamma p}(\right.$

If we can measure the dependence of the effective-cross section on the photon VIRTUALITY

We estimated that with an integrated luminosity of 200 pb^{-1} Q^2 effects can be observed

M. R. and F. A. Ceccopieri, PRD 105 (2022) 1, L011501

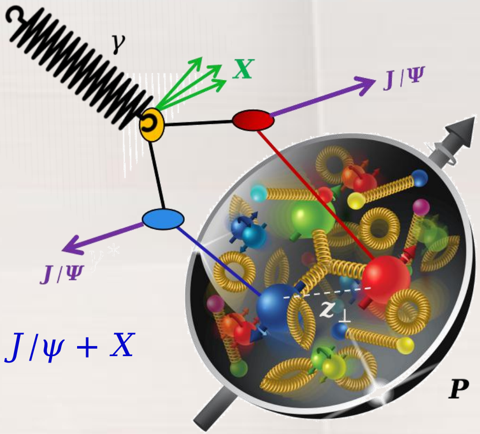
structure of th... he ... pproach

We could access for the first time the <u>mean transverse distance between partons in the proton</u>

M. R. and F. A. Ceccopieri, PRD 105 (2022) 1, L011501

Di-J/Ψ photo-production

F. A. Ceccopieri, H. S. Shao, J. P. Lansberg, M. R. and R. Sangem in prep.



Ilustration of DPS for $\gamma + p \to J/\psi + J/\psi + X$

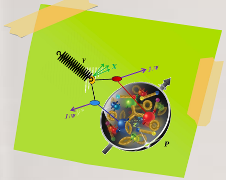
We consider the possibility of **resolved** photon to estimate the DPS cross section in quarkonium-pair photoproduction at the EIC

*Slide from R. Sangem

SPS and DPS cross-sections

F. A. Ceccopieri, H. S. Shao, J. P. Lansberg, M. R. and R. Sangem in prep.

*Slide from R. Sangem



$$\sigma_{SPS}^{(J/\psi,J/\psi)} \propto \sum_{a=g,q} \int dx_{p_a} f_{a/p}(x_{p_a}, \mu) d\hat{\sigma}^{\gamma a \to J/\psi + J/\psi + a}$$

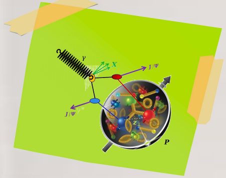
(Unresolved/direct)

$$\sigma_{SPS}^{(J/\psi,J/\psi)} \propto \sum_{a,b=g,q} \int dx_{\gamma_a}\, dx_{p_b} f_{a/\gamma}(x_{\gamma_a}, \mu) f_{b/p}(x_{p_b}, \mu) d\hat{\sigma}^{ab \to J/\psi + J/\psi}$$

(Resolved)

Photon PDF
Proton PDF
Partonic X-section

SPS and DPS cross-sections



F. A. Ceccopieri, H. S. Shao, J. P. Lansberg, M. R. and R. Sangem in prep.

*Slide from R. Sangem

$$\sigma_{SPS}^{(J/\psi,J/\psi)} \propto \sum_{a=g,q} \int dx_{p_a}\, f_{a/p}(x_{p_a},\mu)\, d\hat{\sigma}^{\gamma a \to J/\psi + J/\psi + a}$$

(Unresolved/direct)

$$\sigma_{SPS}^{(J/\psi,J/\psi)} \propto \sum_{a,b=g,q} \int dx_{\gamma_a}\, dx_{p_b}\, f_{a/\gamma}(x_{\gamma_a},\mu)\, f_{b/p}(x_{p_b},\mu)\, d\hat{\sigma}^{ab \to J/\psi + J/\psi}$$

(Resolved)

Photon PDF

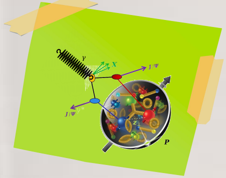
Proton PDF

Partonic X-section

$$\sigma_{DPS}^{(J/\psi,J/\psi)} \propto \frac{1}{2}\frac{1}{\sigma_{eff}^{\gamma p}} \sum_{a,b,c,d} \int dx_{\gamma_a}\, dx_{p_b}\, f_{a/\gamma}(x_{\gamma_a},\mu)\, f_{b/p}(x_{p_b},\mu)\, d\hat{\sigma}_{SPS}^{ab \to J/\psi}(x_{\gamma_a},x_{p_b})$$
$$\times\, dx_{\gamma_c}\, dx_{p_d}\, f_{c/\gamma}(x_{\gamma_c},\mu)\, f_{d/p}(x_{p_d},\mu)\, d\hat{\sigma}_{SPS}^{cd \to J/\psi}(x_{\gamma_c},x_{p_d})$$

Single J/ψ SPS resolved (namely same partonic cross section as hadroproduction)

Di-J/Ψ photo-production @EIC

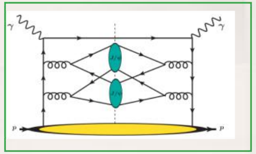


F. A. Ceccopieri, H. S. Shao, J. P. Lansberg, M. R. and R. Sangem in prep.

*Slide from R. Sangem

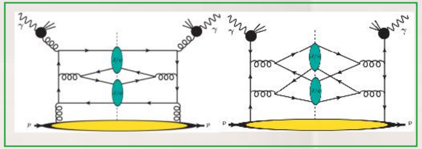
SPS

Unresolved



$\gamma + q \to J/\psi + J/\psi + q$

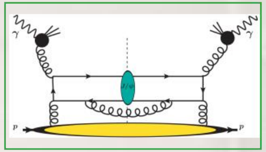
Resolved



$i + j \to J/\psi + J/\psi$ $\quad i, j = g, q$

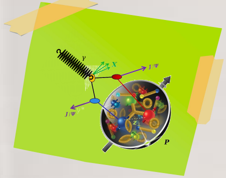
DPS

Resolved



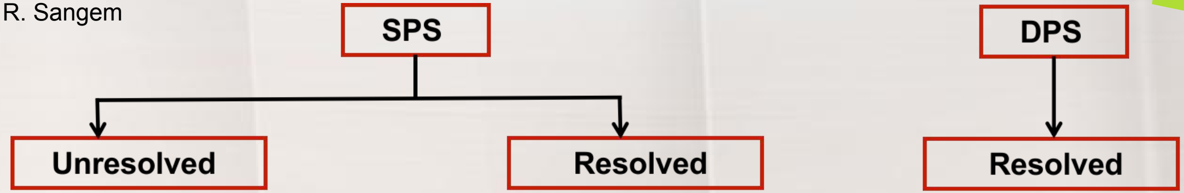
$|g + g \to J/\psi + g|^2$

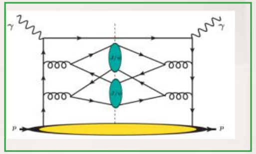
Di-J/Ψ photo-production @EIC



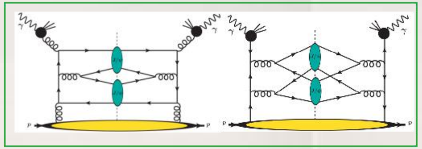
F. A. Ceccopieri, H. S. Shao, J. P. Lansberg, M. R. and R. Sangem in prep.

*Slide from R. Sangem

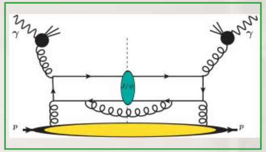




$\gamma + q \to J/\psi + J/\psi + q$



$i + j \to J/\psi + J/\psi$ $\quad i, j = g, q$

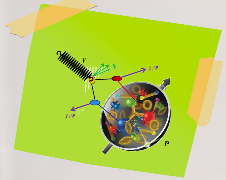


$|g + g \to J/\psi + g|^2$

- GRV photon PDF is used PRD 46, 1973 (1992), while CT18NLO PDF for proton T.J. Hou et al., PRD 103, 014013 (2021)
- HELAC-Onia latest version is used for generating matrix elements HS Shao, CPC 184, 2562 (2013), 198, 238 (2016)
- CO LDMEs are taken from M. Butenschoen and B. A. Kniehl, PRD 84, 051501 (2011)
- We expect at least 600 four-muon events with 100 fb^{-1} luminosity

Di-J/ψ photo-production @EIC

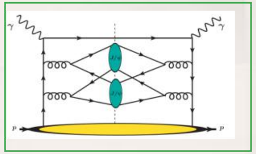
F. A. Ceccopieri, H. S. Shao, J. P. Lansberg, M. R. and R. Sangem in prep.



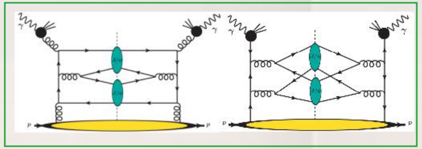
*Slide from R. Sangem

SPS → **Unresolved**, **Resolved**

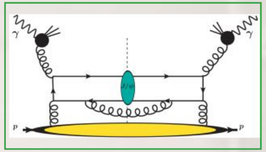
DPS → **Resolved**



$\gamma + q \to J/\psi + J/\psi + q$



$i + j \to J/\psi + J/\psi$ $\quad i, j = g, q$



$|g + g \to J/\psi + g|^2$

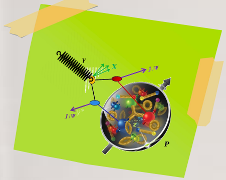
Range of cross sections in CSM $\sqrt{S_{\gamma p}}$=100 GeV

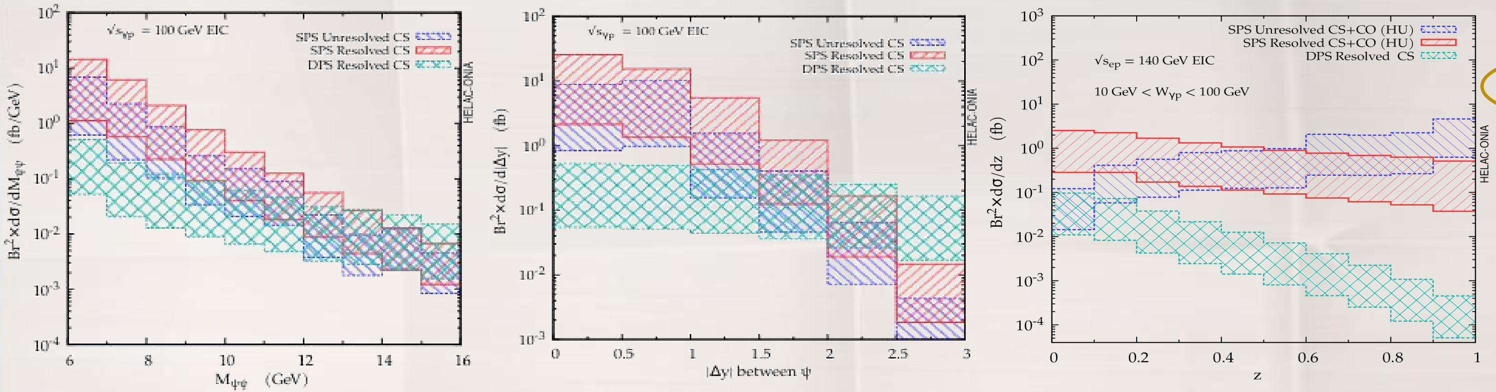
$\sigma_{SPS}^{(J/\psi,J/\psi)} \times Br^2 = 4 - 30$ fb
$\sigma_{DPS}^{(J/\psi,J/\psi)} \times Br^2 = 0.2 - 5$ fb
(Resolved) $\sigma_{eff}^{\gamma p}$ = 10 mb for DPS

$\sigma_{SPS}^{(J/\psi,J/\psi)} \times Br^2 = 2 - 12$ fb (Unresolved)

First predictions

F. A. Ceccopieri, H. S. Shao, J. P. Lansberg, M. R. and R. Sangem in prep.

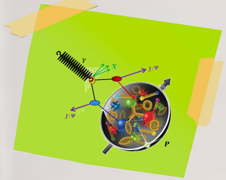




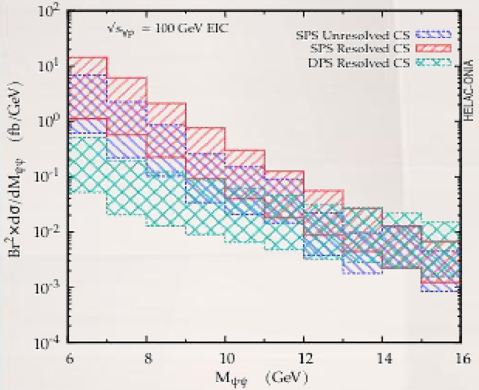
- For $z < 0.1$ resolved SPS dominates over unresolved/direct
- Unique opportunity to study the photon structure
- At larger z one can test quarkonium production mechanism via direct photoproduction
- Resolved case: gluon channel dominates in the low z region, and quark channel at high z
- CS and CO states are considered: CO states contribution is only significant (for some LDMEs) in unresolved but not in the resolved case

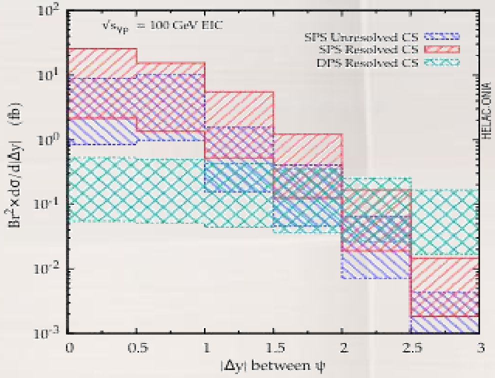
First predictions

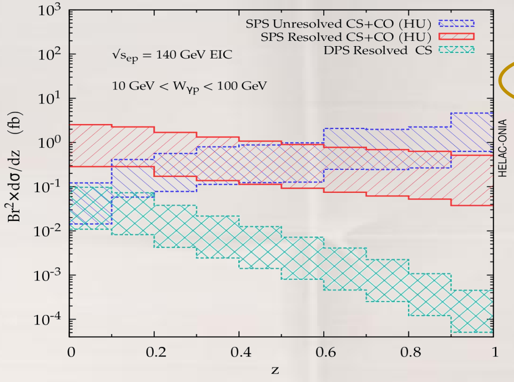
F. A. Ceccopieri, H. S. Shao, J. P. Lansberg, M. R. and R. Sangem in prep.



*Slide from R. Sangem

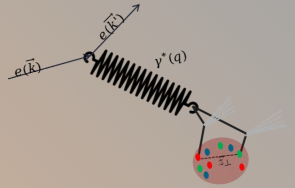






DPS COULD BE NOT NEGLIGIBLE BUT FURTHER ANALYSES OF THE ERRORS ON THE DPS CONTRIBUTION ARE ON GOING

CONCLUSIONS



1) We demonstrated that in p-p collisions only some limited information on the proton can be obtained

2) We proposed to consider DPS initiated via photon-proton interactions by showing that:

* DPS can contribute also in this case. Cross section of the 4 jet photo production strongly affected

* The dependence of $\sigma_{\rm eff}^{\gamma p}(Q^2)$ on the Q^2 can unveil the mean distance of partons in the proton

* We started the **QUARKONIUM Photo-PRODUCTION analysis:**

  - Quarkonium production is a rich channel to probe the parton correlations through DPS

  - We have estimated SPS and DPS cross sections for quarkonium-pair photoproduction at the EIC using the NRQCD framework

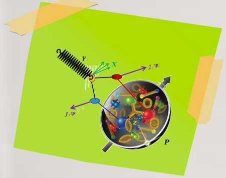
  - DPS total cross section is small compared to the SPS but could be measured if σ_{eff} small

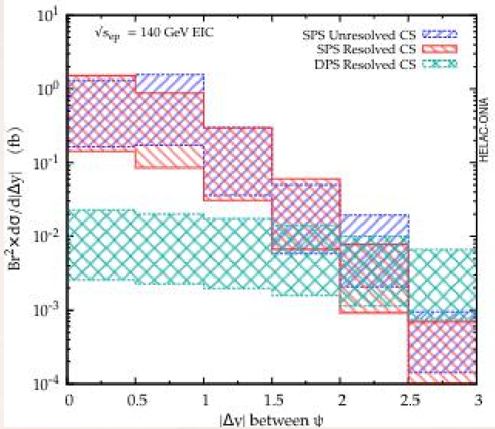
  - Quarkonium-pair photoproduction is a promising channel to probe the gluonic content of the photon structure

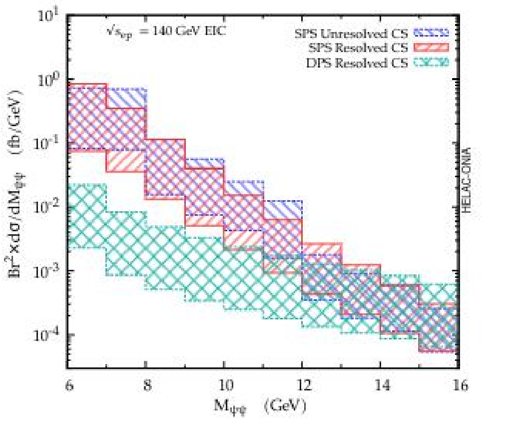
First predictions

F. A. Ceccopieri, H. S. Shao, J. P. Lansberg, M. R. and R. Sangem in prep.

*Slide from R. Sangem

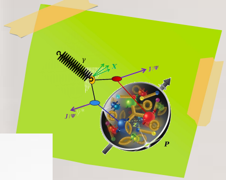






First predictions

F. A. Ceccopieri, H. S. Shao, J. P. Lansberg, M. R. and R. Sangem in prep.



*Slide from R. Sangem

Quarkonium production Mechanism

Three models for describing the formation of Quarkonium and they are successful at different regions

- Color Singlet Model (CSM)

  C.H Chang, Nucl.Phys.B 172 (1980)

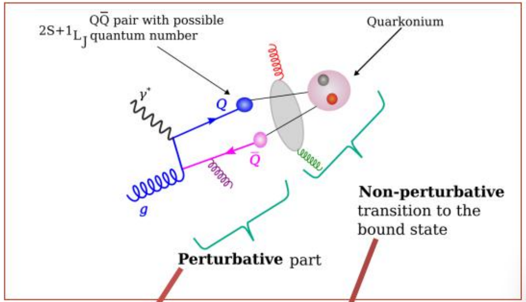
  Baier and R. Ruckl, PLB 102 (1981), Z.Phys.C 19 (1983) 251

- Color Evaporation Model (CEM)

  H. Fritzsch, PL 67B (1977) 217-221

- Non-Relativistic QCD (NRQCD)

  G. T. Bodwin et al, PRD51 (1995)

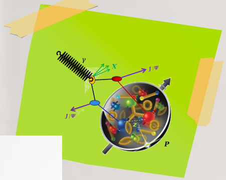


$$d\sigma^{ab\to\mathcal{Q}} = \sum_n d\hat{\sigma}\left[ab \to Q\bar{Q}(n)\right]\langle 0|\mathcal{O}_n^{\mathcal{Q}}|0\rangle$$

3

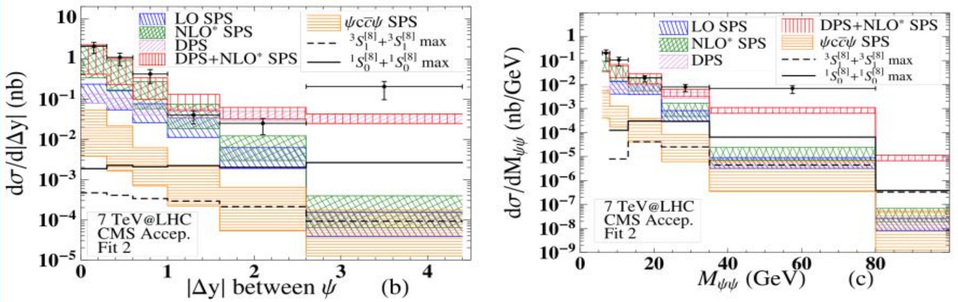
First predictions

F. A. Ceccopieri, H. S. Shao, J. P. Lansberg, M. R. and R. Sangem in prep.



*Slide from R. Sangem

DPS and the LHC data



JP Lansberg, HS Shao, NPB 900 (2015) 273-294

CMS coll. JHEP 09 (2014) 094

- DPS is the simplest explanation for the gap between SPS prediction and the data at large Δy and $M_{\psi\psi}$
- Same observation with the ATLAS data

The effective cross section: a key for the proton structure

To test if in future a dependence of the effective cross section on the photon virtuality could be observed, we considered again the 4 JET photoproduction:

1) We divided the integral of the cross section on Q^2 in two intervals:

$$Q^2 \leqslant 10^{-2} \quad \text{and} \quad 10^{-2} \leqslant Q^2 \leqslant 1 \quad \text{GeV}^2$$

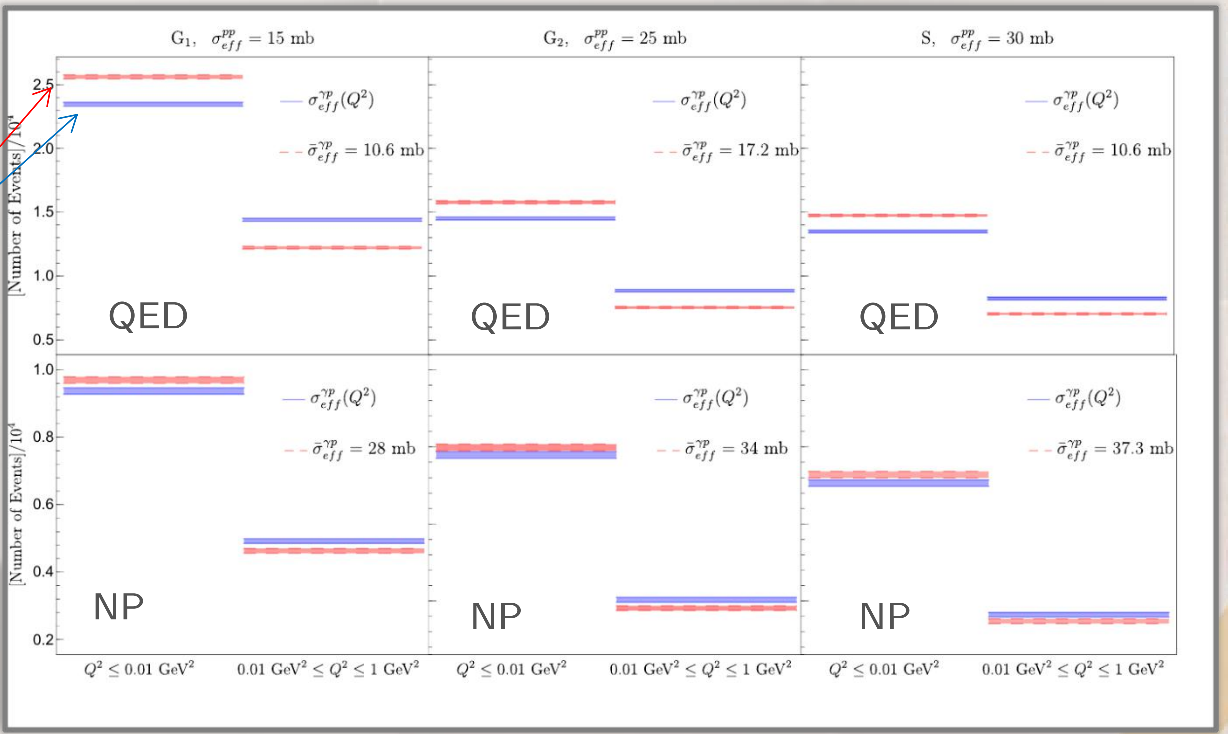
2) We have estimated for each photon and proton models a constant effective cross section $\bar{\sigma}_{\text{eff}}^{\gamma p}$ (with respect to Q^2) such that the total integral of the cross section on Q^2 reproduce the full calculation obtained by means of $\sigma_{\text{eff}}^{\gamma p}(Q^2)$

3) We estimate the minimum luminosity to distinguish the two cases

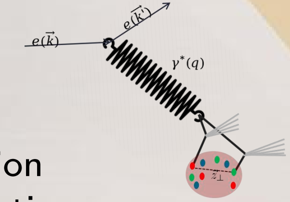
M. R. and F. A. Ceccopieri, PRD 105 (2022) 1, L011501

The effective cross section: a key for the proton structure

With an integrated luminosity of 200 pb^{-1} we can separate:



The γ-p effective cross section



The expression of this quantity is very similar to the proton-proton collision case and can be formally derived by comparing the product of SPS cross sections and the DPS one obtained in **Gaunt, JHEP 01, 042 (2013)** and describing a DPS from a vector bosons splitting with given Q^2 virtuality

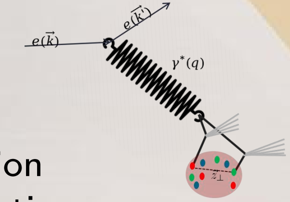
Proton EFF

$$\left[\sigma_{\rm eff}^{\gamma p}(Q^2)\right]^{-1} = \int \frac{d^2 k_\perp}{(2\pi^2)} \boxed{T_p(k_\perp)}\, \underbrace{T_\gamma(k_\perp; Q^2)}$$

This quantity is similar to an EFF

The latter can be formally described within a Light-Front (LF) approach in terms of LF wave functions.

The γ-p effective cross section



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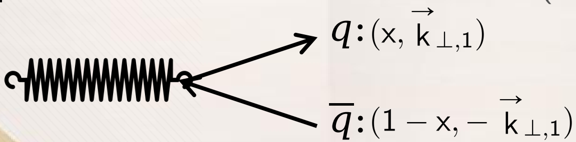
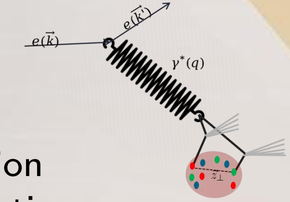
$$\left[\sigma_{\text{eff}}^{\gamma p}(Q^2)\,\right]^{-1} = \int \frac{d^2 k_\perp}{(2\pi^2)} T_p(k_\perp) T_\gamma(k_\perp; Q^2)$$

The full DPS cross section depends on the amplitude of the splitting photon in a $q\,\bar{q}$ pair. The latter can be formally described within a Light-Front (LF) approach in terms of LF wave functions (W.F.):

$$f^{\gamma}_{q,\bar{q}}(x, \tilde{k}_\perp; Q^2) = \int d^2 k_{\perp,1}\, \psi^{\dagger\gamma}_{q\bar{q}}(x, \vec{k}_{\perp,1}; Q^2) \times \psi^{\gamma}_{q\bar{q}}(x, \vec{k}_{\perp,1} + \vec{k}_\perp; Q^2)$$



The γ-p effective cross section

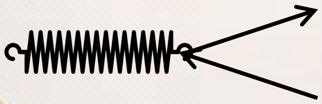


The expression of this quantity is very similar to the proton-proton collision case and can be formally derived by comparing the product of SPS cross sections and the DPS one obtained in **Gaunt, JHEP 01, 042 (2013)** and describing a DPS from a vector bosons splitting with given Q^2 virtuality

$$\left[\sigma_{\rm eff}^{\gamma p}(Q^2)\right]^{-1} = \int \frac{d^2k_\perp}{(2\pi^2)} T_p(k_\perp) T_\gamma(k_\perp; Q^2)$$

$$T_\gamma(k_\perp; Q^2) = \frac{\sum_q \int dx\, f^\gamma_{q,\bar{q}}(x, k_\perp; Q^2)}{\sum_q \int dx\, f^\gamma_{q,\bar{q}}(x, k_\perp = 0; Q^2)}$$

$$f^\gamma_{q,\bar{q}}(x, \tilde{k}_\perp; Q^2) = \int d^2k_{\perp,1}\, \psi^{\dagger\gamma}_{q\bar{q}}(x, \vec{k}_{\perp,1}; Q^2) \times \psi^\gamma_{q\bar{q}}(x, \vec{k}_{\perp,1} + \vec{k}_\perp; Q^2)$$



$q: (x, \vec{k}_{\perp,1})$

$\bar{q}: (1 - x, -\vec{k}_{\perp,1})$

Data and Effective Cross Section

A tool for the comprehension of the role of DPS in hadron-hadron collisions is the so called “effective X-section”.

$$\underbrace{\sigma_{\text{eff}}^{pp} = \frac{m}{2} \frac{\sigma_A^{pp} \sigma_B^{pp}}{\sigma_{\text{DPS}}^{pp}}}_{\text{POCKET FORMULA}}$$

Differential cross section for the process: $pp \longrightarrow A(B) + X$

Differential cross section for a DPS event: $pp \longrightarrow A + B + X$

Data and Effective Cross Section

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POCKET FORMULA

Differential cross section for the process: $pp \longrightarrow A(B) + X$

Differential cross section for a DPS event: $pp \longrightarrow A + B + X$

color factors

PDF

$$\sigma_{eff}(x_1, x_2, x_3, x_4) = \frac{\sum_{i,j,k,l} C_{ik} C_{jl} F_i(x_1) F_j(x_2) F_k(x_3) F_l(x_4)}{\sum_{i,j,k,l} C_{ik} C_{jl} \int d^2 z_\perp \; F_{ij}(x_1, x_3, z_\perp) F_{kl}(x_3, x_4, z_\perp)}$$

M.R., S. Scopetta et al, PLB 752

M. Traini, M.R., S. Scopetta and V. Vento, PLB 768 (2017)

Double PDFs of the proton

$F_{ik}(x_1, x_2, \vec{z}_\perp)$ is unknown. However @LHC kinematics (small x and many partons produced)

$$F_{ik}(x_1, x_2, \vec{z}_\perp) \sim g(x_1, x_2)\, \tilde{T}(\vec{z}_\perp)$$

$\tilde{T}(\vec{z}_\perp)$:
- PROBABILITY DISTRIBUTION OF FINDING TWO PARTONS WITH GIVEN TRANSVERSE DISTANCE
- UNKNOWN: ONLY MODELS

$g(x_1, x_2)$ — DPD:
- SUM RULES AVAILABLE
- MODELS BASED ON SUM RULES
- PDF(x_1)*PDF(x_2)
- [PDF(x_1)*PDF(x_2)] ⊗ pQCD EVOLUTION

2nd uncorrelated scenario

PERTURBATIVE CORRELATIONS

$F_{ik}(x_1, x_2, \vec{z}_\perp)$: Constituent quark models used to grasp basic non perturbative feature

M.R., S. Scopetta et *al*, PRD 87 (2013) 114021
M.R., S, Scopetta et *al*, JHEP 12 (2014) 028

Double PDFs of the proton

Some questions arose:

1) HOW CAN WE BE SURE OF THE ACCURACY OF SUCH APPROXIMATION?

dPDFs are non perturbative in QCD ➡ DPCs cannot be accessed from QCD

2) WHICH INFORMATION ON THE PROTON STRUCTURE COULD BE ACCESSED FROM DPS?

$F_{ij}(x_1, x_2, \vec{z}_\perp)$ is unknown. However @LHC kinematics (small x and many partons produced)

PROBABILITY DISTRIBUTION OF FINDING TWO PARTONS WITH GIVEN TRANSVERSE DISTANCE

UNKNOWN: ONLY MODELS

Constituent quark models used to grasp basic non perturbative feature

SUM RULES AVAILABLE

MODELS BASED ON SUM RULES

$PDF(x_1)*PDF(x_2)$

$[PDF(x_1)*PDF(x_2)] \otimes$ pQCD EVOLUTION

2nd uncorrelated scenario

PERTURBATIVE CORRELATIONS

M.R., S. Scopetta et *al*, PRD 87 (2013) 114021
M.R., S, Scopetta et *al*, JHEP 12 (2014) 028

Double PDFs within the Light-Front

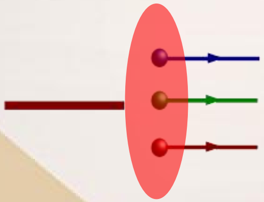
Extending the procedure developed in **S. Boffi, B. Pasquini and M. Traini, Nucl. Phys. B 649, 243 (2003)** for GPDs, we obtained the following expression of the dPDF in momentum space, often called $_2$GPDs:

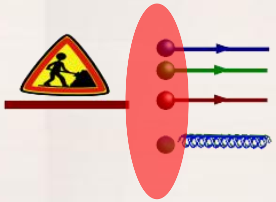
$$F_{ij}(x_1, x_2, k_\perp) = 3(\sqrt{3})^3 \int \prod_{i=1}^{3} d\vec{k}_i \delta\left(\sum_{i=1}^{3} \vec{k}_i\right) \Phi^*(\{\vec{k}_i\}, k_\perp)\Phi(\{\vec{k}_i\}, -k_\perp)$$

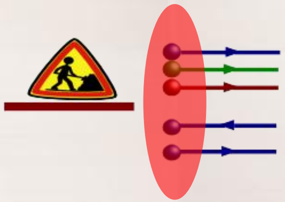
$$\times \quad \delta\left(x_1 - \frac{k_1^+}{P_+}\right)\delta\left(x_2 - \frac{k_2^+}{P_+}\right)$$

Conjugate to $z_\perp$

LF wave-function





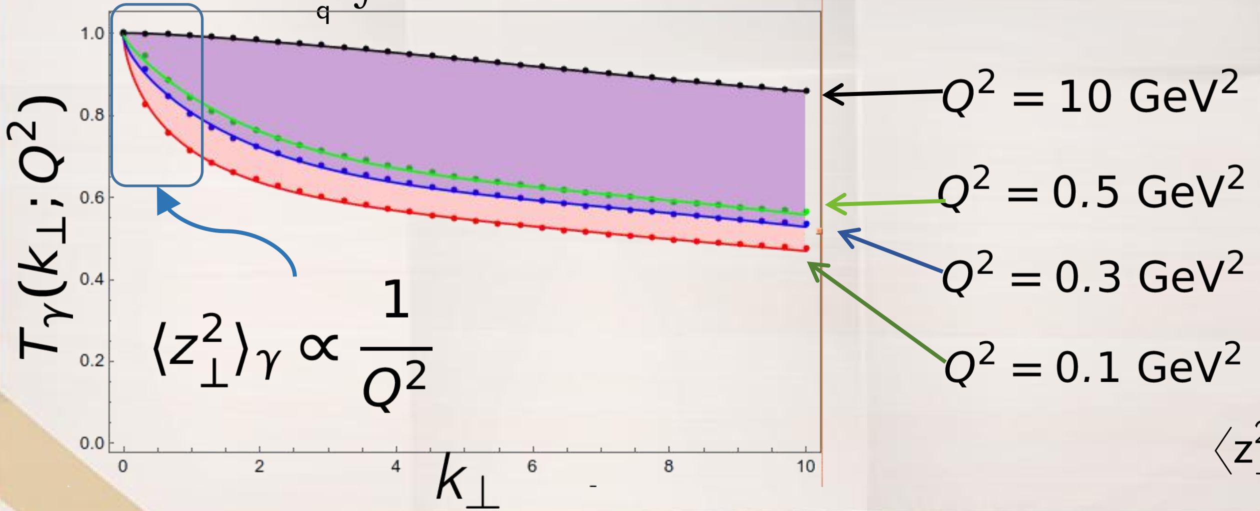


$$\Phi(\{\vec{k}_i\}, \pm k_\perp) = \Phi\left(\vec{k}_1 \pm \frac{\vec{k}_\perp}{2}, \vec{k}_2 \mp \frac{\vec{k}_\perp}{2}, \vec{k}_3\right)$$

More on the LO QED photon EFF and effective x-section

$$T_\gamma(k_\perp; Q^2) = \frac{\sum_q \int dx\, f^\gamma_{q,\bar{q}}(x, k_\perp; Q^2)}{\sum_q \int dx\, f^\gamma_{q,\bar{q}}(x, k_\perp = 0; Q^2)}$$

$$f^\gamma_{q,\bar{q}}(x, \tilde{k}_\perp; Q^2) = \int d^2k_{\perp,1}\, \psi^{\dagger\gamma}_{q\bar{q}}(x, \vec{k}_{\perp,1}; Q^2) \times \psi^\gamma_{q\bar{q}}(x, \vec{k}_{\perp,1} + \vec{k}_\perp; Q^2)$$



$$\langle z_\perp^2 \rangle \propto \frac{d}{k_\perp dk_\perp} T(k_\perp) \Big|_{k_\perp = 0}$$

More on the LO QED photon EFF and effective x-section

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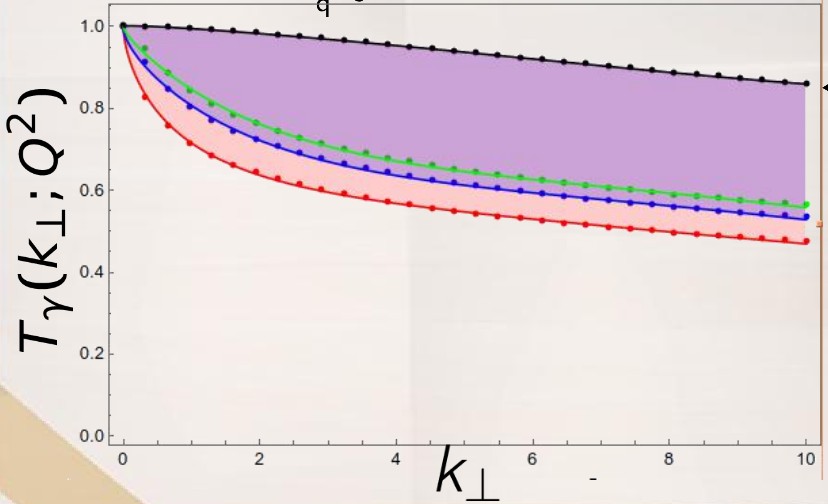


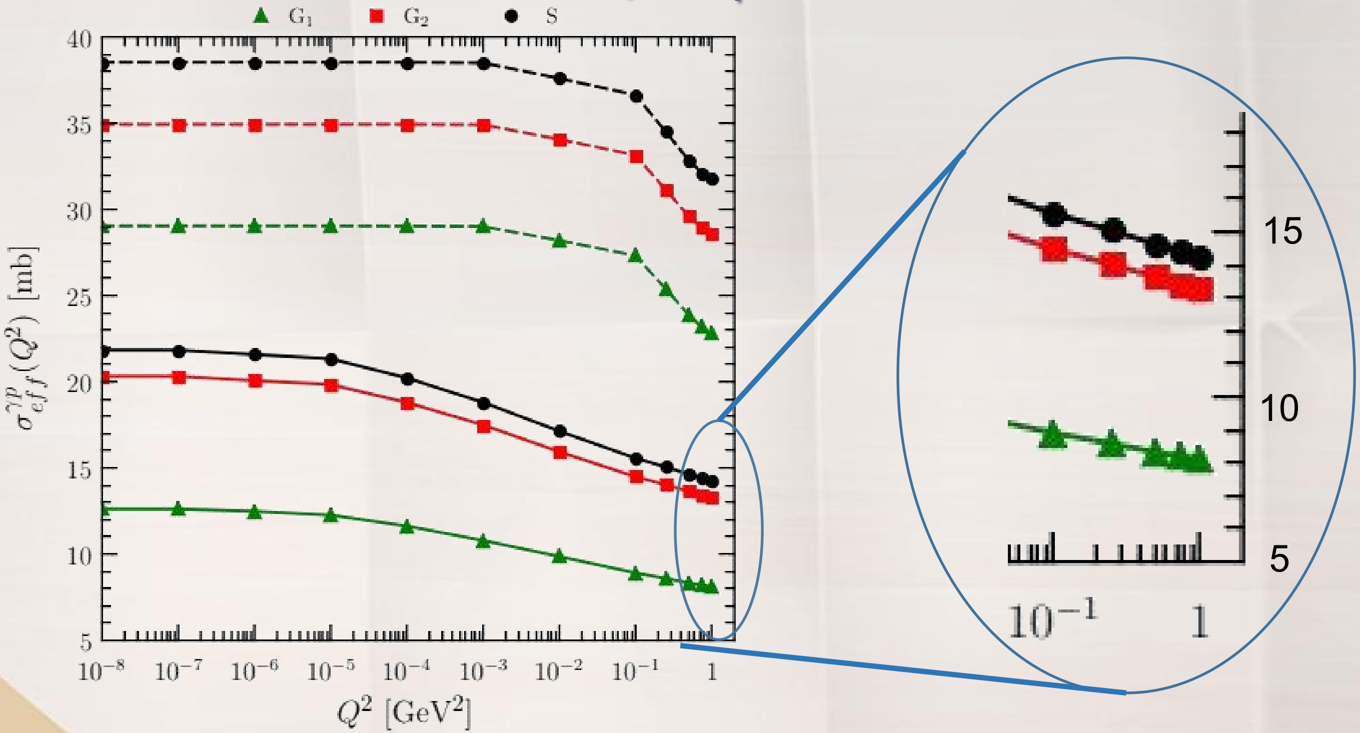
$\leftarrow Q^2 = 10\ \text{GeV}^2$

$$\langle z_\perp^2 \rangle_\gamma \propto \frac{1}{Q^2}$$

Small system ➡ EFF **SLOWLY** decreasing:

$$T_\gamma(k_\perp;Q^2>>1\ \text{GeV}^2) \sim 1$$

More on the LO QED photon EFF and effective x-section



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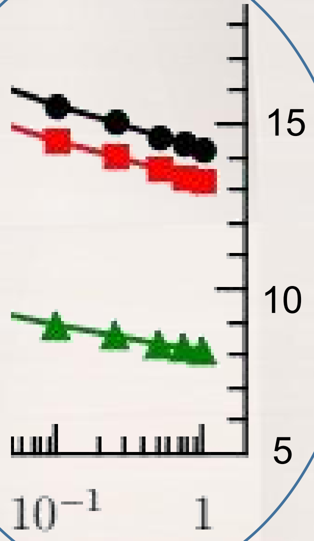
More on the LO QED photon EFF and effective x-section



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$$[\sigma_{eff}^{\gamma p}(Q^2)]^{-1} = \int \frac{d^2k_\perp}{(2\pi)^2} T_p(k_\perp) T_\gamma(k_\perp; Q^2)$$

$$[\sigma_{eff}^{\gamma p}(Q^2)]^{-1} \underset{Q^2>>1}{\sim} \int \frac{d^2k_\perp}{(2\pi)^2} T_p(k_\perp) \times 1$$

For the proton models we have used:

$$\int \frac{d^2k_\perp}{(2\pi)^2} T_p(k_\perp) \sim 2 \int \frac{d^2k_\perp}{(2\pi)^2} T_p(k_\perp)^2$$

$$\sigma_{eff}^{\gamma p}(Q^2 >> 1\ \mathrm{GeV}^2) \sim \sigma_{eff}^{pp}/2$$

More on the LO QED photon EFF and effective x-section

1) Since the photon starts to be a **small** system, the effective-form factor must be similar to a constant (to be properly related to the FT of the probability distribution)

2) as a conseguence, the effective cross section should be of the same order of that for pp collissions.

3) why this two effective x-section are similar if the system are different?

4) a possible explanation can be obtained by considering:

$$\frac{\sigma_{eff}}{3\,\pi} \leq \langle z^2 \rangle \leq \frac{\sigma_{eff}}{\pi}$$

(proven for pp collisions)

→

Inverting this inequality one gets:

$$\langle z_\perp^2 \rangle \leq \sigma_{eff}^{pp} \leq 3\pi \langle z_\perp^2 \rangle$$

More on the LO QED photon EFF and effective x-section

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(proven for pp collisions)

therefore, similar effective x-sections can be related to different **distances,** i.e. **different gemetrical structures!**

(Proton) Model Independent conlcusions

1) in arXiv:2103.1340 we show that high virtual behavior of the effective cross sections correctly follows the result **in J.R. Gaunt JHEP 01, 042 (2013)**, i.e.:

$$\sigma_{eff}^{\gamma p}(Q^2 \to \infty) = \sigma_{1v2}^{pp} = \left[\int \frac{d^2 k_\perp}{(2\pi)^2} T_p(k_\perp) \right]^{-1}$$

2) In Ref. **M.Rinaldi and F.A: Ceccopieri JHEP 09, 097 (2019),** we prove, in a general framework:

$$\frac{\sigma_{\text{eff},2\text{v}1}}{2\pi} \leq \langle b^2 \rangle \leq \frac{2\ \sigma_{\text{eff},2\text{v}1}}{\pi}$$

therefore, by inverting this relation one gets:

$$\frac{\pi}{2} \langle b^2 \rangle \leq \sigma_{eff}^{\gamma p}(Q^2 \to \infty) \leq 2\pi \langle b^2 \rangle$$

(Proton) Model Independent conlcusions

$$\frac{\pi}{2}\langle b^2\rangle \leq \sigma_{eff}^{\gamma p}(Q^2 \to \infty) \leq 2\pi\langle b^2\rangle$$

3) in arXiv:2103.1340, for the moment being we considered proton model producing a (2v2) effective cross section of 15-30 mb (**in new analysis we can relax this condition**).
Now in **M. Rinaldi and F. A. Ceccopieri PRD 97 (2018) 7, 071501**, we prove:

$$\frac{\sigma_{eff}^{pp}}{3\pi} \leq \langle b^2\rangle \leq \frac{\sigma_{eff}^{pp}}{\pi}$$

combining everything:

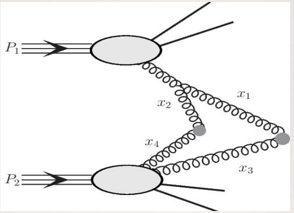
$$\frac{\sigma_{eff}^{pp}}{6} \leq \sigma_{eff}^{\gamma p}(Q^2 \to \infty) \leq 2\sigma_{eff}^{pp}$$

VERIFIED!!

4 Further implementations

Relevant for processes involving heavy particles is the splitting term (1v2 mechanism) i.e.:

$^{*}D_{j_1,j_2}(x_1,x_2) = \int d^2b_\perp \tilde{F}_{j_1,j_2}(x_1,x_2,b_\perp)$

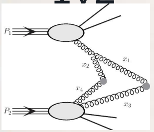


In pQCD evolution: $\frac{dD_{j_1j_2}(x_1,x_2;t)}{dt} = \begin{cases} \text{Homogeneous term (double DGLAP)} \\ + \\ \sum_{j'} F_{j'}(x_1+x_2;t) \underbrace{\frac{1}{x_1+x_2} P_{j'\to j_1j_2}}\left(\frac{x_1}{x_1+x_2}\right) \end{cases}$

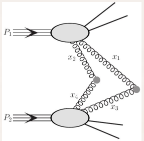
SPLITTING TERM

Gaunt J.R. and Stirling W. J., JHEP 03 (2010)

1v2



$+$



2v2

$$\frac{\sigma_{eff}}{3\pi}\left(1+\frac{3}{2}r_v\right) \le \langle b^2\rangle \le \frac{\sigma_{eff}}{\pi}\left(1+2r_v\right)$$

$$r_v \sim \frac{F^{splitting}_{j_1j_2}(x_1,x_2,k_\perp=0;t)}{F_{j_1j_2}(x_1,x_2,k_\perp=0;t)}$$

with:
$0 \le r_v \le 1$

J.R. Gaunt, R. Maciula and A. Szczurek, PRD 90 (2014) 054017

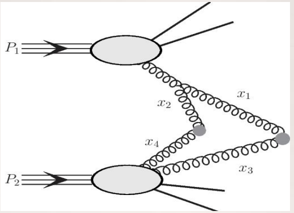
Due to the difficulty in the estimate of the 2 contributions:

Absolute minimum $r_v = 0$ $\quad \frac{\sigma_{eff}}{3\pi} \le \langle b^2\rangle \le \frac{3\ \sigma_{eff}}{\pi}$ $\quad$ Absolute maximum $\mathbf{r_v = 1}$

4 Further implementations

Relevant for processes involving heavy particles is the splitting term (1v2 mechanism) i.e.:

$) = \int d^2b_\perp \tilde{F}_{j_1,j_2}(x_1,x_2,b_\perp)$



In p

ble DGLAP)

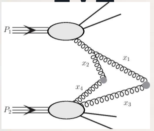
$j_1 j_2 \left(\frac{x_1}{x_1+x_2}\right)$

Gau

M

$\frac{{}^{splitting}_{j_1 j_2}(x_1,x_2,k_\perp=0;t)}{F_{j_1 j_2}(x_1,x_2,k_\perp=0;t)}$

1v2



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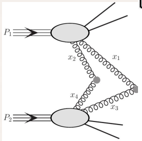
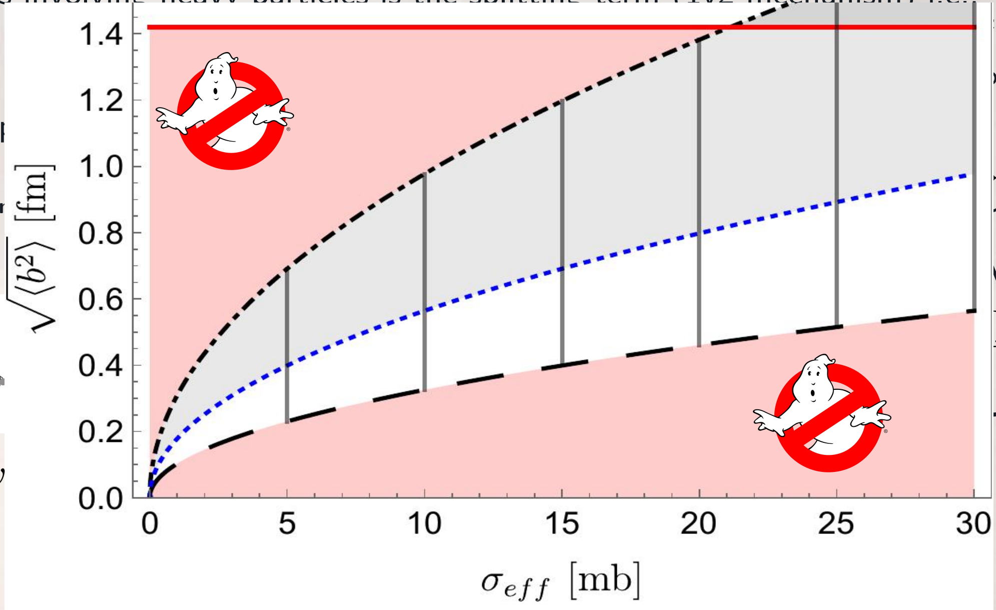




with:

$0 \leq r_v \leq 1$

1) Minimum as function of r_v

$m(r_v)$

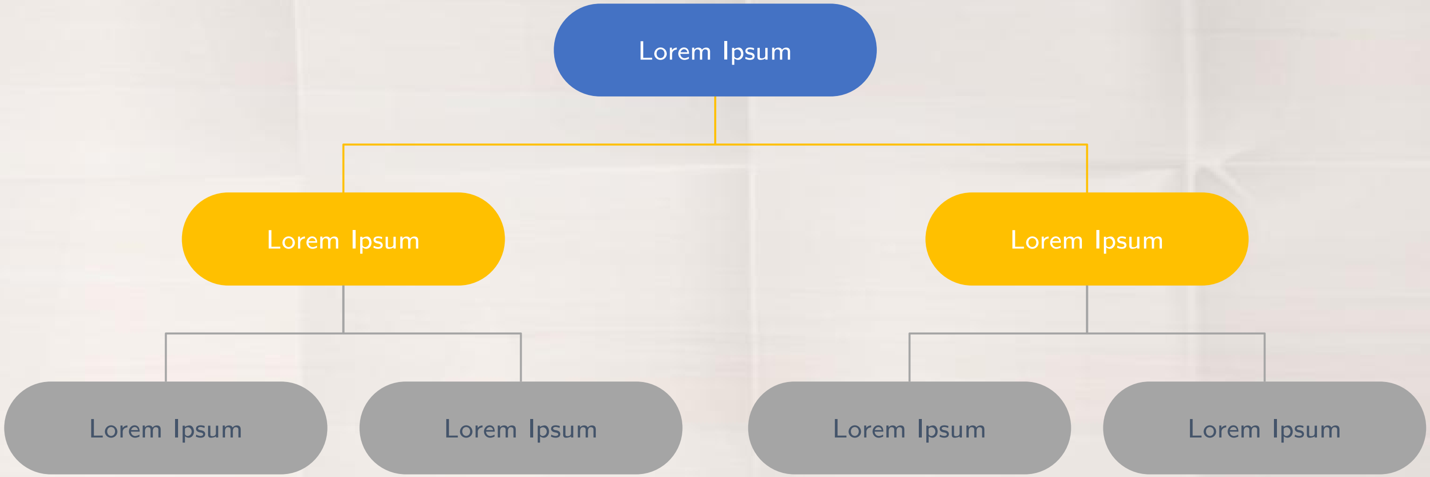
2) Maximum as function of r_v

$M(r_v)$

Absolute minimum $\frac{\sigma_{eff}}{3\pi} \leq \langle b^2 \rangle \leq \frac{3\,\sigma_{eff}}{\pi}$ Absolute maximum

$r_v = 0$ $\quad$ $\mathbf{r_v = 1}$

Use diagrams to explain your ideas



Our process is easy

Vestibulum congue tempus

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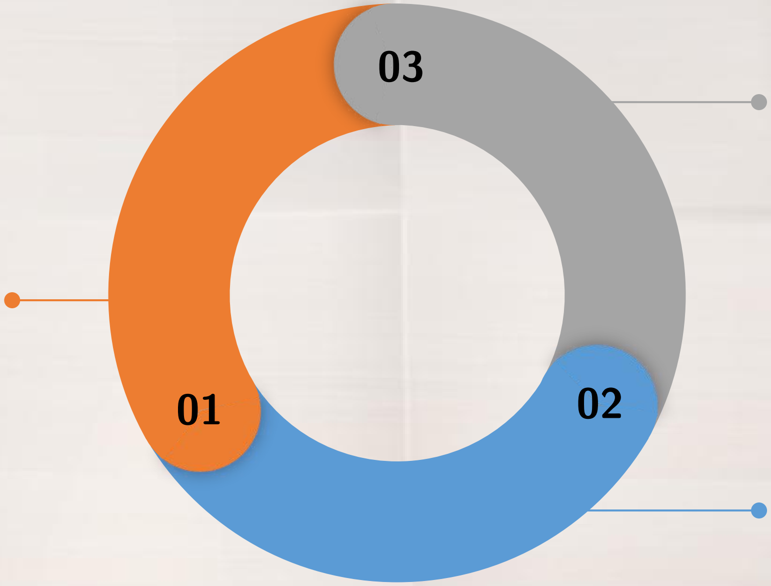

Vestibulum congue tempus

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Vestibulum congue tempus

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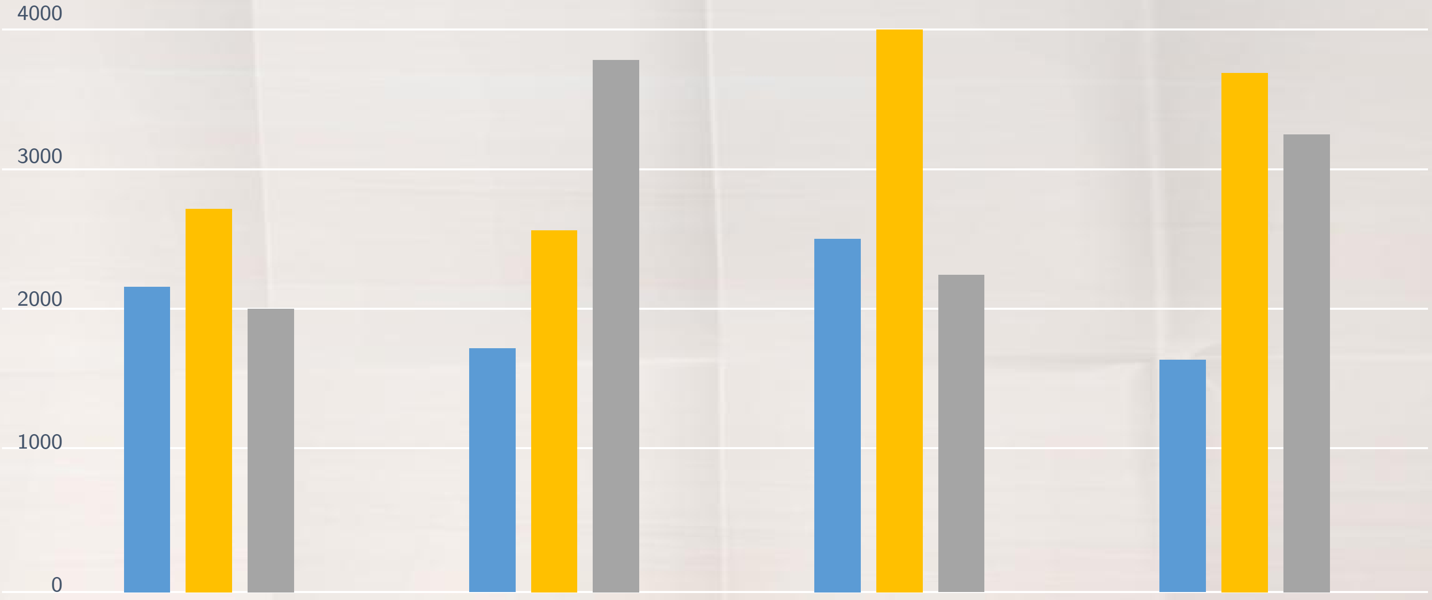

You can insert graphs from Excel or Google Sheets

Desktop project

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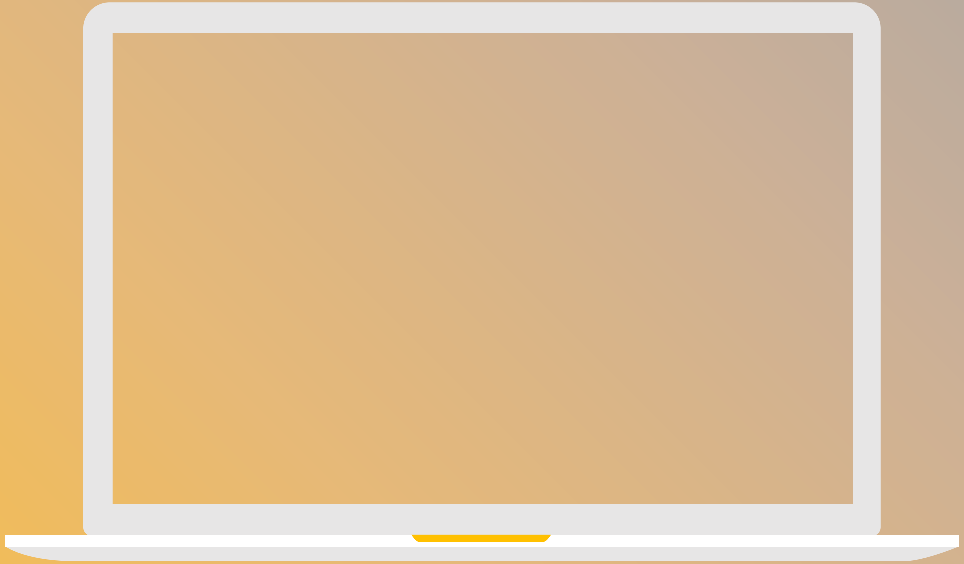
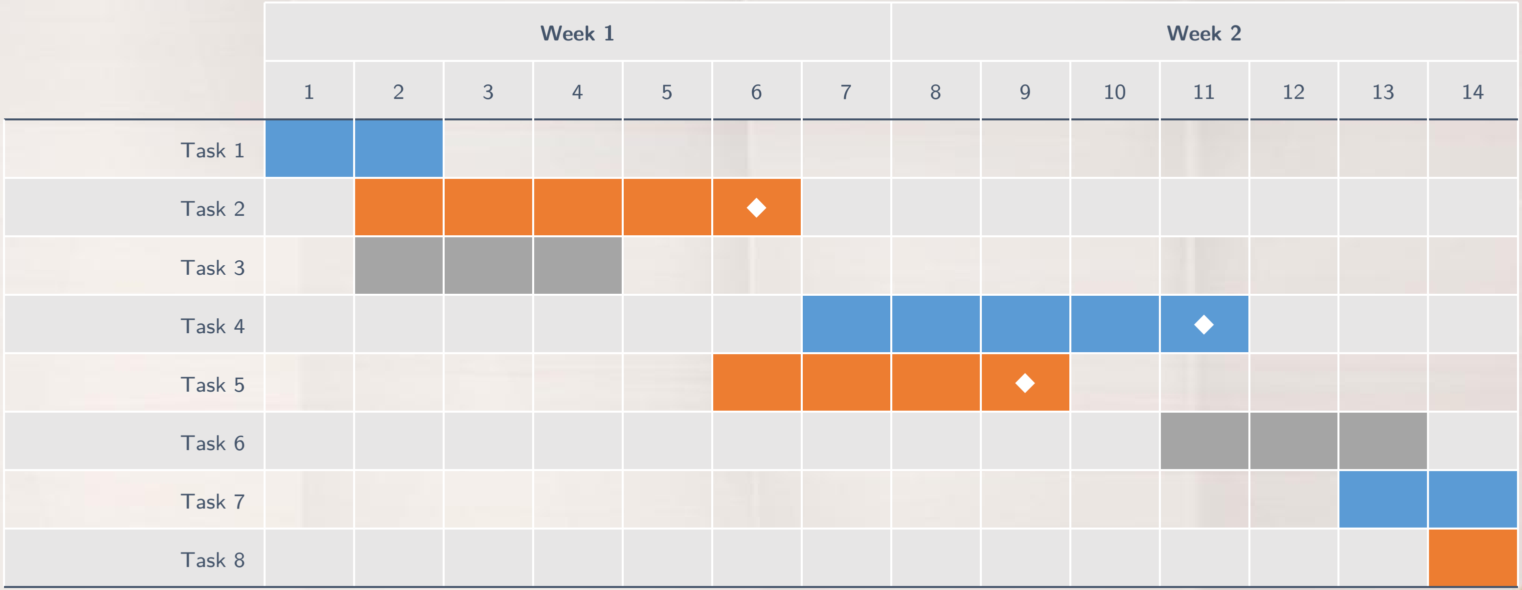


Timeline

- JAN: Blue is the colour of the clear sky and the deep sea
- FEB: Yellow is the color of gold, butter and ripe lemons
- MAR: Red is the colour of danger and courage
- APR: White is the color of milk and fresh snow
- MAY: Black is the color of ebony and of outer space
- JUN: Blue is the colour of the clear sky and the deep sea
- JUL: Yellow is the color of gold, butter and ripe lemons
- AUG: Red is the colour of danger and courage
- SEP: White is the color of milk and fresh snow
- OCT: Black is the color of ebony and of outer space
- NOV: Blue is the colour of the clear sky and the deep sea
- DEC: Yellow is the color of gold, butter and ripe lemons

Gantt chart



Week 1
Week 2
1
2
3
4
5
6
7
8
9
10
11
12
13
14
Task 1
Task 2
Task 3
Task 4
Task 5
Task 6
Task 7
Task 8

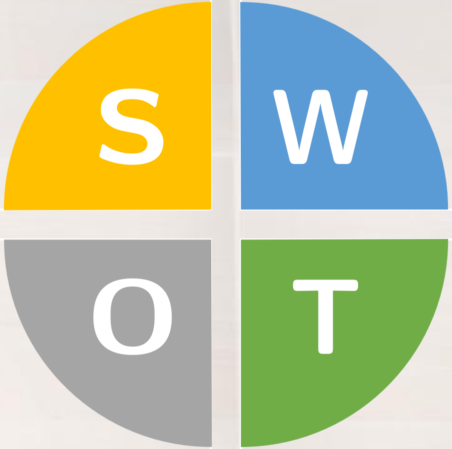
SWOT Analysis

STRENGTHS

Blue is the colour of the clear sky and the deep sea

WEAKNESSES

Yellow is the color of gold, butter and ripe lemons



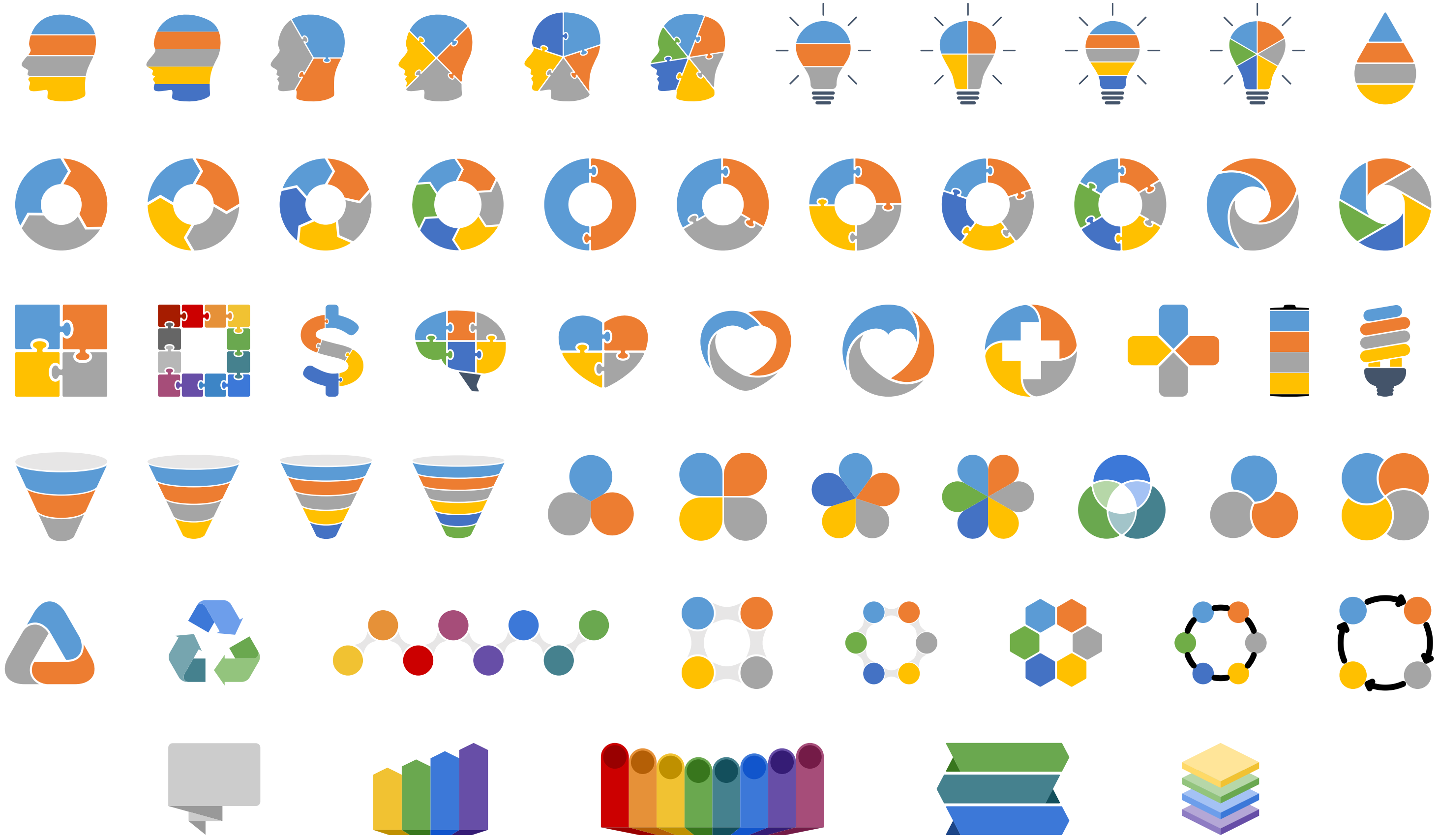
Black is the color of ebony and of outer space

OPPORTUNITIES

White is the color of milk and fresh snow

THREATS

Diagrams and infographics



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