Energy dependence of C-odd color charge correlations in the proton

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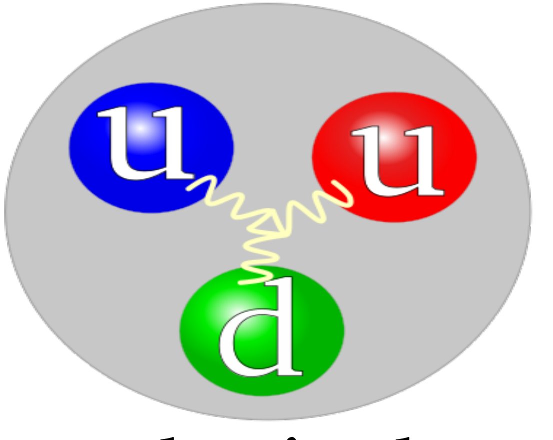
wikipedia



What's that ?

talk based on collaborations with
H. Mäntysaari, R. Paatelainen: 2106.12623, 2210.05390
T. Stebel, V. Skokov: 2001.04516
G. Miller, R. Venugopalan: 1808.02501

Motivation :

* C-odd color charge fluctuations first appear at cubic order in $<\rho^a\rho^b\rho^c>$ correlator ($<\rho^a> \sim \mathrm{tr}\, t^a = 0$; $<\rho^a\rho^b> \sim \mathrm{tr}\, t^a t^b$ is C-even; aside: $<\rho^a\rho^b\rho^c>$ also has C-even terms $\sim i f^{abc}$, not our interest here)

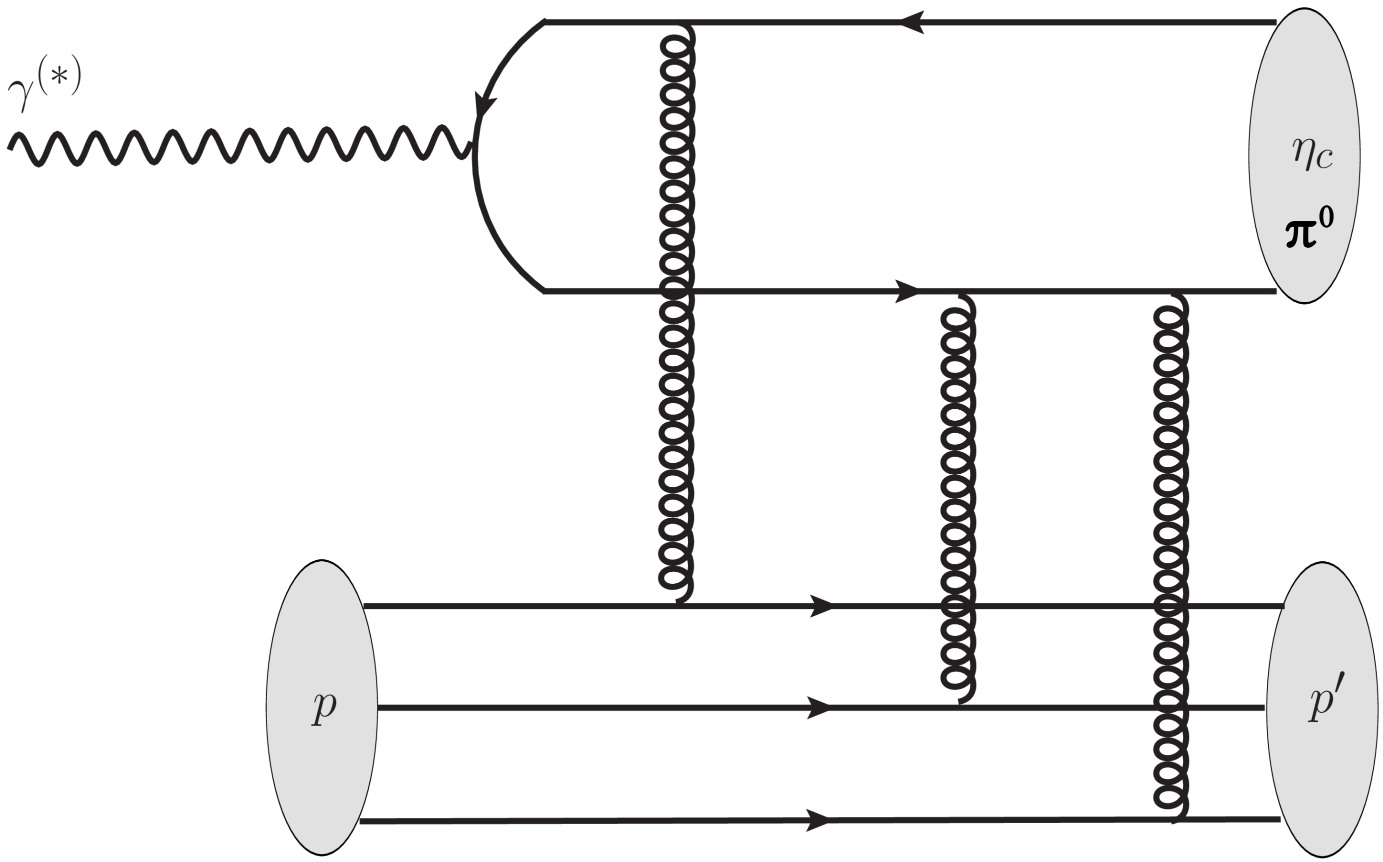
* allows C-odd ggg exchange (Odderon) which appears in a variety of processes; however, low X-sections, see below

It is a basic prediction of QCD but (hard) C-odd ggg exchange has not been clearly seen experimentally, yet

Processes involving the “hard” Odderon :

kinematic regime: hard ggg exchange at x ~ 0.01 – 0.1

* exclusive production of pseudo-scalar mesons in $\gamma^* p \longrightarrow M_{PS}\, p$, large $|t|$
(and high Q^2 in case of π^0 production)

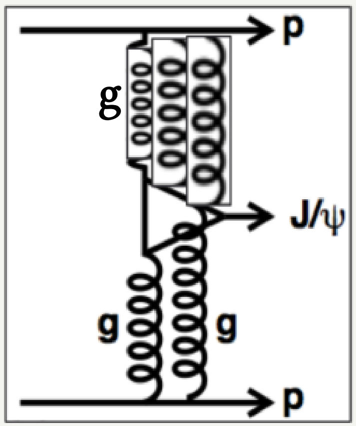


Czyzewski, Kwiecinski, L. Motyka, M. Sadzikowski, PLB 398 (1997);
Engel, Ivanov, Kirschner, Szymanowski, Eur.Phys.J.C4 (1998);
Kilian, Nachtmann, Eur.Phys.J.C5 (1998); A.D, T. Stebel, Phys.Rev.D 99 (2019)

* exclusive production of vector mesons in pp $\longrightarrow M_V\, p$, large $|t|$

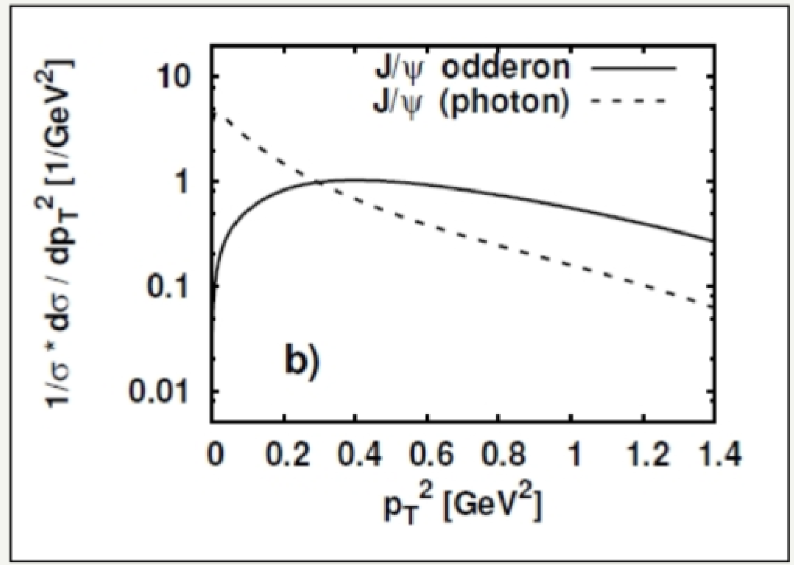
Odderon

Visible in heavy V.M. at high p_T^2?



$$\frac{d\sigma}{dt} \sim e^{bt}$$

Photoproduction: b~6 GeV^{-2}
Proton dissociation b~1 GeV^{-2}
Odderon b small



Bzdak, Motyka, Szymanowski, Cudell
PRD 75 (2007) 094023

$d\sigma^{corr}/dy$	J/ψ	
	odderon	photon
Tevatron	0.3–1.3–5 nb	0.8–5–9 nb
LHC	0.3–0.9–4 nb	2.4–15–27 nb

Given incoherent backgrounds it is likely difficult to attribute a tail to odderon but the comparison of spectra in e-p, pA, AA to pp may be sufficient.

Ronan McNulty, CFNS Workshop: Target fragmentation and diffraction physics with novel processes: Ultraperipheral, electron-ion, and hadron collisions, Feb. 9 – 11, 2022
https://indico.bnl.gov/event/14009/

Fock state description on the light front

The proton on the light front (valence quark Fock state; L.C. time $x^+ = 0$)

$$
\begin{aligned}
|P\rangle \;=\; & \frac{1}{\sqrt{6}} \int \frac{dx_1 dx_2 dx_3}{\sqrt{x_1 x_2 x_3}} \delta(1 - x_1 - x_2 - x_3) \int \frac{d^2k_1 d^2k_2 d^2k_3}{(16\pi^3)^3} 16\pi^3 \delta(\vec{k}_1 + \vec{k}_2 + \vec{k}_3) \\
& \times \psi(k_1, k_2, k_3) \sum_{i_1, i_2, i_3} \epsilon_{i_1 i_2 i_3} |p_1, i_1;\, p_2, i_2;\, p_3, i_3\rangle
\end{aligned}
$$

+ higher Fock states

P. Lepage & Brodsky, 1979 - ....
Brodsky, Pauli, Pinsky, PR (1998)

* start from effective, non-perturbative 3q Fock space amplitude to describe "large"-x structure of the proton (by the way: it's an *entangled state*)

* add corrections (such as |qqqg>) as needed

$\langle\rho^a\rho^b\rho^c\rangle$ correlator (C odd part, LO)

does not vanish (color charge fluct. not Gaussian) :

$$\left\langle \rho^a(\vec q_1)\,\rho^b(\vec q_2)\,\rho^c(\vec q_3)\,\right\rangle_{K_\perp}\Big|_{\mathcal{C}=-} \;\equiv\; \frac{g^3}{4}\,d^{abc}\,G_3^-(\vec q_1,\vec q_2,\vec q_3)$$

$$G_3^-(\vec q_1,\vec q_2,\vec q_3) \;=\; \int [dx_i][dp_i]$$

1-body, positive = $H_q(K_\perp^2)$ →

$$\Big[\psi^*(\vec p_1+(1-x_1)\vec K_\perp,\vec p_2-x_2\vec K_\perp,\vec p_3-x_3\vec K_\perp)$$

$$-\psi^*(\vec p_1-\vec q_1-x_1\vec K_\perp,\vec p_2+\vec q_1+(1-x_2)\vec K_\perp,\vec p_3-x_3\vec K_\perp)$$

2-body, negative →

$$-\psi^*(\vec p_1+\vec q_2+(1-x_1)\vec K_\perp,\vec p_2-\vec q_2-x_2\vec K_\perp,\vec p_3-x_3\vec K_\perp)$$

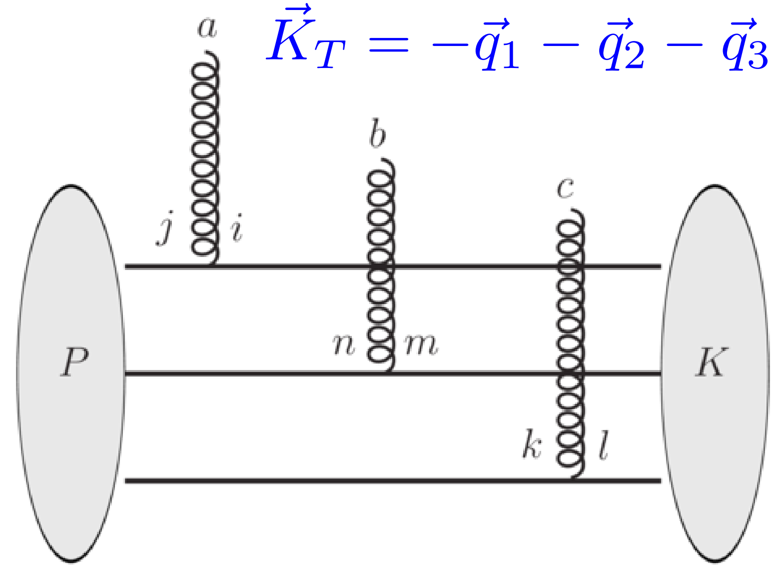
$$-\psi^*(\vec p_1-\vec q_1-\vec q_2-x_1\vec K_\perp,\vec p_2+\vec q_1+\vec q_2+(1-x_2)\vec K_\perp,\vec p_3-x_3\vec K_\perp)$$

3-body, positive →

$$+2\,\psi^*(\vec p_1-\vec q_1-x_1\vec K_\perp,\vec p_2+\vec q_1+\vec q_2+(1-x_2)\vec K_\perp,\vec p_3-\vec q_2-x_3\vec K_\perp)$$

$$\Big]\,\psi(\vec p_1,\vec p_2,\vec p_3)$$

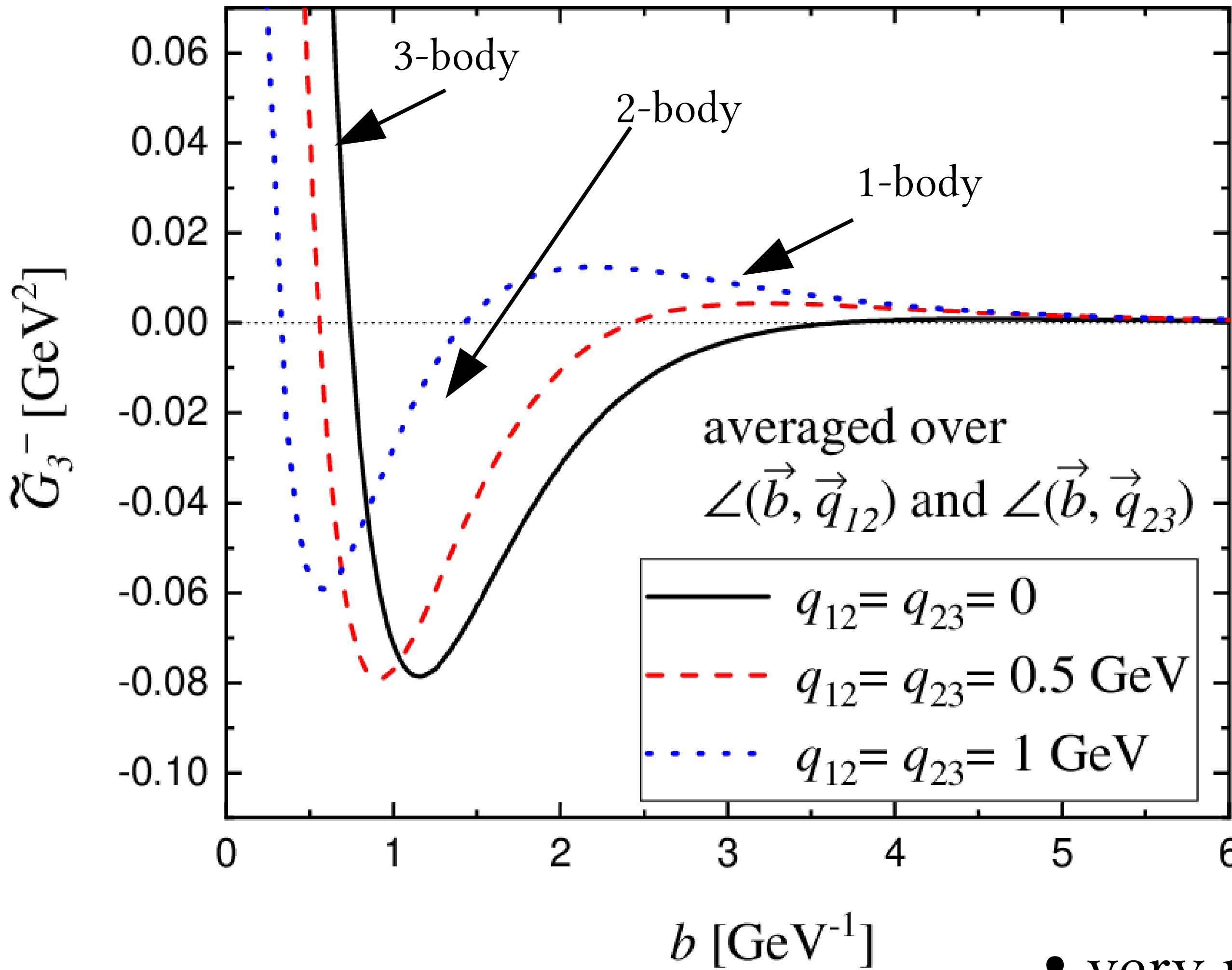
$$\vec K_T = -\vec q_1-\vec q_2-\vec q_3$$



* 1-, 2- and 3-body matrix elements, sum vanishes when either $q_i \longrightarrow 0$ (Ward identities)

* "3-body" diagrams not (power-) suppressed when $\vec q_1 \sim \vec q_2 \sim \vec q_3 \sim -\vec K_T/3 \gg \Lambda_{\rm QCD}$ but actually dominant !

A.D., T. Stebel, V. Skokov, arXiv:2001.04516

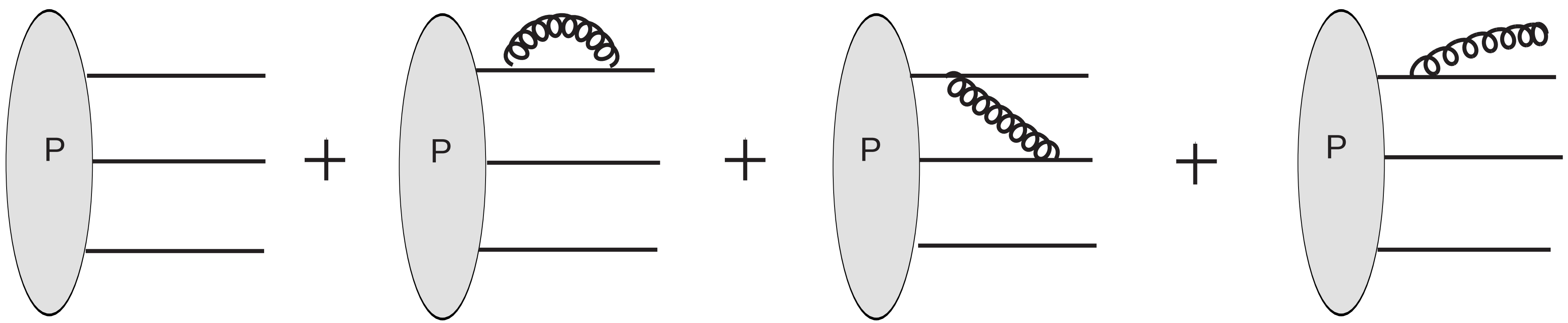


here $q_{ij} = q_i - q_j$ are relative transverse momenta of gluon probes

- very non-trivial structure; non-monotonic, sign changes etc
- diverges for $b \rightarrow 0$ due to contribution from high K_T

Now to $|P\rangle \sim \psi_{qqq}|qqq\rangle + \psi_{qqqg}|qqqg\rangle$

computed in LF perturbation theory (in LC gauge),
1-gluon emission / exchange,
w/o employing small-x approximation

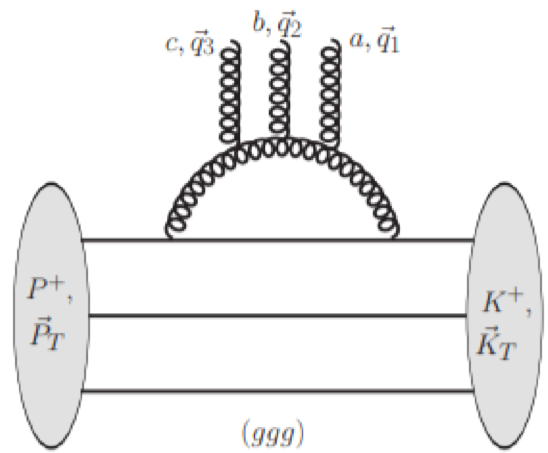


1^{st} perturbative correction to $<\rho^a\rho^b\rho^c>$ correlator

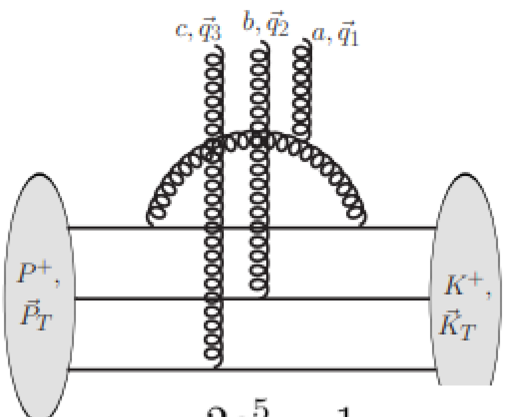
* C-odd contribution to dipole scattering amplitude, "initial condition"
  (small-x evol: Kovchegov, Szymanowski, Wallon, PLB 2004;
  Hatta, Iancu, Itakura, L. McLerran, NPA2005;
  Lappi, Ramnath, Rummukainen, Weigert, PRD 2016)

* Two sample diagrams (~100 more listed in arXiv:2106.12623)

* dependence on cutoff x for gluon + mom.



~ T^a_{bc}, C even



~d^{abc}, C odd
probes couple to higher-x quarks too

$$\frac{2g^5}{3\cdot 16\pi^3}\,\frac{1}{2}N_c\,\mathrm{tr}\left(t^a t^b t^c + t^a t^c t^b\right)\int[\mathrm{d}x_i]\int\left[\mathrm{d}^2k_i\right]\,\Psi_{qqq}(x_1,\vec{k}_1;x_2,\vec{k}_2;x_3,\vec{k}_3)$$
$$\Psi^*_{qqq}(x_1,\vec{k}_1+x_1\vec{q}-\vec{q}_1;x_2,\vec{k}_2+x_2\vec{q}-\vec{q}_2;x_3,\vec{k}_3+x_3\vec{q}-\vec{q}_3)$$
$$\frac{(2\pi)^{D-1}}{2p_1^+}\int\frac{\widetilde{\mathrm{d}k}_g}{2(p_1^+-k_g^+)}\langle S|\hat{\psi}_{q\to qg}(\vec{p}_1;\vec{p}_1-\vec{k}_g,\vec{k}_g)\,\hat{\psi}^*_{q\to qg}(\vec{p}_1-\vec{q}_1;\vec{p}_1-\vec{k}_g,\vec{k}_g-\vec{q}_1)|S\rangle$$

Eikonal dipole scattering amplitude :

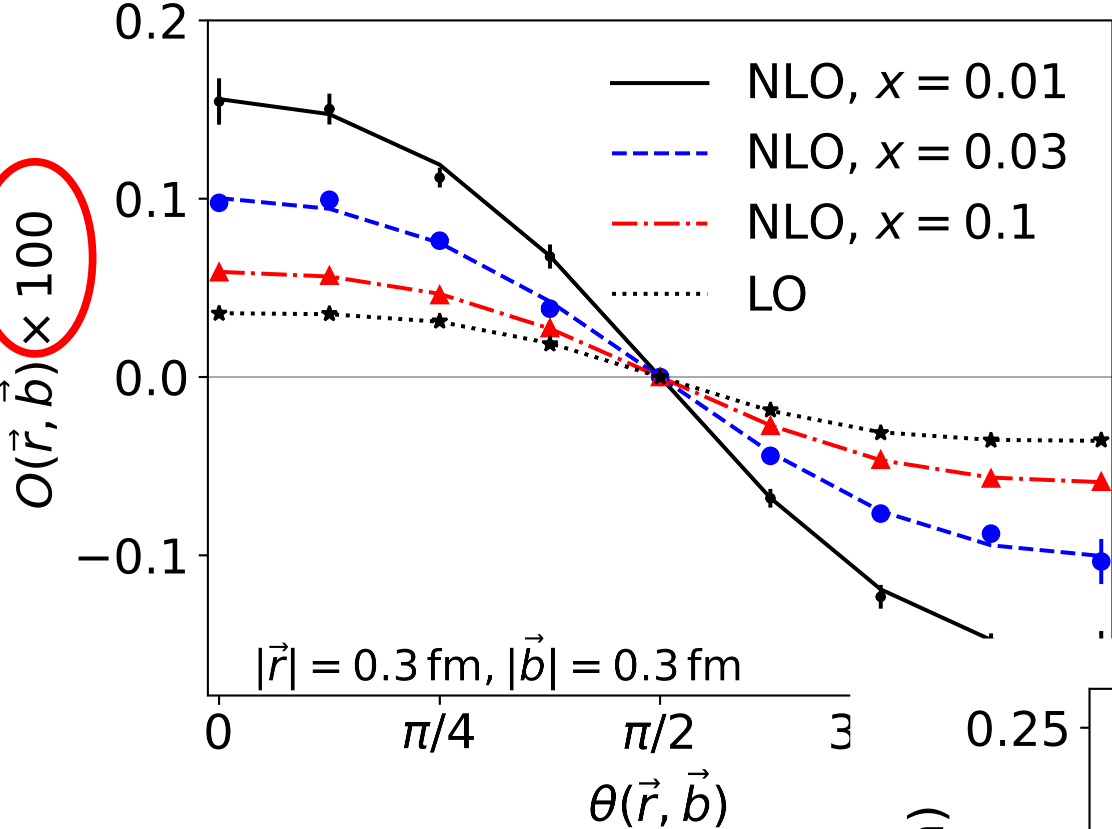
$$\mathcal{S}(\vec{x},\vec{y}) = \frac{1}{N_c}\left\langle \mathrm{tr}\, U(\vec{x})\, U^\dagger(\vec{y})\right\rangle \quad , \quad U(\vec{x}) = \mathcal{P} e^{-ig\int \mathrm{d}x^- A^{+a}(x^-,\vec{x})\, t^a}$$

Imaginary part (C and P odd) in the weak scattering regime :

$$\mathrm{Im}\, S(\vec{r},\vec{b}) = O(\vec{r},\vec{b}) = -\frac{5}{18} g^6 \frac{1}{3} \int_{q_1,q_2,q_3} \frac{1}{q_1^2}\frac{1}{q_2^2}\frac{1}{q_3^2} \sin(\vec{b}\cdot\vec{K})\, G_3^-(\vec{q}_1,\vec{q}_2,\vec{q}_3)$$
$$\left[\sum_{i=1,2,3} \sin\left(\vec{r}\cdot\vec{q}_i + \frac{1}{2}\vec{r}\cdot\vec{K}\right) - \sin\left(\frac{1}{2}\vec{r}\cdot\vec{K}\right)\right]$$

(this is actually a fairly tough 15d integral, for each $\vec{r},\vec{b}$)

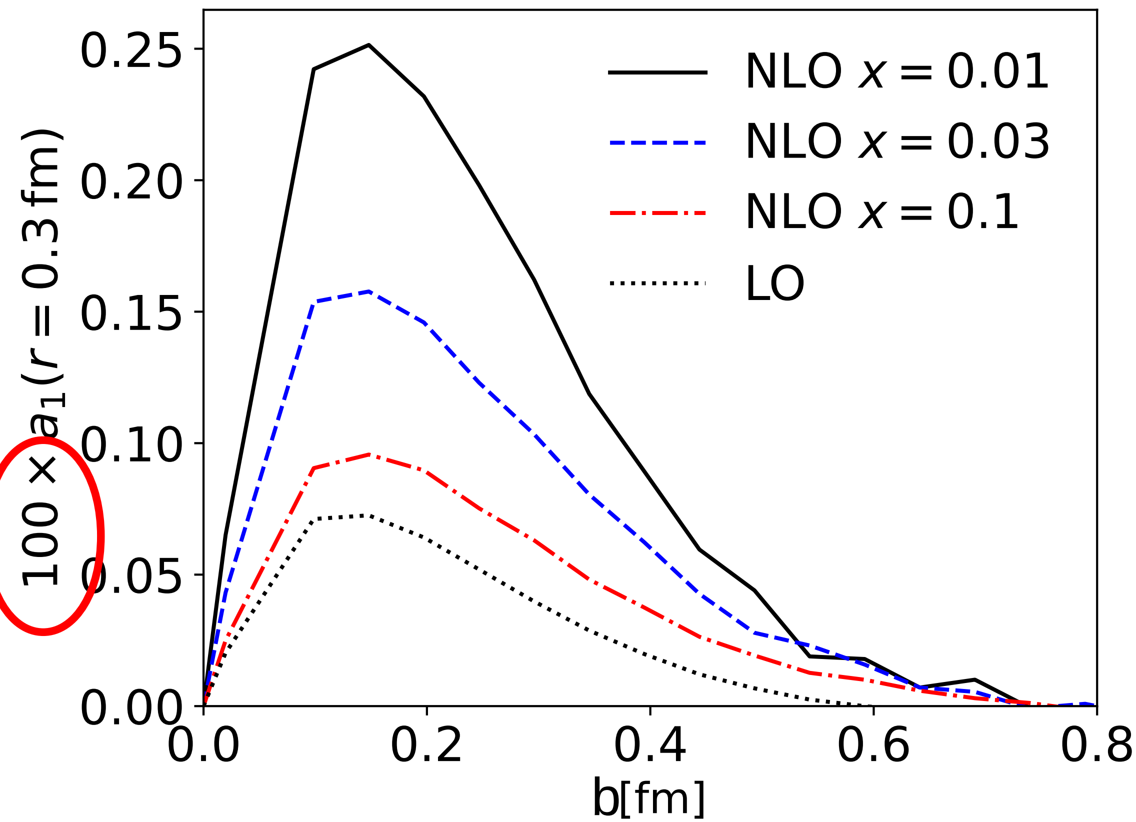
Results to be shown obtained with $\alpha_s = 0.2$, $m_{\mathrm{col}} = 0.2$ GeV
more details in 2210.05390



* dominant angular dependence

$$O(\vec{r}, \vec{b}) = a_1(r, b)\ \cos\theta$$

* note the *increase* of a_1 when gluon is added, and x decreases



* dominant b is much smaller than for Re S: $b_{RMS} \sim 0.6$ fm from HERA J/Ψ data

(Kowalski, Motyka, Watt, PRD 2006)

* note magnitude of a1: scale ~100 times smaller than for Re S

Yet smaller x ... add more soft gluons to proton

$$S_Y(\vec{x},\vec{y}) = \int \mathrm{D}A^+ \, W_Y[A^+] \, \frac{1}{N_c} \mathrm{tr}\, U(\vec{x}) U^\dagger(\vec{y})$$

$$\partial_Y W_Y = -H_{\mathrm{JIMWLK}} \, W_Y$$

numerical JIMWLK evolution of Odderon:
Lappi et al, PRD 2016

large-Nc mean-field approx for small |1-S| :

$$\partial_Y O(\vec{x},\vec{y}) = \frac{\alpha_s N_c}{2\pi^2} \int \mathrm{d}^2\vec{z} \, \frac{(\vec{x}-\vec{y})^2}{(\vec{x}-\vec{z})^2 \, (\vec{z}-\vec{y})^2} \, [O(\vec{x},\vec{z}) + O(\vec{z},\vec{y}) - O(\vec{x},\vec{y})]$$

* for schematic initial conditions, O(r) decreases with x
(see Lappi et al; Yao, Hagiwara, Hatta: PLB 2019)

* asymptotically: energy independent Bartels, Lipatov, Vacca Odderon
(see Kovchegov, Szymanowaki, Wallon: PLB 2004)

small-x RG requires

i) small-x kinematics, sure...

ii) but also, that there is only a "small change" of the weight functional $W_Y[A^+]$ with each addtl soft g

iii) going from $|qqq\rangle \longrightarrow |qqq\rangle + |qqqg\rangle$ is a big modification though if x <~ 0.1 and/or k_T is large

$\longrightarrow$ qualitatively different x-dependence of $O = \mathrm{Im}\, S$

Summary

* computed correlator $<\rho^a\rho^b\rho^c>$ of three color charge operators in a model proton (with "reasonable" quark x, $\mathbf{k}_T$ distributions, color and momentum correlations at x > 0.1)
  + first perturbative (1-gluon emission / exchange) correction, numerically small at x~0.1, increases towards lower x

* color charge correlators on the light front exhibit non-trivial structure as a function of **b**, **r**, their relative azimuth, and x

* if $<\rho^3>_{C=--} \neq 0$ is seen ⟶ evidence for C-odd, non-Gaussian color charge correlations in the proton at sub-femtometer scales !

* O(**b**, **r**) = Im S predicted to increase as x: ~0.1 ⟶ ~0.01, should eventually turn into Odderon evolution predicted by small-x RG. However, it is ~ 100 times smaller than gg exchange, 1 – Re S.

Thank you !

Backup Slides

Model LFwf for the |qqq> state of the proton:

(Brodsky & Schlumpf, PLB 329, 1994)

$$\begin{aligned}\psi_{\rm H.O.}(x_1,\vec{k}_1;x_2,\vec{k}_2;x_3,\vec{k}_3) &= N_{\rm H.O.}\exp(-\mathcal{M}^2/2\beta^2)\ , \\ \psi_{\rm Power}(x_1,\vec{k}_1;x_2,\vec{k}_2;x_3,\vec{k}_3) &= N_{\rm Power}(1+\mathcal{M}^2/\beta^2)^{-p}\ .\end{aligned}$$

$$\mathcal{M}^2 = \sum_{i=1}^{3} \frac{\vec{k}^2_{\perp i} + m^2}{x_i}$$

m = 0.26 GeV, β = 0.55 for H.O. wf

m = 0.263, β = 0.607, p = 3.5 for PWR wf

With these parameters they fit:

- proton radius $R^2 = -6\ \frac{dF_1(Q^2)}{dQ^2}\Big|_{Q^2=0} = (0.76\ {\rm fm})^2$
- proton / neutron magnetic moments $1 + F_2(Q^2 \to 0) = 2.81\ /\ -1.66$
- axial vector coupling g_A = 1.25