





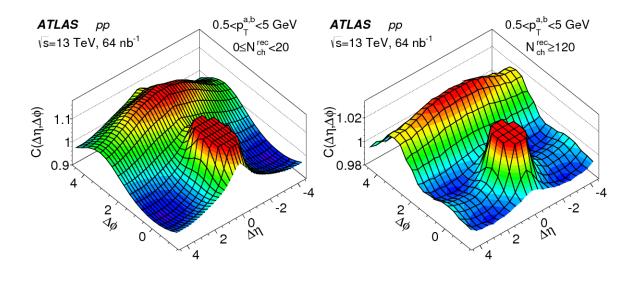


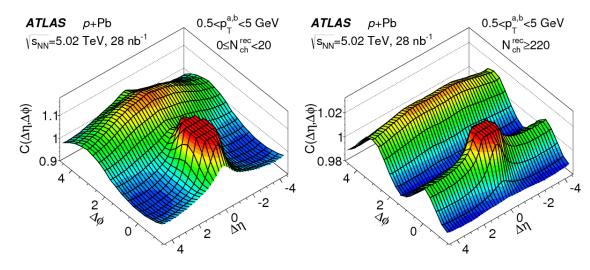


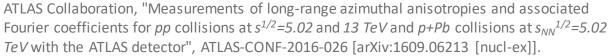
SURFACING ODD HARMONICS IN TWO PARTICLE CORRELATIONS WITHIN THE COLOR GLASS CONDENSATE APPROACH

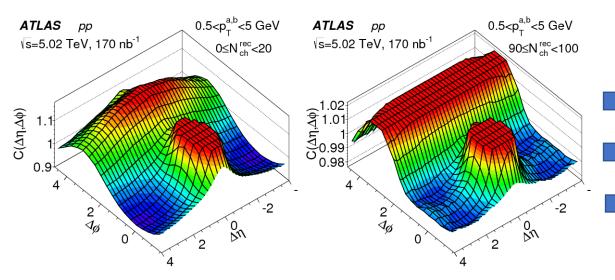
Víctor Vila

in collaboration with Cyrille Marquet and Anderson Kendi h International Workshop on Multiple Partonic Interactions at the LHC, 14-18 November 2022









One of the main unveilings of the CMS collaboration.

First noticed in relativistic heavy-ion data from RHIC.

An incentive to seek the origin of particle correlations.



Mechanisms behind the ridge in pp, pA and AA collisions:

- HICs: collective behaviour of the gluons cloud's final state.
- Small systems: hydrodynamical evolution logic consistent with data.
  - Particle correlations: roots in the very early stages of the collision.



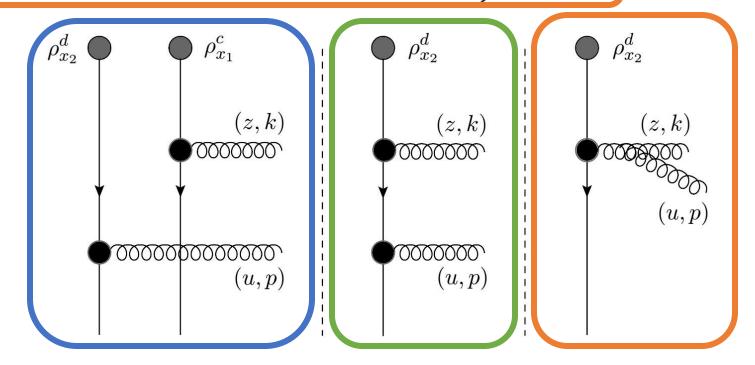
- Extremely effective.
- Anisotropic distribution of particles motivates decomposition in Fourier harmonics with the corresponding coefficients  $v_n$ :
  - Rise of even Fourier harmonics.
  - **ABSENCE OF ODD AZIMUTHAL HARMONICS.**

$$A_{ij}^{ab}(k,p) = \int_{u,z} e^{ik\cdot z + ip\cdot u} \left\{ \int_{x_1,x_2} \left\{ f_i(z - x_1)[S_z - S_{x_1}]^{ac} \rho_{x_1}^c \right\} \left\{ f_j(u - x_2)[S_u - S_{x_2}]^{bd} \rho_{x_2}^d \right\} \right\}$$

$$-\frac{1}{2} \int_{x_1} f_i(z - x_1) f_j(u - x_1) \Big\{ [S_z - S_{x_1}] \bar{\rho}_{x_1} [S_u^{\dagger} + S_{x_1}^{\dagger}] \Big\}^{ab} \Big\}$$

$$+ \int_{x_1} f_i(z-u) f_j(u-x_1) \left\{ [S_z - S_u] \bar{\rho}_{x_1} S_u^{\dagger} \right\}^{ab} \right\},$$

A. Kovner and M. Lublinsky, "Angular Correlations in Gluon Production at High Energy", Phys.Rev. D83 (2011) 034017 [arXiv:1012.3398 [hep-ph]].



$$\frac{dN}{d^2pd^2kd\eta d\xi} = \boxed{\sigma^4} + \boxed{\sigma^3} + \boxed{\sigma^2}_{P,T}$$

A. Kovner and M. Lublinsky, "Angular Correlations in Gluon Production at High Energy", Phys.Rev. D83 (2011) 034017 [arXiv:1012.3398 [hep-ph]].

$$\sigma^{4} = \int_{u,z,\bar{u},\bar{z}} e^{ik\cdot(z-\bar{z})+ip\cdot(u-\bar{u})} \int_{x_{1},x_{2},\bar{x}_{1},\bar{x}_{2}} f(\bar{z}-\bar{x}_{1}) \cdot f(z-x_{1}) \ f(\bar{u}-\bar{x}_{2}) \cdot f(u-x_{2})$$

$$\times \left\{ \rho_{x_{1}} [S_{z}^{\dagger} - S_{x_{1}}^{\dagger}] [S_{\bar{z}} - S_{\bar{x}_{1}}] \rho_{\bar{x}_{1}} \right\} \left\{ \rho_{x_{2}} [S_{u}^{\dagger} - S_{x_{2}}^{\dagger}] [S_{\bar{u}} - S_{\bar{x}_{2}}] \rho_{\bar{x}_{2}} \right\}$$

$$\sigma^{3} = \int_{u,z,\bar{u},\bar{z}} e^{ik\cdot(z-\bar{z})+ip\cdot(u-\bar{u})} \int_{x_{1},\bar{x}_{1},x_{2}(\bar{x}_{2})} \\ -\frac{1}{2} f(\bar{z}-\bar{x}_{1}) \cdot f(z-x_{1}) f(\bar{u}-\bar{x}_{1}) \cdot f(u-x_{2}) \\ \times \operatorname{Tr} \left\{ \bar{\rho}_{x_{1}} [S_{z}^{\dagger} - S_{x_{1}}^{\dagger}] [S_{\bar{z}} - S_{\bar{x}_{1}}] \bar{\rho}_{\bar{x}_{1}} [S_{\bar{u}}^{\dagger} + S_{\bar{x}_{1}}^{\dagger}] [S_{u} - S_{x_{2}}] \bar{\rho}_{x_{2}} \right\} \\ +\frac{1}{2} f(\bar{z}-\bar{x}_{1}) \cdot f(z-x_{1}) f(\bar{u}-\bar{x}_{2}) \cdot f(u-x_{1}) \\ \times \operatorname{Tr} \left\{ \bar{\rho}_{\bar{x}_{2}} [S_{\bar{u}}^{\dagger} - S_{\bar{x}_{2}}^{\dagger}] [S_{u} + S_{x_{1}}] \bar{\rho}_{x_{1}} [S_{z}^{\dagger} - S_{x_{1}}^{\dagger}] [S_{\bar{z}} - S_{\bar{x}_{1}}] \bar{\rho}_{\bar{x}_{1}} \right\} \\ +f(\bar{z}-\bar{u}) \cdot f(z-x_{1}) f(\bar{u}-\bar{x}_{1}) \cdot f(u-x_{2}) \\ \times \operatorname{Tr} \left\{ \bar{\rho}_{x_{1}} [S_{z}^{\dagger} - S_{x_{1}}^{\dagger}] [S_{\bar{z}} - S_{\bar{u}}] \bar{\rho}_{\bar{x}_{1}} S_{\bar{u}}^{\dagger} [S_{u} - S_{x_{2}}] \bar{\rho}_{x_{2}} \right\} \\ -f(\bar{z}-\bar{x}_{1}) \cdot f(z-u) f(\bar{u}-\bar{x}_{2}) \cdot f(u-x_{1}) \\ \times \operatorname{Tr} \left\{ \bar{\rho}_{\bar{x}_{2}} [S_{\bar{u}}^{\dagger} - S_{\bar{x}_{2}}^{\dagger}] S_{u} \bar{\rho}_{x_{1}} [S_{z}^{\dagger} - S_{u}^{\dagger}] [S_{\bar{z}} - S_{\bar{x}_{1}}] \bar{\rho}_{\bar{x}_{1}} \right\}$$

$$\sigma^{2} = \int_{u,z,\bar{u},\bar{z}} e^{ik\cdot(z-\bar{z})+ip\cdot(u-\bar{u})} \int_{x_{1},\bar{x}_{1}} \frac{1}{4} f(\bar{z}-\bar{x}_{1}) \cdot f(z-x_{1}) f(\bar{u}-\bar{x}_{1}) \cdot f(u-x_{1})$$

$$\times \operatorname{Tr} \left\{ [S_{u}+S_{x_{1}}] \bar{\rho}_{x_{1}} [S_{z}^{\dagger}-S_{x_{1}}^{\dagger}] [S_{\bar{z}}-S_{\bar{x}_{1}}] \bar{\rho}_{\bar{x}_{1}} [S_{\bar{u}}^{\dagger}+S_{\bar{x}_{1}}^{\dagger}] \right\}$$

$$+ f(\bar{z}-\bar{u}) \cdot f(z-u) f(\bar{u}-\bar{x}_{1}) \cdot f(u-x_{1})$$

$$\times \operatorname{Tr} \left\{ S_{u} \bar{\rho}_{x_{1}} [S_{z}^{\dagger}-S_{u}^{\dagger}] [S_{\bar{z}}-S_{\bar{u}}] \bar{\rho}_{\bar{x}_{1}} S_{\bar{u}}^{\dagger} \right\}$$

$$- \frac{1}{2} f(\bar{z}-\bar{x}_{1}) \cdot f(z-u) f(\bar{u}-\bar{x}_{1}) \cdot f(u-x_{1})$$

$$\times \operatorname{Tr} \left\{ S_{u} \bar{\rho}_{x_{1}} [S_{z}^{\dagger}-S_{u}^{\dagger}] [S_{\bar{z}}-S_{\bar{x}_{1}}] \bar{\rho}_{\bar{x}_{1}} [S_{\bar{u}}^{\dagger}+S_{\bar{x}_{1}}^{\dagger}] \right\}$$

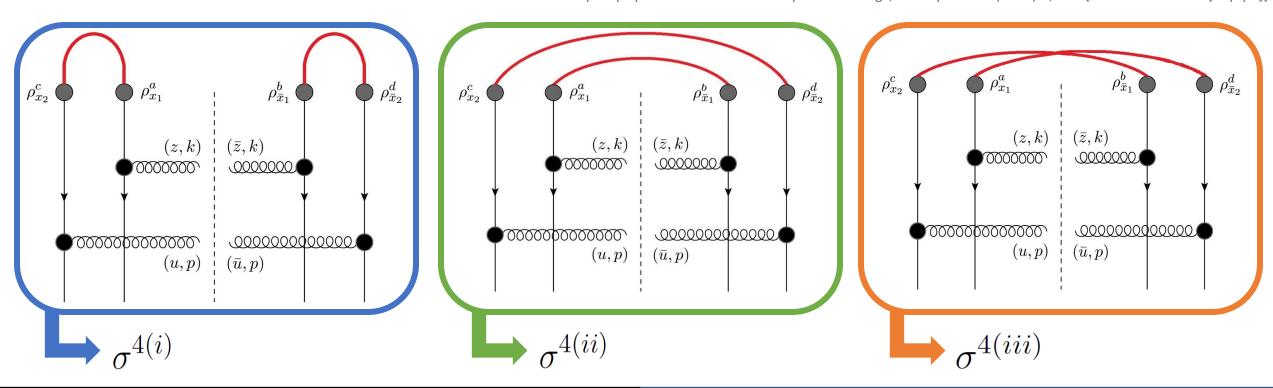
$$- \frac{1}{2} f(\bar{z}-\bar{u}) \cdot f(z-x_{1}) f(\bar{u}-\bar{x}_{1}) \cdot f(u-x_{1})$$

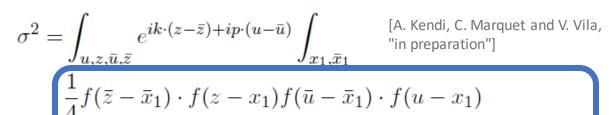
$$\times \operatorname{Tr} \left\{ [S_{u}+S_{x_{1}}] \bar{\rho}_{x_{1}} [S_{z}^{\dagger}-S_{x_{1}}^{\dagger}] [S_{\bar{z}}-S_{\bar{u}}] \bar{\rho}_{\bar{x}_{1}} S_{\bar{u}}^{\dagger} \right\}$$

$$\left\langle \hat{\rho}_{x_2}^c \hat{\rho}_{x_1}^a \hat{\rho}_{\bar{x}_2}^d \hat{\rho}_{\bar{x}_1}^b \right\rangle_P = \left\langle \hat{\rho}_{x_2}^c \hat{\rho}_{x_1}^a \right\rangle_P \left\langle \hat{\rho}_{\bar{x}_2}^d \hat{\rho}_{\bar{x}_1}^b \right\rangle_P + \left\langle \hat{\rho}_{x_2}^c \hat{\rho}_{\bar{x}_1}^d \right\rangle_P \left\langle \hat{\rho}_{x_1}^a \hat{\rho}_{\bar{x}_1}^b \right\rangle_P + \left\langle \hat{\rho}_{x_2}^c \hat{\rho}_{\bar{x}_1}^d \right\rangle_P \left\langle \hat{\rho}_{x_1}^a \hat{\rho}_{\bar{x}_2}^b \right\rangle_P = \delta^{ab} \mu^2 \delta^{(2)}(x_1 - x_2) + \left\langle \hat{\rho}_{x_2}^c \hat{\rho}_{\bar{x}_1}^b \right\rangle_P \left\langle \hat{\rho}_{x_1}^a \hat{\rho}_{\bar{x}_2}^d \right\rangle_P$$

## **MV MODEL PROJECTILE AVERAGING**

T. Altinoluk, N. Armesto, A. Kovner and M. Lublinsky, "Double and triple inclusive gluon production at mid rapidity: quantum interference in p-A scattering", Eur. Phys. J. C78 (2018) 9, 702 [arXiv:1805.07739 [hep-ph]].





$$\times \operatorname{Tr} \left\{ [S_u + S_{x_1}] \bar{\rho}_{x_1} [S_z^{\dagger} - S_{x_1}^{\dagger}] [S_{\bar{z}} - S_{\bar{x}_1}] \bar{\rho}_{\bar{x}_1} [S_{\bar{u}}^{\dagger} + S_{\bar{x}_1}^{\dagger}] \right\}$$

$$+ f(\bar{z} - \bar{u}) \cdot f(z - u) f(\bar{u} - \bar{x}_1) \cdot f(u - x_1)$$

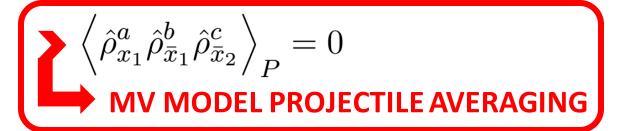
$$\times \operatorname{Tr} \left\{ S_u \bar{\rho}_{x_1} [S_z^{\dagger} - S_u^{\dagger}] [S_{\bar{z}} - S_{\bar{u}}] \bar{\rho}_{\bar{x}_1} S_{\bar{u}}^{\dagger} \right\}$$

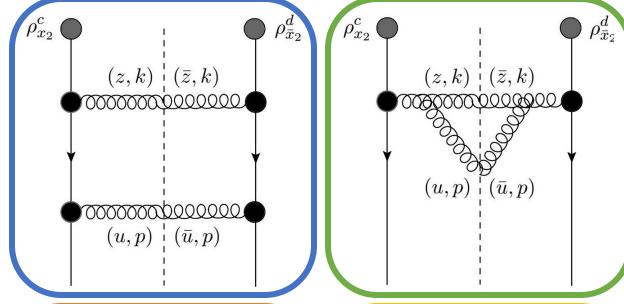
$$-\frac{1}{2}f(\bar{z}-\bar{x}_1)\cdot f(z-u)f(\bar{u}-\bar{x}_1)\cdot f(u-x_1)$$

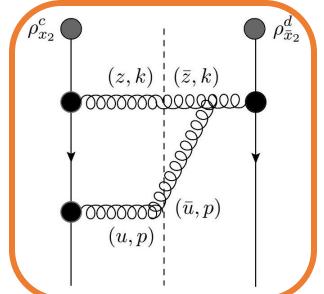
$$\times \operatorname{Tr} \left\{ S_{u} \bar{\rho}_{x_{1}} [S_{z}^{\dagger} - S_{u}^{\dagger}] [S_{\bar{z}} - S_{\bar{x}_{1}}] \bar{\rho}_{\bar{x}_{1}} [S_{\bar{u}}^{\dagger} + S_{\bar{x}_{1}}^{\dagger}] \right\}$$

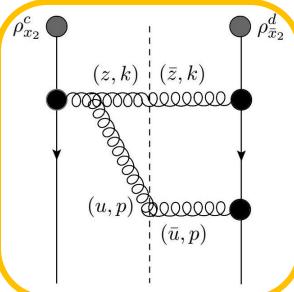
$$-\frac{1}{2}f(\bar{z}-\bar{u})\cdot f(z-x_1)f(\bar{u}-\bar{x}_1)\cdot f(u-x_1)$$

$$\times \operatorname{Tr} \left\{ [S_{u} + S_{x_{1}}] \bar{\rho}_{x_{1}} [S_{z}^{\dagger} - S_{x_{1}}^{\dagger}] [S_{\bar{z}} - S_{\bar{u}}] \bar{\rho}_{\bar{x}_{1}} S_{\bar{u}}^{\dagger} \right\}$$









$$\begin{split} \tilde{D}(\bar{u},u) &= \left\langle \frac{1}{N^2 - 1} \mathrm{Tr} \left[ S^{\dagger}(\bar{u}) S(u) \right] \right\rangle_T \\ \tilde{Q}(\bar{u},u,z,\bar{z}) &= \left\langle \frac{1}{N^2 - 1} \mathrm{Tr} \left[ S^{\dagger}(\bar{u}) S(u) S^{\dagger}(z) S(\bar{z}) \right] \right\rangle_T \end{split}$$

**AEA** 

LARGE N<sub>C</sub>

$$\tilde{D}(\bar{u}, u) = D^2(\bar{u}, u)$$

$$\tilde{Q}(\bar{u}, u, z, \bar{z}) \approx D^2(\bar{u}, u)D^2(z, \bar{z}) + D^2(\bar{u}, \bar{z})D^2(u, z)$$

$$\sigma^{4(ii)} = \int_{u,z,\bar{u},\bar{z}} e^{ik\cdot(z-\bar{z})+ip\cdot(u-\bar{u})} \int_{x_1,x_2} f(\bar{z}-x_1) \cdot f(z-x_1) \ f(\bar{u}-x_2) \cdot f(u-x_2)$$

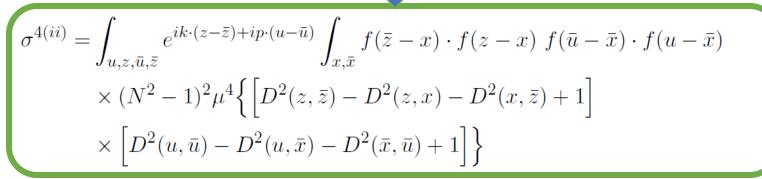
$$\times (N^2-1)^2 \mu^4 \Big\{ \tilde{D}(z,\bar{z}) \tilde{D}(u,\bar{u}) - \tilde{D}(z,\bar{z}) \tilde{D}(u,x_2) - \tilde{D}(z,\bar{z}) \tilde{D}(x_2,\bar{u})$$

$$+ \tilde{D}(z,\bar{z}) - \tilde{D}(x_1,\bar{z}) \tilde{D}(u,\bar{u}) + \tilde{D}(x_1,\bar{z}) \tilde{D}(u,x_2) + \tilde{D}(x_1,\bar{z}) \tilde{D}(x_2,\bar{u})$$

$$- \tilde{D}(x_1,\bar{z}) - \tilde{D}(z,x_1) \tilde{D}(u,\bar{u}) + \tilde{D}(z,x_1) \tilde{D}(u,x_2) + \tilde{D}(z,x_1) \tilde{D}(x_2,\bar{u})$$

$$- \tilde{D}(z,x_1) + \tilde{D}(u,\bar{u}) - \tilde{D}(u,x_2) - \tilde{D}(x_2,\bar{u}) + 1 \Big\}$$

$$f(x-y) = \frac{g_s}{2\pi} \frac{(x-y)^i}{(x-y)^2} = \frac{g_s}{2\pi} \int \frac{d^2q}{2\pi i} \frac{q^i}{q^2} e^{iq \cdot (x-y)}$$
$$D(r) = \int \frac{d^2q}{\pi} e^{-iq \cdot r} \frac{1}{Q_s^2} e^{-\frac{q^2}{Q_s^2}}$$
$$D^2(r) = \int \frac{d^2q}{2\pi} e^{-iq \cdot r} \frac{1}{Q_s^2} e^{-\frac{q^2}{2Q_s^2}}$$



THE WEIZSACKER-WILLIAMS FIELDS AND **GBW MODEL FOR THE DIPOLES** 

$$\sigma^{4([i+iii].I)} \approx \alpha_s^2 (4\pi)^2 (N^2 - 1) \mu^4 S_{\perp} \frac{2\pi}{Q_s^2} e^{-\frac{(k+p)^2}{4Q_s^2}}$$

$$\times \left[ \frac{2\left(k^4 + p^4 + 2\left(k \cdot p\right)^2\right)}{k^2 p^2 \left(k - p\right)^4} + \frac{8Q_s^2 \left(k + p\right)^4}{k^2 p^2 \left(k - p\right)^6} + \frac{64Q_s^4 \left(k^4 + 4(k \cdot p)^2 + p^4 + 8(k \cdot p)(k^2 + p^2) + 14k^2 p^2\right)}{k^2 p^2 \left(k - p\right)^8} \right]$$

$$+ \{(p \to -p)\}$$

$$\sigma^{4([i+iii].II)} \approx \alpha_s^2 (4\pi)^2 (N^2 - 1) \mu^4 S_{\perp} (2\pi)^2 \left[ \delta^{(2)} (k+p) + \delta^{(2)} (k-p) \right]$$
$$\times \frac{4}{k^{12}} e^{-\frac{k^2}{Q_s^2}} \left( k^4 + k^2 e^{\frac{k^2}{2Q_s^2}} Q_s^2 + 4 e^{\frac{k^2}{2Q_s^2}} Q_s^4 \right)^2$$

$$\sigma^{4(ii)} = \alpha_s^2 (4\pi)^2 (N^2 - 1)^2 \mu^4 S_{\perp}^2$$

$$\times e^{-\frac{k^2}{2Q_s^2}} \left\{ \frac{2}{k^2} - \frac{1}{k^2} e^{\frac{k^2}{2Q_s^2}} + \frac{1}{2Q_s^2} \left[ \text{Ei} \left( \frac{k^2}{2Q_s^2} \right) - \text{Ei} \left( \frac{k^2 \lambda}{2Q_s^2} \right) \right] \right\}$$

$$\times \{ (k \to p) \}$$

- THE FULLY CORRELATED PIECE **ENCAPSULATING TWO EFFECTS:** 
  - **BOSE ENHANCEMENT OF GLUONS** IN THE WAVE FUNCTION OF THE INCOMING HADRONS
  - **HANBURY-BROWN-TWISS** INTERFERENCE EFFECT IN THE **EMISSION OF GLUONS**

[A. Kendi, C. Marquet and V. Vila, "in preparation"]

THE UNCORRELATED PIECE **CORRESPONDING TO THE SQUARE** OF THE SINGLE INCLUSIVE GLUON **PRODUCTION CROSS-SECTION** 

$$\sigma^{2(i)} = \alpha_s^2 (4\pi)^2 N^3 \mu^2 S_{\perp} \frac{1}{k^6 p^6} e^{-\frac{k^2 + p^2}{2Q_s^2}}$$

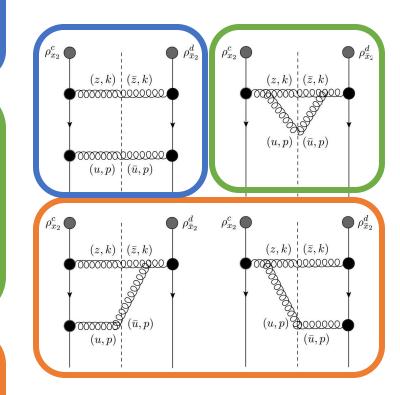
$$\times \left[ k^4 + e^{\frac{k^2}{2Q_s^2}} k^2 Q_s^2 + 4e^{\frac{k^2}{2Q_s^2}} Q_s^4 \right] \left[ \left( 2e^{\frac{p^2}{2Q_s^2}} - 1 \right) p^4 + e^{\frac{p^2}{2Q_s^2}} p^2 Q_s^2 + 4e^{\frac{p^2}{2Q_s^2}} Q_s^4 \right]$$

$$\sigma^{2(ii)} = \alpha_s^2 (4\pi)^2 N^3 \mu^2 S_{\perp} \left\{ \frac{1}{(2\pi)^2 Q_s^4} e^{-\frac{k^2 + p^2}{2Q_s^2}} \int_{s_1, s_2} \frac{1}{s_1^2} \frac{1}{(s_1 - s_2)^2} e^{-\frac{s_1^2 + 2k \cdot s_1}{2Q_s^2}} e^{-\frac{s_2^2 - 2p \cdot s_2}{2Q_s^2}} \right.$$

$$+ \frac{1}{2\pi^2 Q_s^4} e^{-\frac{k^2 + p^2}{2Q_s^2}} \int_{s_1, s_2} \frac{s_1^i}{s_1^2} \frac{1}{s_2^2} e^{-\frac{2s_1^2 + 2(k - p) \cdot s_1}{2Q_s^2}} e^{-\frac{s_2^2 - 2(s_1 - p) \cdot s_2}{2Q_s^2}}$$

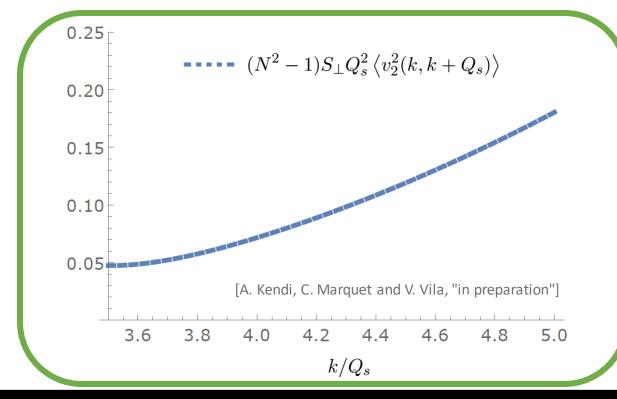
$$+ \frac{1}{(k + p)^2} \left[ 1 + \frac{1}{2} \frac{2^3 Q_s^2}{(k + p)^2} + \frac{1}{2} \frac{2^6 Q_s^4}{(k + p)^4} \right] \right\}$$

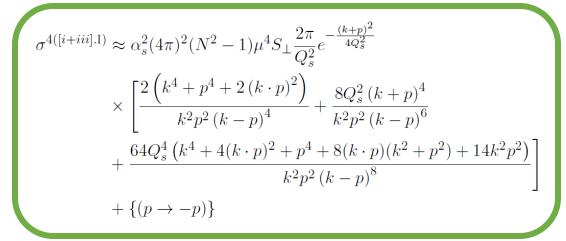
$$\sigma^{2[(iii)+(iv)]} = \alpha_s^2 (4\pi)^2 N^3 \mu^2 S_{\perp} \left\{ \frac{1}{(2\pi)^2 Q_s^4} e^{-\frac{k^2 + p^2}{2Q_s^2}} \int_{s_1, s_2} \left[ \left( \frac{s_1^i}{s_1^2} + \frac{k^i}{k^2} \right)^2 \left( \frac{s_2^j}{s_2^2} + \frac{p^j}{p^2} \right) - \frac{1}{k^2} \frac{p^j}{p^2} \right] \times e^{-\frac{s_1^2 + 2k \cdot s_1}{2Q_s^2}} e^{-\frac{s_2^2 - 2p \cdot s_2}{2Q_s^2}} + \frac{1}{k^2} \frac{p \cdot (k+p)}{p^2 (k+p)^2} \left( e^{-\frac{(k+p)^2}{4Q_s^2}} - 1 \right) \right\}$$



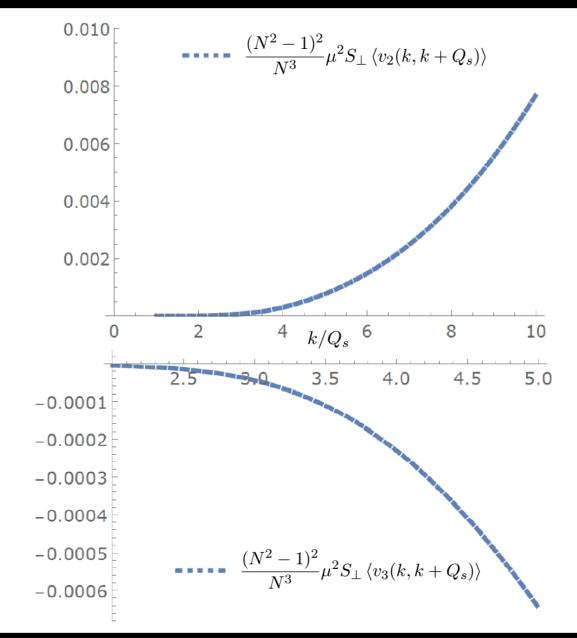
[A. Kendi, C. Marquet and V. Vila, "in preparation"]

$$v_n^2(k,p) = \frac{\int d\phi_k d\phi_p e^{in(\phi_k - \phi_p)} \frac{d^2 N^{(2)}}{d^2 k d^2 p}}{\int d\phi_k d\phi_p \frac{d^2 N^{(2)}}{d^2 k d^2 p}}$$





- **BOSE CORRELATIONS REGIME: VERY** WEAKLY DEPENDENCE ON THE **MOMENTUM**
- **SLOW INCREASE TOWARDS LARGE MOMENTA**
- **BOSE CORRELATED PART SCALES WITH** THE SAME POWER OF MOMENTUM AS **UNCORRELATED PIECE**



$$\begin{split} \sigma^{2[(iii)+(iv)]} &= \alpha_s^2 (4\pi)^2 N^3 \mu^2 S_\perp \left\{ \frac{1}{(2\pi)^2 Q_s^4} e^{-\frac{k^2+p^2}{2Q_s^2}} \int_{s_1,s_2} \left[ \left( \frac{s_1^i}{s_1^2} + \frac{k^i}{k^2} \right)^2 \left( \frac{s_2^j}{s_2^2} + \frac{p^j}{p^2} \right) - \frac{1}{k^2} \frac{p^j}{p^2} \right] \\ &\times e^{-\frac{s_1^2 + 2k \cdot s_1}{2Q_s^2}} e^{-\frac{s_2^2 - 2p \cdot s_2}{2Q_s^2}} \\ &- \left[ \frac{1}{k^2} \frac{p \cdot (k+p)}{p^2 (k+p)^2} \left( e^{-\frac{(k+p)^2}{4Q_s^2}} - 1 \right) \right\} \end{split}$$

- THE STUDIED PIECE REVEALS BOTH **EVEN AND ODD HARMONICS**
- SAME PATTERN AS THE BOSE CORRELATED PIECE FOR  $v_2$
- **ODD HARMONICS SURFACING FROM** THIS SMALL CONTRIBUTION
- TO BE COMPLETED WITH THE **REMAINING PIECES (WORK IN PROGRESS**)

[A. Kendi, C. Marguet and V. Vila, "in preparation"]

- We calculate the double inclusive gluon production cross-section within the approach of the dense-dilute CGC via going beyond the Glasma Graph approximation.
- We retrieve the contribution that is responsible for independent production of two gluons as well as the single inclusive spectrum.
- As a novelty, we provide the quantum correction accounting for the two gluons being correlated in the incoming wave function.
- The numerical evaluation of the Bose type contribution for the two-gluons independent production configuration confirms the already studied behaviour of  $v_2$ .
- One of the pieces of the quantum correction  $\sigma^2$  reveals the presence of both even and odd azimuthal harmonics [full numerical evaluation still in progress].

## THANK YOU VERY MUCH FOR YOUR ATTENTION!