NLO DGLAP splitting kernels for color non-singlet DPDs

Florian Fabry – florian.fabry@desy.de

Deutsches Elektronen-Synchrotron, DESY

work in collaboration with Markus Diehl and Alexey Vladimirov

MPI@LHC workshop 2022

November 17, 2022

Setting the stage

- Color non-singlet DPDs suppressed by Sudakov factor ([Artru, Mekhfi; 1988], [Manohar, Waalewijn; 2012])

  → rapidity dependence, see later

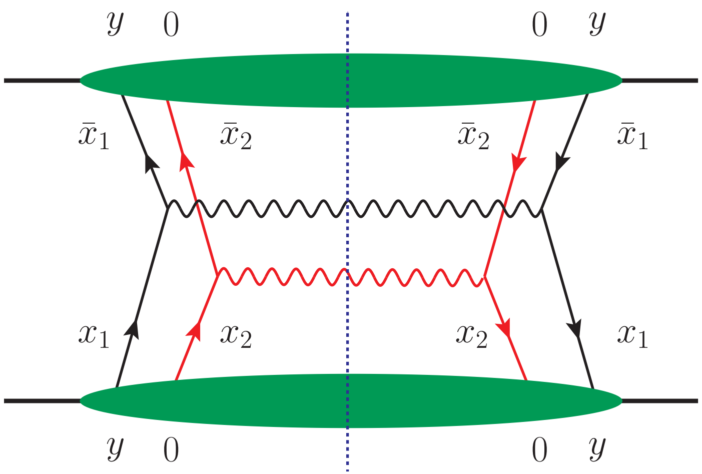
- Also suppressed after DGLAP evolution?

  Not necessarily! (see previous talk, [Blok, Mehl; 2022])

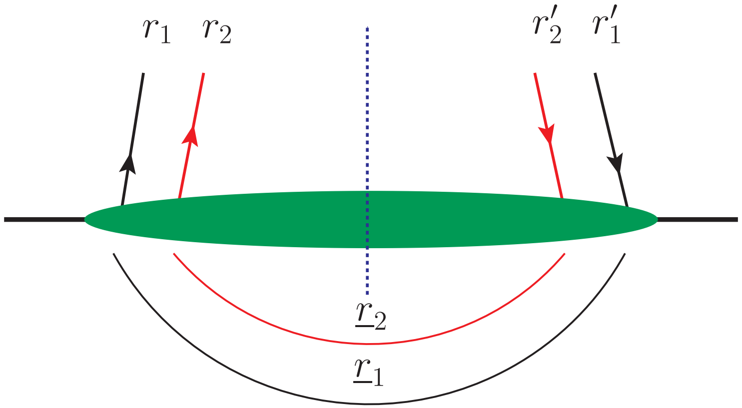
- Until now, DGLAP evolution available only at LO…

- Let's go to NLO!

Double parton distributions (DPDs)



LO double Drell-Yan production
adapted from [Diehl, Gaunt; 2017]



DPD color indices
adapted from [Diehl, Gaunt; 2017]

Matrix element structure very similar to the one of PDFs:

$$F^{\underline{r_1}\underline{r_2}}_{a_1 a_2}(x_i, \boldsymbol{y}, \mu_i, \zeta) \propto \int dy^- \frac{dz_1^-}{2\pi}\frac{dz_2^-}{2\pi} e^{i(x_1 z_1^- + x_2 z_2^-)p^+} \times \langle p| \mathcal{O}^{r_1}_{a_1}(y, z_1, \mu_1, \zeta)\mathcal{O}^{r_2}_{a_2}(0, z_2, \mu_2, \zeta) |p\rangle \Big|_{y^+=z_1^+=z_2^+=0,\ \boldsymbol{z}_1=\boldsymbol{z}_2=0}$$

Operators inside matrix element have the same structure as the ones inside PDFs

DGLAP evolution of DPDs

- Collinear, *y*-dependent DPDs evolve with a DGLAP equation

$$\frac{d}{d\ln\mu_1}\,{}^{R_1R_2}F_{a_1a_2}(x_i,\mu_i,\zeta) = 2\sum_{b,R'} {}^{R_1\overline{R}'}P_{a_1b}(x_1',\mu_1,x_1^2\zeta) \underset{x_1}{\otimes} {}^{R'R_2}F_{ba_2}(x_1,x_2,\mu_i,\zeta)$$

(analoguous equation for μ_2)

Note: finite distance *y* acts as a UV-cutoff for otherwise existing divergences in interactions between the two partons

- Diagonal after projection onto irreducible representations with the help of color projector:

$${}^{R_1R_2}F_{a_1a_2} \propto P^{r_1r_2}_{\overline{R}_1\overline{R}_2}\,F^{\overline{r}_1\overline{r}_2}_{a_1a_2}$$

$$3\otimes\overline{3} = 1\oplus 8$$

$$8\otimes 8 = 1\oplus 8_A\oplus 8_S\oplus 10\oplus\overline{10}\oplus 27$$

- New feature: rapidity dependence!

Rapidity dependence of DPDs

- Rapidity divergences compensated by a soft factor, same structure as for TMDs!
- Rapidity dependence in DPDs:

$$\frac{\partial}{\partial \ln \zeta} {}^{R_1 R_2}F_{a_1 a_2}(x_i, \boldsymbol{y}; \mu_i, \zeta) = \frac{1}{2} {}^{R_1}J(\boldsymbol{y}; \mu_i) \; {}^{R_1 R_2}F_{a_1 a_2}(x_i, \boldsymbol{y}; \mu_i, \zeta)$$

- Rapidity dependence in splitting kernels:

$$\frac{\partial}{\partial \ln \zeta} {}^{RR'}P_{ab}(x, \mu_1, \zeta) = -\frac{1}{4} \delta_{R\overline{R}'} \delta_{ab} \delta(1-x) \; {}^{R}\gamma_J(\mu)$$

→ additional rapidity term in color non-singlet splitting kernels,

for color singlet: ${}^{1}J = 0, \; {}^{1}\gamma_J = 0$

Sudakov factor inside DPDs

- Isolate (x-independent) rapidity dependence:

$$
{}^{R_1R_2}F_{a_1a_2}(x_i,\boldsymbol{y};\mu_i,\zeta) = \exp\left[{}^{R_1}J(\boldsymbol{y},\mu_i)\log\frac{\sqrt{\zeta}}{\mu_y} + \int_{\mu_y}^{\mu_1}\frac{d\mu}{\mu}\,{}^{R_1}\gamma_J(\mu)\log\frac{\mu}{\mu_y} + (\mu_1\leftrightarrow\mu_2)\right] \times \widehat{F}_{a_1a_2,\mu_0,\mu_y^2}(x_i,\boldsymbol{y};\mu_i),\;\; \mu_y = \frac{2e^{-\gamma_E}}{y^*},\;\; \zeta\sim\mathcal{O}(Q)
$$

See talk by Peter Plößl

- Let's get an idea of the effect of the Sudakov factor. Expand only to LO and use RGE of CS kernel. The last term determines behavior.

$$
\exp\left[{}^{R_1}J(\mu_y,\mu_y)\log\frac{\sqrt{\zeta}}{\mu_y} + \frac{\pi\,{}^{R_1}\gamma_J^{(0)}}{\beta_0}\sum_{i=1,2}\left(\log\frac{\mu_i}{\mu_y} - \left[\log\log\frac{\mu_i}{\Lambda} - \log\log\frac{\mu_y}{\Lambda}\right]\log\frac{\sqrt{\zeta}}{\Lambda}\right)\right]
$$

Logs inside CS kernel vanish at this scale configuration.

Vanishes for color singlet

Suppression!

Vanishes for $\mu_y = \mu_i$ → no/weak suppression at certain scales! (see also [Blok, Mehl; 2022])

Sudakov factor inside DPDs

- Main takeaway: Sudakov suppression can be small/absent!
- Thus, understanding effect of remaining (*x*-dependent) log and DGLAP evolution becomes even more important.
- DGLAP equation for “reduced” DPD:

$$\frac{\partial}{\partial \log \mu_1} {}^{R_1 R_2}\widehat{F}_{a_1 a_2,\, \mu_0, \zeta_0}(x_1, x_2, \boldsymbol{y}; \mu_1, \mu_2)$$

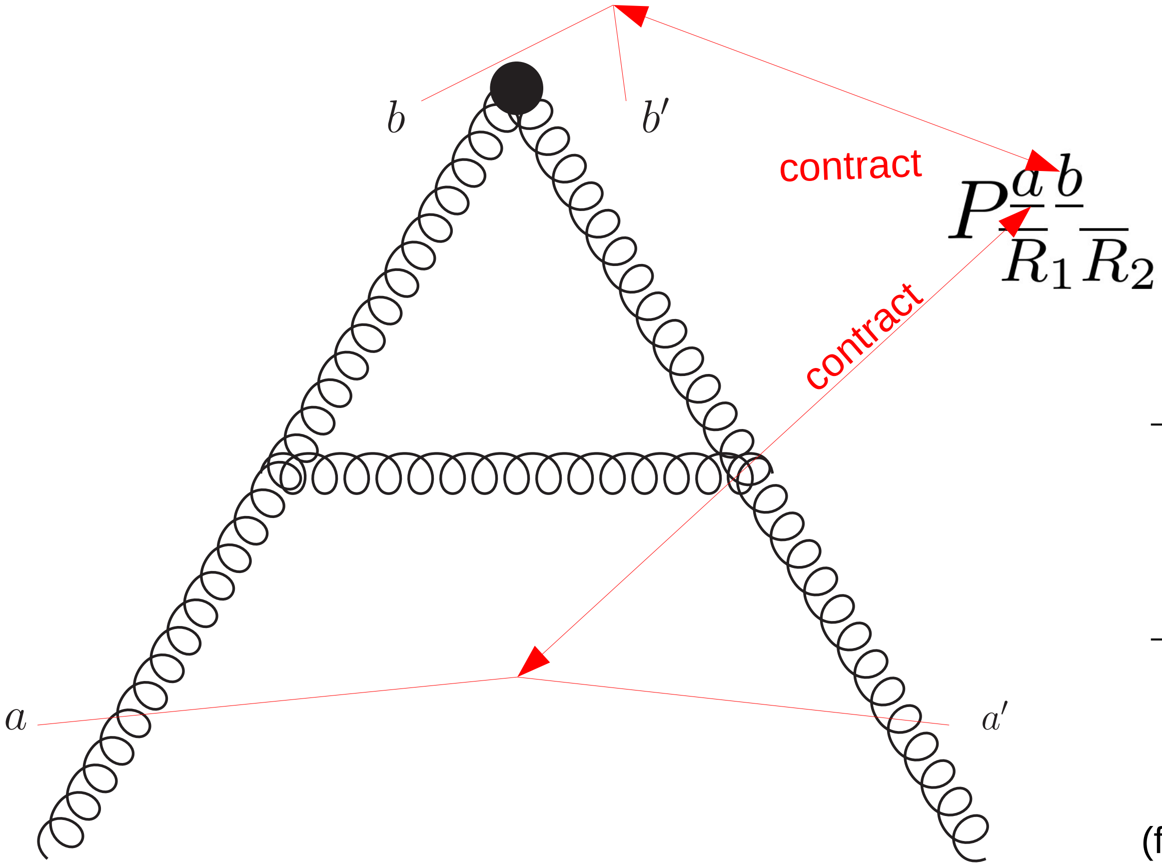
absent in color singlet

$$= \underbrace{- {}^{R_1}\gamma_J(\mu_1) \log x_1 \, {}^{R_1 R_2}\widehat{F}_{a_1 a_2,\, \mu_0, \zeta_0}(x_1, x_2, \boldsymbol{y}; \mu_1, \mu_2)}$$

$$+ 2 \sum_{b_1, R_1'} {}^{R_1 \overline{R}_1'} P_{a_1 b_1}(x_1'; \mu_1, \mu_1^2) \underset{x_1}{\otimes} {}^{R_1' R_2}\widehat{F}_{b_1 a_2,\, \mu_0, \zeta_0}(x_1', x_2, \boldsymbol{y}; \mu_1, \mu_2)$$

No *x*-dependence in rapidity variable, Mellin convolution as we know it

LO colored DGLAP kernels



→ Leads to a global "color" factor for all non-*δ(1-x)* terms:

$$^{R_1 R_2}P^{(0)}_{ab}(x) = c_{ab}(R_1 R_2)\,{}^{11}P^{(0)}_{ab}(x)$$

→ *δ(1-x)* terms stay as they are, because color projectors are normalized to unity

(first done in [Diehl, et. al.; 2011], also used in [Blok, et.al.; 2022])

NLO colored DGLAP kernels

- More graphs → more color factors → no global factor, but more involved structure!
- Need to regulate rapidity divergencies
- Calculated using two methods:

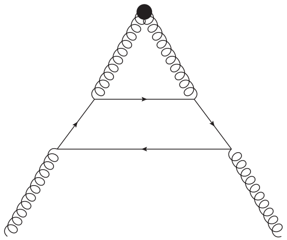
  1. Based on existing results of DGLAP kernels for PDFs
  2. Based on short distance matching of TMD operators projected onto color non-singlet representations

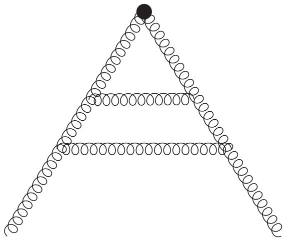
1. Method:
Extraction from graph-by-graph results

based on [Curci, Furmanski, Petronzio; 1980], [Ellis, Vogelsang; 1996], [Vogelsang; 1996] and [Vogelsang; 1997]
(special thanks to Werner Vogelsang for help)

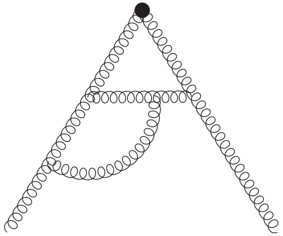
Approach

Terms					
	$(\mathrm{b_q})$	$(\mathrm{b_g})$	(c)	(d)	...
$p_{gg}(x)\ln^2(x)$		-2	4		
$p_{gg}(x)\ln^2(1-x)$			4		
$p_{gg}(x)\ln(x)\ln(1-x)$			16		
$p_{gg}(x)I_0\Big(\ln(1-x)+\ln(x)\Big)$			24		
$p_{gg}(x)\ln(x)$			$-22/3$	$4/3$	
...	...				





...



(tables based on results from the publications on the previous slide)

Approach

Combine with

RR'	11	AA	SS	$27\,27$
global factor	1	1	1	1
graph ($\mathrm{b_q}$)	$-\frac{1}{2N}n_f$	0	$-\frac{1}{N}n_f$	$\frac{1}{2}n_f$
graph ($\mathrm{b_g}$)	$\frac{N^2}{2}$	0	0	$\frac{5}{2}$
graph (c)	$\frac{N^2}{2}$	$\frac{1}{4}N^2$	$\frac{1}{4}N^2$	$-\frac{3}{2}$
graph (d)	$\frac{N}{2}n_f$	$\frac{1}{4}Nn_f$	$\frac{1}{4}Nn_f$	$-\frac{1}{2}n_f$

... ...

(need to recalculate 4-gluon vertex diagrams from scratch)

Limitations: the rapidity dependence

- Calculational method tailored for collinear PDFs

  → No sensitivity to rapidity divergences,
  i.e. no extraction of $\delta(1-x)$ terms possible

- Either invent new scheme that also regulates rapidity divergences, or make use of existing literature

  → TMD matrix elements!

- *serves also a cross check for all the non-$\delta(1-x)$ terms*

2. Method: Extraction from projected TMD matrix elements

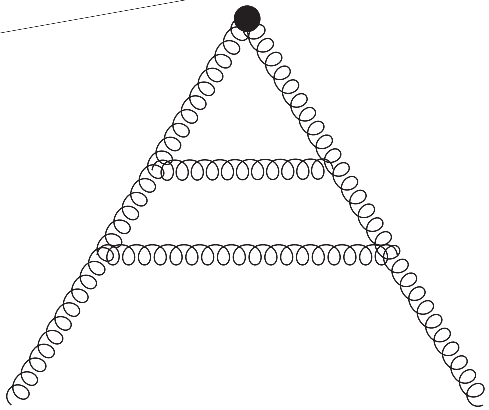
based on [Echevarria, Scimemi, Vladimirov; 2016]
and [Gutierrez-Reyes, Scimemi, Vladimirov; 2018]

Approach

Use short distance matching formula between collinear PDF and TMD matrix elements to have access to rapidity regulator.

$$ {}^{RR'}\widehat{\mathcal{M}}_{ab}(x,\boldsymbol{z},\mu,\zeta) = \sum_{c,R''} {}^{R\overline{R}''}C_{ac}(x',\boldsymbol{z},\mu,x^2\zeta) \underset{x}{\otimes} {}^{R''R'}\mathcal{M}_{cb}(x',\mu,\zeta) $$

again obtained from evaluating color projected diagrams like



Extract splitting kernels from single pole at NNLO after calculating the matrix elements up to desired order.

Results

- Kernels can be decomposed such that

$$
{}^{RR'}P_{ab}(x,\zeta_p/\mu^2) = \underline{{}^{RR'}P_{ab,\text{real}}(x)} + \left(\delta_{R\overline{R}'}\delta_{ab}P_{a,\text{sing}} + \underline{{}^{RR'}P_{ab,\text{non-sing}}} - \frac{1}{4}\delta_{R\overline{R}'}\delta_{ab}\,{}^{R}\underline{\gamma_J}\ln\frac{\zeta_p}{\mu^2}\right)\delta(1-x)
$$

- Part from real graphs calculated with both methods
- Casimir scaling of NLO anomalous dimension same as at LO!

$$
{}^{10}\gamma_J^{(1)} = {}^{\overline{10}}\gamma_J^{(1)} = 2\,{}^{8}\gamma_J^{(1)}\Big|_{N=3} = 134 - 6\pi^2 - \frac{20}{3}n_f\,,
$$

$$
{}^{27}\gamma_J^{(1)} = \frac{8}{3}\,{}^{8}\gamma_J^{(1)}\Big|_{N=3}.
$$

- Identical Casimir scaling also for the additional “non-sing” terms

Summary

- For the first time, obtained all colored NLO DGLAP kernels, for unpolarized, longitudinal and transversity distributions
- All non-$\delta(1-x)$ terms are cross-checked with two completely independent methods
- Also obtained the NLO anomalous dimension of the CS-kernel for higher-than-octet representations

Stay tuned for numerical results!

Thank you for your attention!

Back up: Color Projectors

$$P^{a\,b}_{\underline{a}\,\underline{b}\;11} = \frac{1}{N^2-1}\delta^{aa'}\delta^{bb'}$$

$$P^{\underline{a}\,\underline{b}}_{AA} = \frac{1}{N} f^{aa'c} f^{bb'c}$$

$$P^{\underline{a}\,\underline{b}}_{SS} = \frac{N}{N^2-4} d^{aa'c} d^{bb'c}$$

$$P^{\underline{a}\,\underline{b}}_{AS} = \frac{1}{\sqrt{N^2-4}} f^{aa'c} d^{bb'c}$$

$$P^{\underline{a}\,\underline{b}}_{SA} = \frac{1}{\sqrt{N^2-4}} d^{aa'c} f^{bb'c}$$

$$P^{\underline{a}\,\underline{b}}_{10\,\overline{10}} = \frac{1}{4}\left(\delta^{ab}\delta^{a'b'} - \delta^{ab'}\delta^{a'b}\right) - \frac{1}{2} P^{\underline{a}\,\underline{b}}_{AA} - \frac{i}{4}\left(d^{abc} f^{a'b'c} + f^{abc} d^{a'b'c}\right)$$

$$P^{\underline{a}\,\underline{b}}_{\overline{10}\,10} = \frac{1}{4}\left(\delta^{ab}\delta^{a'b'} - \delta^{ab'}\delta^{a'b}\right) - \frac{1}{2} P^{\underline{a}\,\underline{b}}_{AA} + \frac{i}{4}\left(d^{abc} f^{a'b'c} + f^{abc} d^{a'b'c}\right)$$

$$P^{\underline{a}\,\underline{b}}_{27\,27} = \frac{1}{2}\left(\delta^{ab}\delta^{a'b'} + \delta^{ab'}\delta^{a'b}\right) - P^{\underline{a}\,\underline{b}}_{SS} - P^{\underline{a}\,\underline{b}}_{11}$$

DGLAP evolution channels

- Color singlet: Identical splitting kernels as for PDFs

- Two flavor-singlet combinations in the color octet:
  - $\sum_{q=u,d,s,\ldots} \left({}^{8}q + {}^{8}\overline{q}\right)$ mixes with symmetric gluon
  - $\sum_{q=u,d,s,\ldots} \left({}^{8}q - {}^{8}\overline{q}\right)$ mixes with antisymmetric gluon

- Gluon decuplet and 27-multiplet evolve independently from quark distributions

Backup: Flavor singlet evolution equations

- Two flavor-singlet combinations in the color octet:

$$\frac{d}{d\ln\mu_1}\begin{pmatrix} {}^{R_1R_2}F_{\Sigma^+ a_2} \\ {}^{R_3R_4}F_{g a_4} \end{pmatrix} = 2\begin{pmatrix} {}^{R_1R_1}P_{\Sigma^+\Sigma^+} & n_f\,{}^{R_1R_3}P_{\Sigma^+ g} \\ {}^{R_3R_1}P_{g\Sigma^+} & {}^{R_3R_3}P_{gg} \end{pmatrix} \underset{x_1}{\otimes} \begin{pmatrix} {}^{R_1R_2}F_{\Sigma^+ a_2} \\ {}^{R_3R_4}F_{g a_4} \end{pmatrix}, \quad R_1R_3 = 11, 8S$$

$$\frac{d}{d\ln\mu_1}\begin{pmatrix} {}^{8R_2}F_{\Sigma^- a_2} \\ {}^{AR_4}F_{g a_4} \end{pmatrix} = 2\begin{pmatrix} {}^{88}P_{\Sigma^-\Sigma^-} & n_f\,{}^{8A}P_{\Sigma^- g} \\ {}^{A8}P_{g\Sigma^-} & {}^{AA}P_{gg} \end{pmatrix} \underset{x_1}{\otimes} \begin{pmatrix} {}^{8R_2}F_{\Sigma^- a_2} \\ {}^{AR_4}F_{g a_4} \end{pmatrix}$$

Backup: Method overview

	Unpol.	Longit./ Helicity	Transv.
Method 1: Graphs	✓	✓	✓
Method 2: Matching	✓	✗	✓

Matrix elements not available to us in the form we need

Backup: Longitudinal kernels: scheme change

γ_5 matrix does not anti-commute with all γ matrices in dim. reg. with more than 4 space-time dimension.

- This leads to additional terms that violate scale independence of a combination of non-singlet distributions (see [Vogelsang; 1996]).
- Get rid of these terms with a scheme change on twist-2 operator level:

$$^{R}\mathcal{O}_{\Delta q_i} = {}^{R}\widetilde{Z} \otimes {}^{R}\mathcal{O}_{\Delta q_i,\overline{\mathrm{MS}}}$$

$$^{R}\mathcal{O}_{\Delta \bar{q}_i} = {}^{R}\widetilde{Z} \otimes {}^{R}\mathcal{O}_{\Delta \bar{q}_i,\overline{\mathrm{MS}}}$$

This leads to:

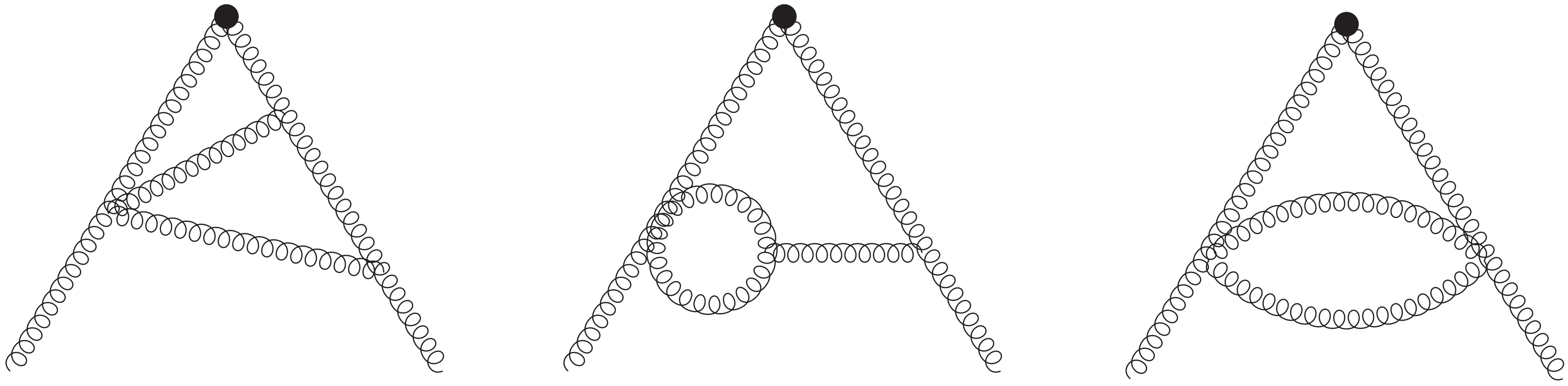
$$^{RR}P_{\Delta}^{\pm,(0)} = {}^{RR}P_{\Delta,\overline{\mathrm{MS}}}^{\pm,(0)}$$

$$^{RR}P_{\Delta}^{\pm,(1)} = {}^{RR}P_{\Delta,\overline{\mathrm{MS}}}^{\pm,(1)} - \frac{1}{2}\beta_0 {}^{R}\widetilde{Z}^{(1)}$$

$$^{RR'}P_{\Delta\Sigma^{\pm}\Delta g}^{(1)} = {}^{RR'}P_{\Delta\Sigma^{\pm}\Delta g,\overline{\mathrm{MS}}}^{(1)} + {}^{R}\widetilde{Z}^{(1)} \otimes {}^{RR'}P_{\Delta\Sigma^{\pm}\Delta g}^{(0)}$$

$$^{R'R}P_{\Delta g\Delta\Sigma^{\pm}}^{(1)} = {}^{R'R}P_{\Delta g\Delta\Sigma^{\pm},\overline{\mathrm{MS}}}^{(1)} - {}^{R'R}P_{\Delta g\Delta\Sigma^{\pm}}^{(0)} \otimes {}^{R}\widetilde{Z}^{(1)}$$

What about 4-vertex graphs?



Color structure of gluon 4-vertex does not factorize:

$$f^{e'bc}f^{e'ea}\left(g^{\nu'\beta'}g^{\mu\alpha'} - g^{\nu'\mu}g^{\alpha'\beta'}\right)$$
$$+\, f^{e'be}f^{e'ca}\left(g^{\nu'\alpha'}g^{\beta'\mu} - g^{\nu'\mu}g^{\alpha'\beta'}\right)$$
$$+\, f^{e'ba}f^{e'ce}\left(g^{\nu'\alpha'}g^{\beta'\mu} - g^{\nu'\beta'}g^{\mu\alpha'}\right)$$

Calculate by hand, using the methods illustrated in [Ellis, Vogelsang; 1996]

Calculating 4-vertex graphs

Master formula:

$$^{RR'}\Gamma_{(\mathrm{k})}(x) = \mathrm{PP}\left\{ \frac{1}{D-2} \int_0^{Q^2} d|k^2| \int dPS^{(2)} \frac{\mathcal{N}^{RR'}_{(\mathrm{k})}(x)}{2|k^2|^2} \right\}$$

- $^{RR'}\Gamma_{(\mathrm{k})}(x)$ → prop. to splitting kernel
- PP → extracts single pole of expansion in ϵ
- $\int_0^{Q^2} d|k^2|$ → Integral over momentum flowing out of the top of the graph → upper cut-off at hard scale
- $\frac{\mathcal{N}^{RR'}_{(\mathrm{k})}(x)}{2|k^2|^2}$ → Graph with integral over residual phase space

- Equivalent to taking the UV part of the graph (scaleless integrals vanish in dim. reg.)
  - Splitting kernels as anomalous dimensions connected to renormalisation factors of PDF/DPD operators
  - See [Collins, Rogers, Sate; 2021] for a comparison of both approaches

Method 2: Approach

- Make use of short distance matching formula between collinear PDF and TMD matrix elements to have access to rapidity regulator:

$$ {}^{RR'}\widehat{\mathcal{M}}_{ab}(x,\boldsymbol{z},\mu,\zeta) = \sum_{c,R''} {}^{R\overline{R}''}C_{ac}(x',\boldsymbol{z},\mu,x^2\zeta) \underset{x}{\otimes} {}^{R''R'}\mathcal{M}_{cb}(x',\mu,\zeta) $$

- R.h.s. calculated with the help of the δ-regulator inside Wilson line [Echevarria et. al.]:

$$ W_{rs}(\xi,v) = \mathcal{P}\exp\left\{-ig\,t^b_{rs}\int_0^\infty ds\,v\cdot A^b(\xi+sv)e^{-\delta^+ s}\right\} $$

in eikonal propagators: $\dfrac{1}{(k_1^+ - i\delta^+)(k_2^+ - 2i\delta^+)\dots(k_n^+ - ni\delta^+)}$

Method 2: Limitations(?)

- Colored matrix elements only available for unpol. and transv. case

  → δ(1-x) terms are missing for longitudinal kernels

- However, these terms come from kinematical regions in which gluons become soft → interaction can be approximated by eikonal coupling, which is spin independent

- This is validated
  - … for all polarizations in color singlet case

    → known for a long time in the literature
  - … for unpol. and transv. in all color representations (our results)

Back up: Kernels in their full glory

$$^{88}P_{qq}^{V,(1)}(x) = c_{qq}(88)\left\{ ^{11}P_{qq}^{V,(1)}(x) - \frac{C_F C_A}{4}\left[\Big(2p_{qq}(x) - (1+x)\Big)\ln^2(x) + (8-4x)\ln(x) + 6(1-x)\right]\right\}$$

$$^{88}P_{q\bar{q}}^{V,(1)}(x) = (N^2+1)\, c_{qq}(88)\, ^{11}P_{q\bar{q}}^{V,(1)}(x)$$

$$^{88}P_{qq}^{S,(1)}(x) = -c_{qq}(88)(N^2-2)^{11}P_{qq}^{S,(1)}(x)$$

$$^{88}P_{q\bar{q}}^{S,(1)}(x) = 2c_{qq}(88)^{11}P_{q\bar{q}}^{S,(1)}(x)$$

Back up: Kernels in their full glory

$$
{}^{8A}P^{(1)}_{\Sigma^- g}(x) = c_{qg}(8A)\Bigg\{ {}^{11}P^{(1)}_{\Sigma^+ g}(x) + \frac{1}{2}C_A\Bigg[\left(3x - p_{\Sigma^\pm g}(x) + \frac{3}{2}\right)\ln^2(x) + \frac{1}{3}\left(-89x - 22p_{\Sigma^\pm g}(x) + 4\right)\ln(x) + \frac{109}{9}p_{\Sigma^\pm g}(x) - 2S_2(x)p_{\Sigma^\pm g}(-x) + \frac{83}{9}x - \frac{172}{9} - \frac{20}{9x}\Bigg]\Bigg\}
$$

$$
{}^{8S}P^{(1)}_{\Sigma^+ g}(x) = \frac{c_{qg}(8S)}{c_{qg}(8A)}\,{}^{8A}P^{(1)}_{\Sigma^- g}(x)
$$

Back up: Kernels in their full glory

$$
{}^{A8}P^{(1)}_{g\Sigma^-}(x) = c_{gq}(A8)\Bigg\{{}^{11}P^{(1)}_{g\Sigma^+}(x) + \frac{C_F C_A}{18}\Bigg[-\Big(\frac{27}{2}x + 9p_{g\Sigma^\pm}(x) + 27\Big)\ln^2(x) + \Big(24x^2 + 27x + 135\Big)\ln(x) + 58p_{g\Sigma^\pm}(x) - 18S_2(x)p_{g\Sigma^\pm}(-x) - 44x^2 - 58x + 44\Bigg]\Bigg\}
$$

$$
{}^{S8}P^{(1)}_{g\Sigma^+}(x) = \frac{c_{gq}(S8)}{c_{gq}(A8)}\,{}^{A8}P^{(1)}_{g\Sigma^-}(x)
$$

Back up: Kernels in their full glory

$$
\begin{aligned}
{}^{AA}P_{gg}^{(1)}(x) = c_{gg}(AA)\Bigg\{ C_A^2 \Bigg[& 2(1+x)\ln^2(x) - 4p_{gg}(x)\ln(x)\ln(1-x) \\
& + \frac{1}{3}\Big(-22x^2 + 14x - 4\Big)\ln(x) \\
& + \frac{1}{9}\Big(67 - 3\pi^2\Big)p_{gg}(x) + 6(1-x)\Bigg] \\
+ C_A n_f \Bigg[& -\frac{1}{2}(1+x)\ln^2(x) - \frac{1}{6}\Big(19x + 13\Big) - \frac{10}{9}p_{gg}(x) \\
& + \frac{28}{9}x^2 + x - 3 - \frac{10}{9x}\Bigg]\Bigg\}
\end{aligned}
$$

Back up: Kernels in their full glory

$$
{}^{SS}P_{gg}^{(1)}(x) = \frac{c_{gg}(SS)}{c_{gg}(AA)}\,{}^{AA}P_{gg}^{(1)}(x)
$$

$$
+ c_{gg}(SS)\left(\frac{1}{2}C_A - C_F\right)n_f\left\{(1+x)\ln^2(x) + (5x+3)\ln(x) - \frac{10}{3}x^2 - 4x + 8 - \frac{2}{3x}\right\}
$$

$$
{}^{AS}P_{gg}^{(1)}(x) = {}^{SA}P_{gg}^{(1)}(x) = 0
$$

$$
{}^{10\,\overline{10}}P_{gg}^{(1)}(x) = {}^{\overline{10}\,10}P_{gg}^{(1)}(x) = 0
$$

Back up: Kernels in their full glory

$$
\begin{aligned}
{}^{27\,27}P_{gg}^{(1)}(x) = c_{gg}(27\,27)\Bigg\{ & \Big[-15p_{gg}(x) - 12(1+x)\Big]\ln^2(x) - 36p_{gg}(x)\ln(x)\ln(1-x) \\
& + \Big[44x^2 + 57x + 93\Big]\ln(x) - \Big[3\pi^2 - 67\Big]p_{gg}(x) \\
& - 30S_2(x)p_{gg}(-x) - \frac{335}{3}x^2 - \frac{117}{2}(1-x) + \frac{335}{3x} \\
& + n_f\Big[-2(1+x)\ln(x) - \frac{10}{3}p_{gg}(x) + \frac{13}{3}x^2 + 3(1-x) - \frac{13}{3x}\Big]\Bigg\}
\end{aligned}
$$

Back up: Kernels in their full glory

$$
{}^{RR'}P_{ab}(x,\zeta_p/\mu^2) = {}^{RR'}P_{ab,\mathrm{real}}(x) + \left(\delta_{R\overline{R}'}\delta_{ab}P_{a,\mathrm{sing}} + {}^{RR'}P_{ab,\text{non-sing}} - \frac{1}{4}\delta_{R\overline{R}'}\delta_{ab}{}^{R}\gamma_J \ln\frac{\zeta_p}{\mu^2}\right)\delta(1-x).
$$

$$
\begin{aligned}
{}^{88}P^{V,(1)}_{qq,\text{non-sing}} &= {}^{AA}P^{(1)}_{gg,\text{non-sing}} = {}^{SS}P^{(1)}_{gg,\text{non-sing}} \\
&= C_A^2\left\{\frac{101}{54} - \frac{11}{144}\pi^2 - \frac{7}{4}\zeta_3\right\} + C_A n_f\left\{\frac{1}{72}\pi^2 - \frac{7}{27}\right\} \\
{}^{10\,\overline{10}}P^{(1)}_{gg,\text{non-sing}} &= {}^{\overline{10}\,10}P^{(1)}_{gg,\text{non-sing}} = 2\,{}^{AA}P^{(1)}_{gg,\text{non-sing}}\Big|_{N=3} \\
&= \frac{101}{3} - \frac{11}{8}\pi^2 - \frac{63}{2}\zeta_3 + n_f\left\{\frac{1}{12}\pi^2 - \frac{14}{9}\right\} \\
{}^{27\,27}P^{(1)}_{gg,\text{non-sing}} &= \frac{4}{3}\,{}^{10\,\overline{10}}P_{gg,\text{non-sing}}.
\end{aligned}
$$

Back up: Kernels in their full glory

$$
{}^{10}\gamma_J^{(1)} = {}^{\overline{10}}\gamma_J^{(1)} = 2\,{}^{8}\gamma_J^{(1)}\Big|_{N=3} = 134 - 6\pi^2 - \frac{20}{3}n_f\,,
$$

$$
{}^{27}\gamma_J^{(1)} = \frac{8}{3}\,{}^{8}\gamma_J^{(1)}\Big|_{N=3}.
$$

Backup: rapidity dependence in DGLAP evolution

$$
\begin{aligned}
&{}^{R_1R_2}F_{a_1a_2}(x_i, \boldsymbol{y}; \mu_i, \zeta_p) \\
&= \exp\left[{}^{R_1}J(\boldsymbol{y}; \mu_i)\, \log \frac{\sqrt{\zeta_p}}{\sqrt{\zeta_0}}\right] {}^{R_1R_2}F_{a_1a_2}(x_i, \boldsymbol{y}; \mu_i, \zeta_0) \\
&= \exp\left[{}^{R_1}J(\boldsymbol{y}; \mu_i)\, \log \frac{\sqrt{\zeta_p}}{\sqrt{\zeta_0}} - \int_{\mu_0}^{\mu_1} \frac{d\mu}{\mu}\, {}^{R_1}\gamma_J(\mu)\, \log \frac{\sqrt{\zeta_0}}{\mu} - \int_{\mu_0}^{\mu_2} \frac{d\mu}{\mu}\, {}^{R_1}\gamma_J(\mu)\, \log \frac{\sqrt{\zeta_0}}{\mu}\right] \\
&\quad \times {}^{R_1R_2}\widehat{F}_{a_1a_2,\, \mu_0, \zeta_0}(x_i, \boldsymbol{y}; \mu_i)\,,
\end{aligned}
$$