Maximally entangled proton and charged hadron multiplicity in Deep Inelastic Scattering

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Based on
Eur.Phys.J.C 82 (2022) 2, 111

M. Hentschinski, KK

2207.09430

M. Hentschinski, KK, R. Straka

Motivation

Entropy and low x dynamics (and hadronic collisions) attracts considerable theoretical interest since it

- constraints the growth of PDFs with energy through quantum bounds
- links to other areas (thermodynamics, gravity, quantum information, conformal field theory)
- possible links of entropy to saturation

Various approaches to entropy in the low x limit : entropy of gluon density, thermodynamic entropy, momentum space entanglement, Wehr entropy,...

K. Kutak '11, I. Zahed '12, R. Peschanski '13,

A. Kovner, M. Lublinsky '15, D. Kharzeev, E. Levin'17, A. Kovner, M. Lublinsky, M. Serino '18,

Hagiwara, Y, Hatta, B. Xiao,Yuan'18, N. Armesto, F. Dominguez, A. Kovner, M. Lublinsky, V. Skokov'19

Z. Tu, D. Kharzeev, T. Ulrich '20, C. Akkaya, H. Duan , A. Kovner, V. Skokov '20

K. Zhang, K. Hao, D. Kharzeev, V. Korepin' 21, E. Levin, D. Kharzeev '21, H. Duan , A. Kovner, V. Skokov '21

D. Kharzeev '21; M. Hentschinski, K. Kutak '21,

Dvali, Venugopalan' 21; Liu, 4 x Liu, Nowak, Zahed '22 ;

A. Dumitriu,Kolbusz '22; Kou, Wang, Chen '22

Boltzman and von Neuman entropy formulas – reminder

The entropy S of macrostate is given by the log of number W of distinct microstates that compose it

$$S = -\sum_{i=1}^{W} p(i) \ln p(i)$$

Gibbs entropy

For uniform distribution $p(i) = \frac{1}{W}$ the entropy is maximal

Boltzmann entropy

$$S = \ln W$$

Since partons are introduced as the microscopic constituents that compose the macroscopic state of the proton, it is natural to evaluate the corresponding entropy or entropy corresponding to parton density.

K. Kutak '11
A. Kovner, M. Lublinsky '15
Kharzeev, Levin '17

But proton as a whole is a pure state and the von Neuman entropy is 0. Can we get any nontrivial result?

For pure state (one state) density matrix is

$$\rho = |\psi\rangle\langle\psi|$$

$$S_{VN} = -Tr[\rho \ln \rho] = -1 \ln 1 = 0$$

For mixed state i.e. classical statistical mixture

$$\rho = \sum p(i) |\psi_i\rangle\langle\psi_i|$$

$$S_{VN} \neq 0$$

Kharzeev, Levin '17

Entanglement entropy in DIS

The composite system is described by $\mathcal{H}_A \otimes \mathcal{H}_B$

$|\Psi_{AB}\rangle$ in $A \cap B$

physical state in A
physical state in B

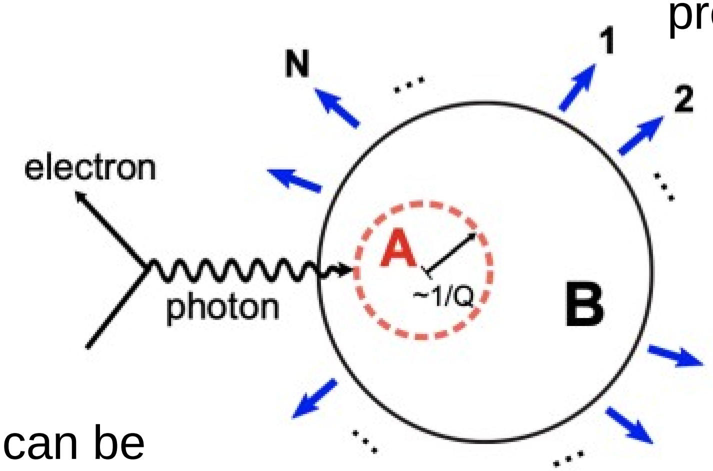
$$|\Psi_{AB}\rangle = \sum_{i,j} c_{ij} |\varphi_i^A\rangle \otimes |\varphi_j^B\rangle$$

entangled
if the product can not be expressed as separable product state

separable
if the product can be expressed as separable product state

$$|\Psi_{AB}\rangle = |\varphi^A\rangle \otimes |\varphi^B\rangle$$

proton's rest frame



$\mathcal{H}_B$ of dimension n_B.

$\mathcal{H}_A$ of dimension n_A

Kharzeev, Levin '17

The Schmidt decomposition allows to write

$$|\Psi_{AB}\rangle = \sum \alpha_n |\Psi_n^A\rangle |\Psi_n^B\rangle$$

orthonormal states belonging to A

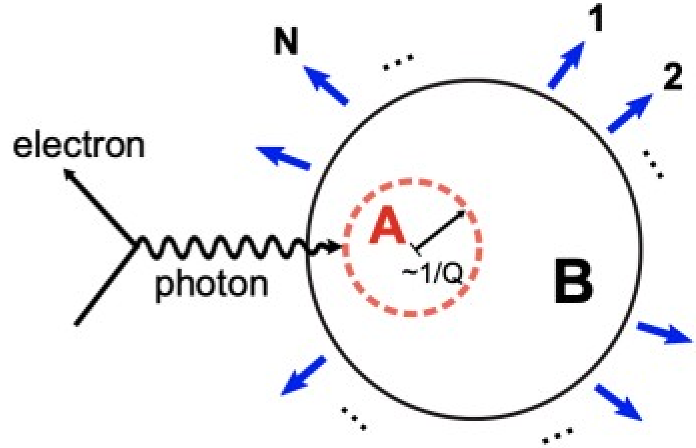
orthonormal states bolonging to B

related to matrix C

Entanglement entropy in DIS

Interaction leads to decoherence.

$$|\Psi_{AB}\rangle = \sum_n \alpha_n |\Psi_n^A\rangle |\Psi_n^B\rangle$$



Density matrix

$$\rho_{AB} = |\Psi_{AB}\rangle\langle\Psi_{AB}|$$

proton's rest frame

$$\rho_A = \mathrm{tr}_B \rho_{AB} = \sum_n \alpha_n^2 |\Psi_n^A\rangle\langle\Psi_n^A|$$

The density matrix of the mixed state probed in region A

$\alpha_n^2 \equiv p_n$ probability of state with n partons (dipoles)

Kharzeev, Levin '17

$$S = -\sum_n p_n \ln p_n$$

entropy results from the entanglement between the regions A and B, and can thus be interpreted as the entanglement entropy. Entropy of region A is the same as entropy in region B.

Remark: spectral decomposition guarantees that there is a basis where the density matrix is diagonal

Partonic, dipole cascade

$$p_n = P_n$$

$$\frac{dP_n(Y)}{dY} = -\lambda n P_n(Y) + (n-1)\lambda P_{n-1}(Y)$$

set of partons is described by set of dipoles with fixed sizes ,Y is rapidity and is related to energy Mueller 95, Lublinsky, Levin '03

$$P_n(Y) = e^{-\lambda Y}\left(1 - e^{-\lambda Y}\right)^{n-1}$$

depletion of the probability to find n dipoles due to the splitting into (n + 1) dipoles.

the growth due to the splitting of (n − 1) dipoles into n dipoles.

$$S = -\sum_n p_n \ln p_n$$

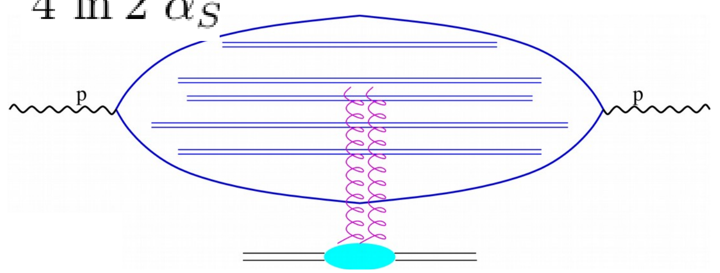
$$S(Y) = \ln\left(e^{\lambda Y} - 1\right) + e^{\lambda Y} \ln\left(\frac{1}{1 - e^{-\lambda Y}}\right)$$

See also Kovner, Levin, Lublinsky, JHEP 05 (2022) 019;

$S(Y) \approx \lambda Y$ where $Y = \ln 1/x$

$$xg(x) = \langle n \rangle = \sum_n n P_n(Y) = \left(\frac{1}{x}\right)^{\lambda}$$

$4 \ln 2\, \bar{\alpha}_S$



Kharzeev, Levin '17

Assumptions

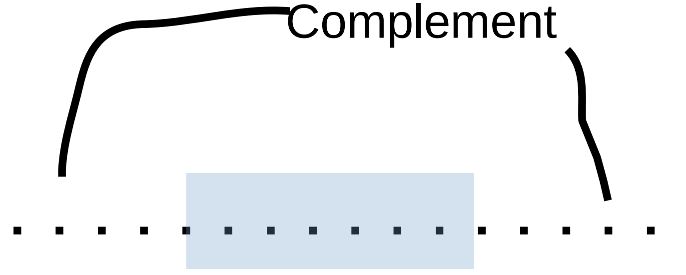
$S(x) = \ln(xg(x))$ and eventually

$$S(x, Q) = \ln(xg(x, Q))$$

See also Liu, Nowak, Zahed

See also
Zahed '12
N. Armesto, F. Dominguez, A. Kovner, M. Lublinsky, V. Skokov'19
Nowak, Liu, Zahed '22

Comments



spnis in 1 D

Region A of length L

$$S = \frac{c}{3} \ln \frac{L}{\epsilon}$$

Entanglement entropy obtained from CFT calculations as well as from gravity using Ryu-Takayanagi formula

Relation to Kharzeev-Levin formula

$$L = (mx)^{-1} \qquad \epsilon \equiv 1/m$$

Length of region probed in DIS

Proton’s Compton wave length

See also
Callan, Wilczek ‘94
Calabrese, Cardy ‘04

and lectures by
Headrick

$$S = \ln \left(\frac{1}{x}\right)^{1/3}$$

Studied also in the context of 2 d QCD

Liu, Nowak, Zahed, ‘22

Casini, Huerta, Hosco ‘05

KL entropy formula - interpretation

At low x partonic microstates have equal probabilities $P_n(Y) = e^{-\lambda Y}\left(1 - e^{-\lambda Y}\right)^{n-1}$

In this equipartitioned state the entropy is maximal – the partonic state at small x is maximally entangled.

In terms of information theory as Shanon entropy:

- equipartitioning in the maximally entangled state means that all “signals” with different number of partons are equally likely

- it is impossible to predict how many partons will be detected in a give event.

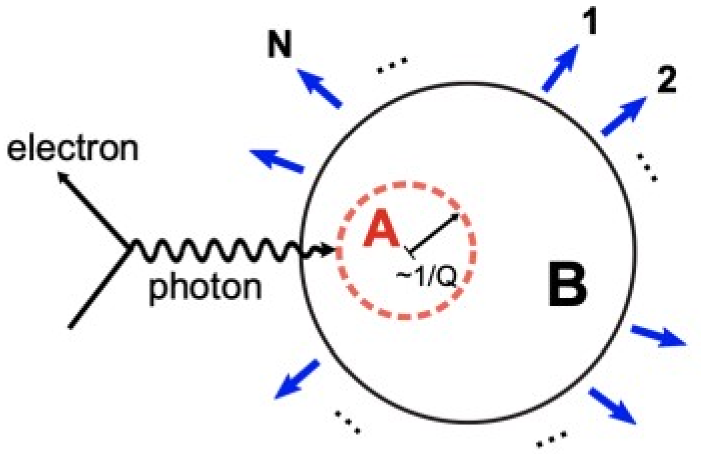
- structure function at small x should become universal for all hadrons.

Furthermore entropy of the final state hadrons can not be smaller than entropy of partons.

Entanglement entropy – calculation and measurement

For DIS at high energies, this entanglement entropy can be calculated using

$$S(x, Q^2) = \ln \left\langle n \left(\ln \frac{1}{x}, Q \right) \right\rangle$$

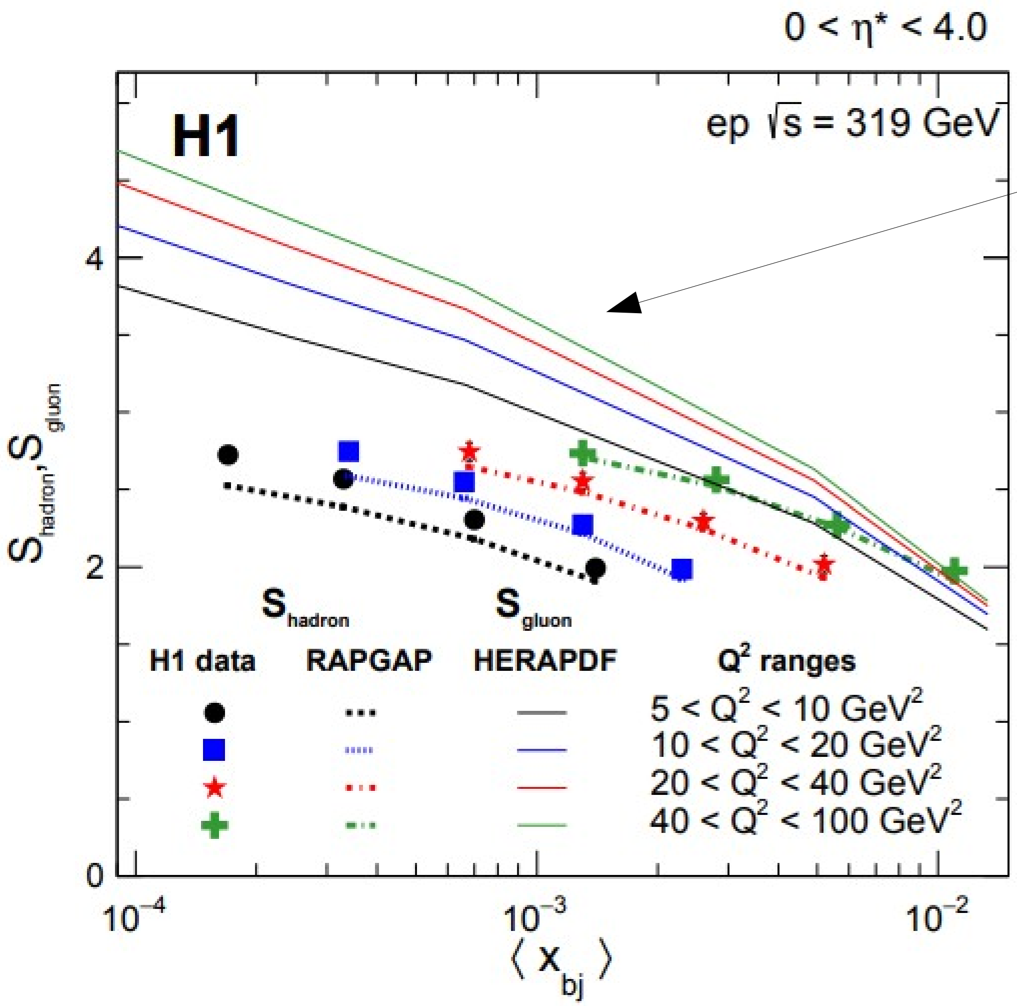


$$S_{hadron} = \sum P(N) \ln P(N)$$

The charged particle multiplicity distribution is measured in either the current fragmentation region or the target fragmentation region.

Fraction of events with charged hadron. Measurement performed in rapidity bins.

Monte Carlo, KL formula, and data



HERA pdf used

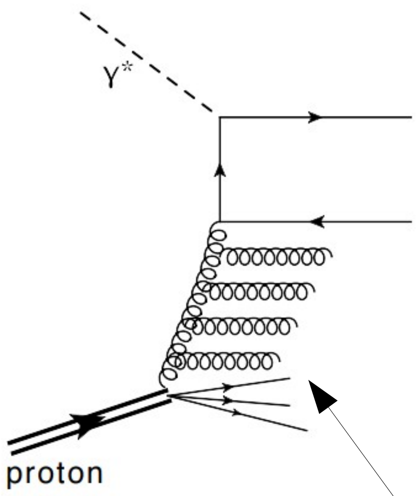
See also Kharzeev and Levin
to use quarks instead of gluons
Phys. Rev. D 104, 031503 (2021)

H1

Eur.Phys.J.C 81 (2021) 3, 212

See also Z. Tu, D. Kharzeev, T. Ulrich ’20
for calculations of EE in p-p.

Gluon and quark distribution



In the linear regime obeys BFKL equation. In our calculations we use NLO BFKL with kinematical improvements and running coupling. The gluon density has been fitted to F_2 data (exact kinematics was used)

Hentschinski, Sabio-Vera, Salas.
Phys.Rev.D 87 (2013) 7, 076005
Phys.Rev.Lett. 110 (2013) 4, 041601

We calculate the sea quarks distribution using

$$x\Sigma(x,Q) = P_{qg}(Q,\mathbf{k}) \otimes \mathcal{F}(x,\mathbf{k}^2)$$

$$xg(x,Q) = \int_0^{Q^2} d\mathbf{k}^2 \mathcal{F}(x,\mathbf{k}^2)$$

Transverse momentum dependent splitting function
Catani, Hautmann
Nucl.Phys. B427 (1994) 475-524

Other methods for resummation:
KMS (Kwiecinski, Martin, Stasto);
CCSS (Colferai, Ciafaloni, Staśto, Salam)

Gluon distribution

NLO BFKL with collinear resummation

$$\mathcal{F}\left(x,\boldsymbol{k}^2,Q\right)=\frac{1}{\boldsymbol{k}^2}\int_{\frac{1}{2}-i\infty}^{\frac{1}{2}+i\infty}\frac{d\gamma}{2\pi i}\ \hat{g}\left(x,\frac{Q^2}{Q_0^2},\gamma\right)\ \left(\frac{\boldsymbol{k}^2}{Q_0^2}\right)^{\gamma}$$

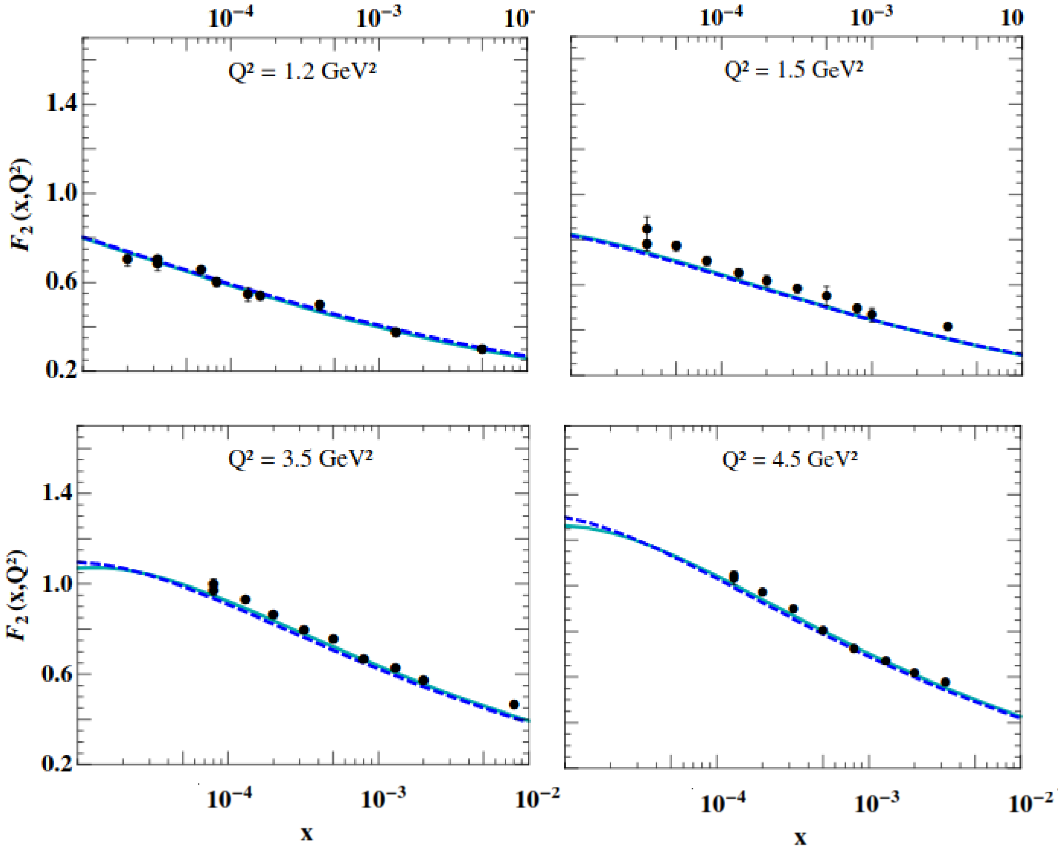
$$\hat{g}\left(x,\frac{Q^2}{Q_0^2}\gamma\right)=\frac{\mathcal{C}\cdot\Gamma(\delta-\gamma)}{\pi\Gamma(\delta)}\cdot\left(\frac{1}{x}\right)^{\chi(\gamma,Q,Q)}\left\{1+\frac{\bar{\alpha}_s^2\beta_0\chi_0\left(\gamma\right)}{8N_c}\log\left(\frac{1}{x}\right)\left[-\psi\left(\delta-\gamma\right)+\log\frac{Q^2}{Q_0^2}-\partial_\gamma\right]\right\}$$

the low x growth

Hentschinski, Sabio-Vera, Salas.
Phys.Rev.D 87 (2013) 7, 076005
Phys.Rev.Lett. 110 (2013) 4, 041601

F_2 from HSS fit

F_2 data description



Hentschinski, Sabio-Vera, Salas.
Phys.Rev.D 87 (2013) 7, 076005
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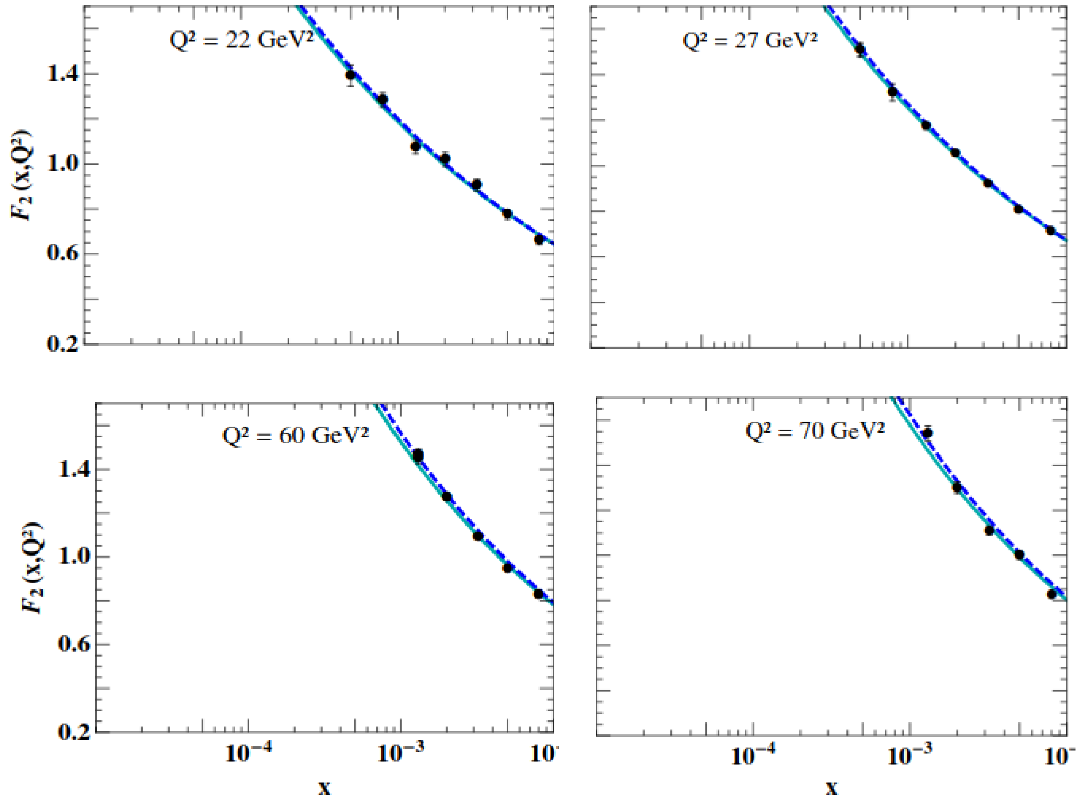
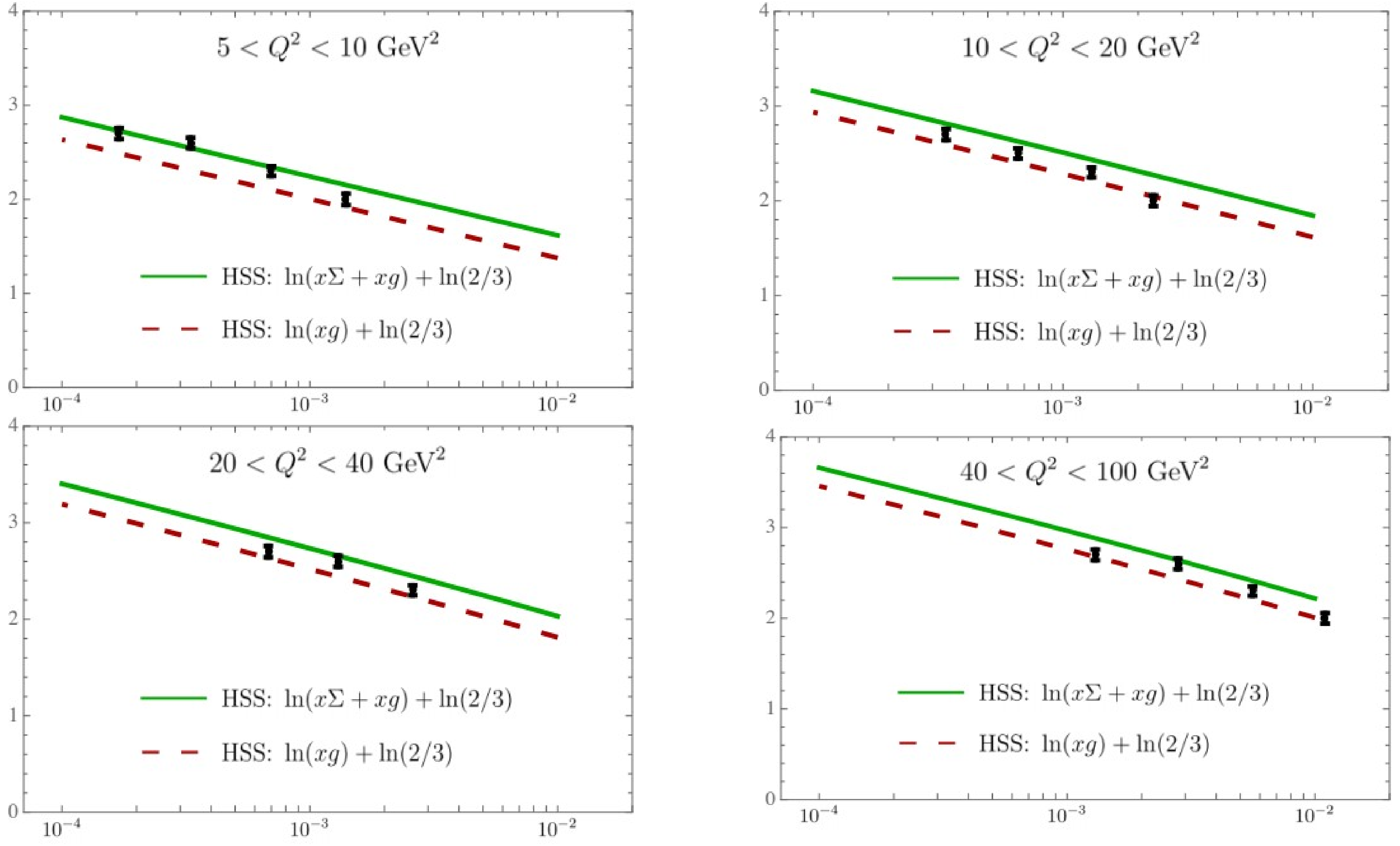


Hentschinski, Sabio-Vera, Salas.
Phys.Rev.D 87 (2013) 7, 076005
Phys.Rev.Lett. 110 (2013) 4, 041601

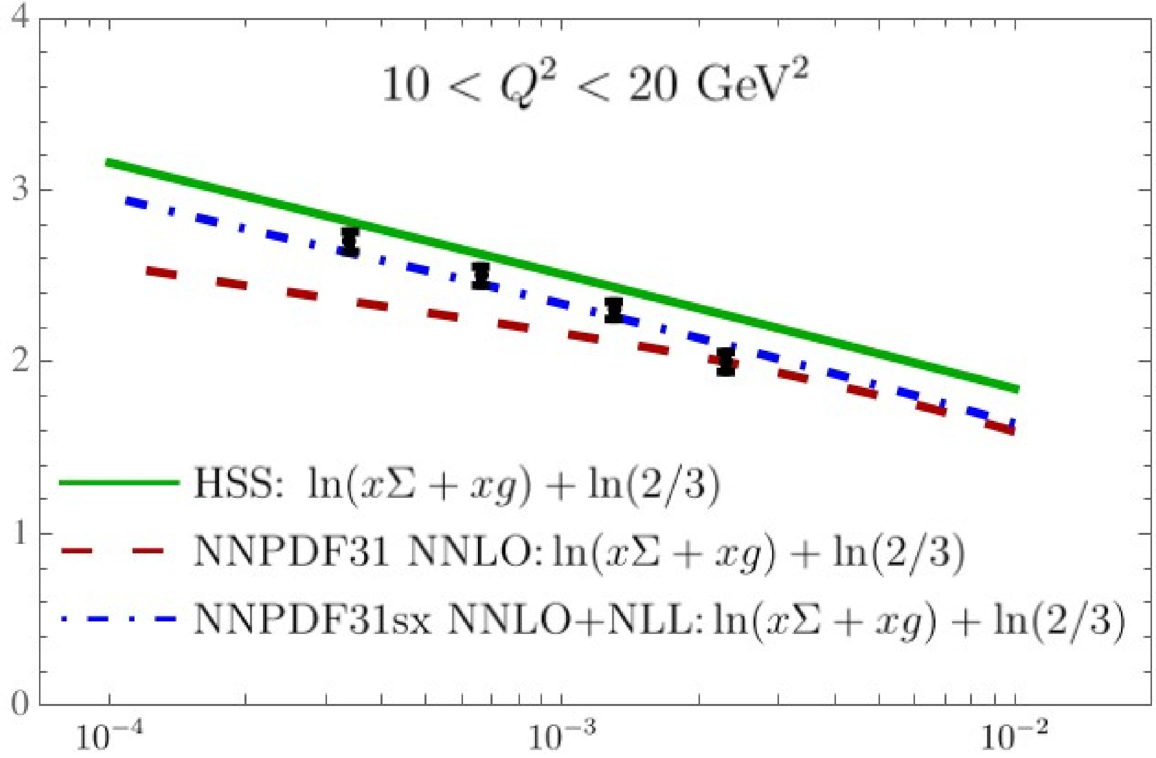
Results 1



Hint that the general idea works. Gluon dominates over quarks.
One has to also take into account that only charged pions were measured i.e 2/3 of partons contribute

Results 2 – shape

Eur.Phys.J.C 82 (2022) 2, 111 Hentschinski, Kutak



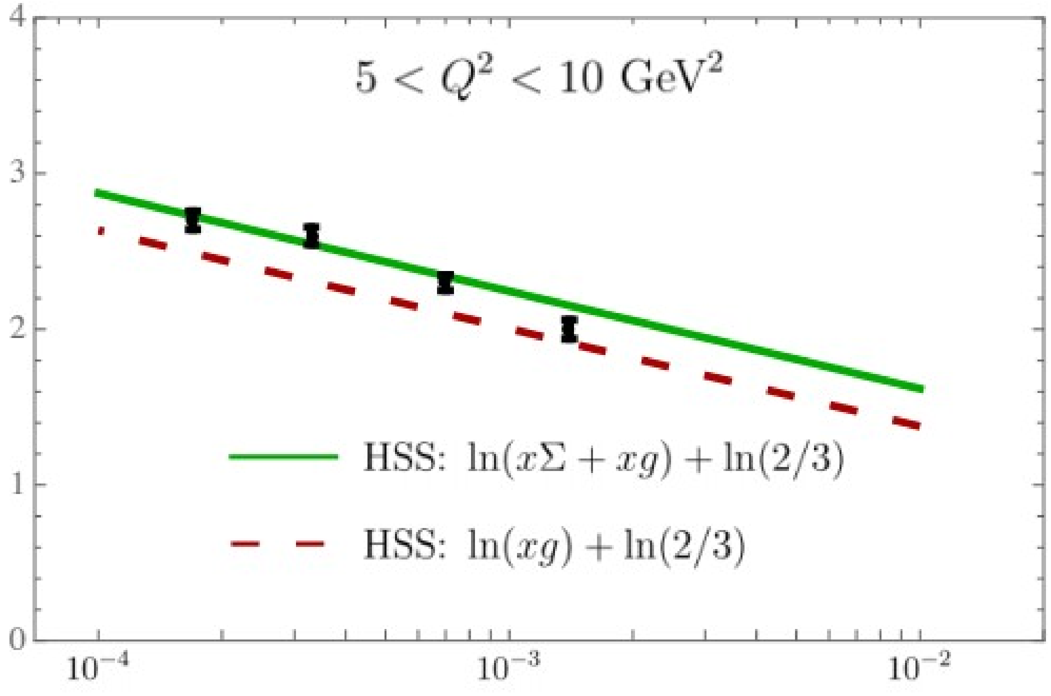
Low x resummation is essential

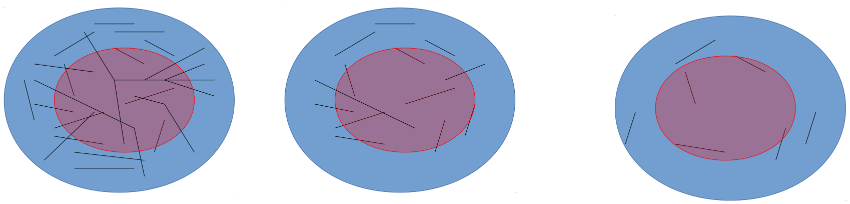
HSS gluon density used
i.e. NLO BFKL + kinematical Improvements
Hentschinski, Sabio-Vera, Salas.
Phys.Rev.D 87 (2013) 7, 076005
Phys.Rev.Lett. 110 (2013) 4, 041601

NNPDF 31 → DGLAP
NNPDF 31sx → DGLAP + low x resummation

Large uncertainities of pdfs. In this study we did not take them into account.

Dipoles and mechanism of entanglement





segments – dipoles, color singlets
maximally entangled states

red circle – resolved area defined by photon

entanglement arises because of dipoles that are partially in the red circle and partially in blue.

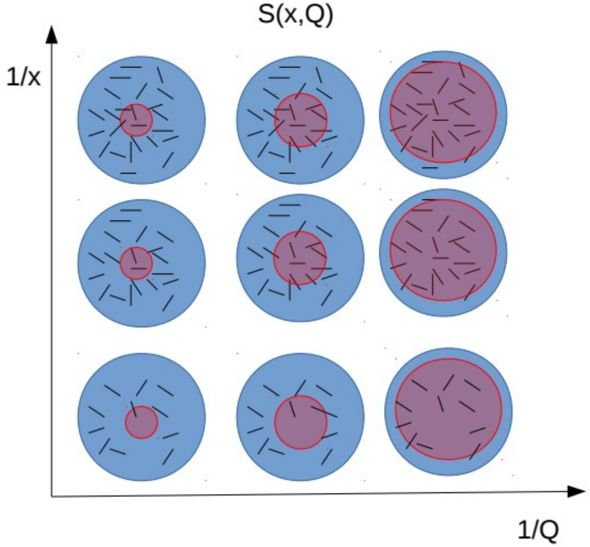
The broken dipoles contribute to final state hadron multiplicity and entropy of proton

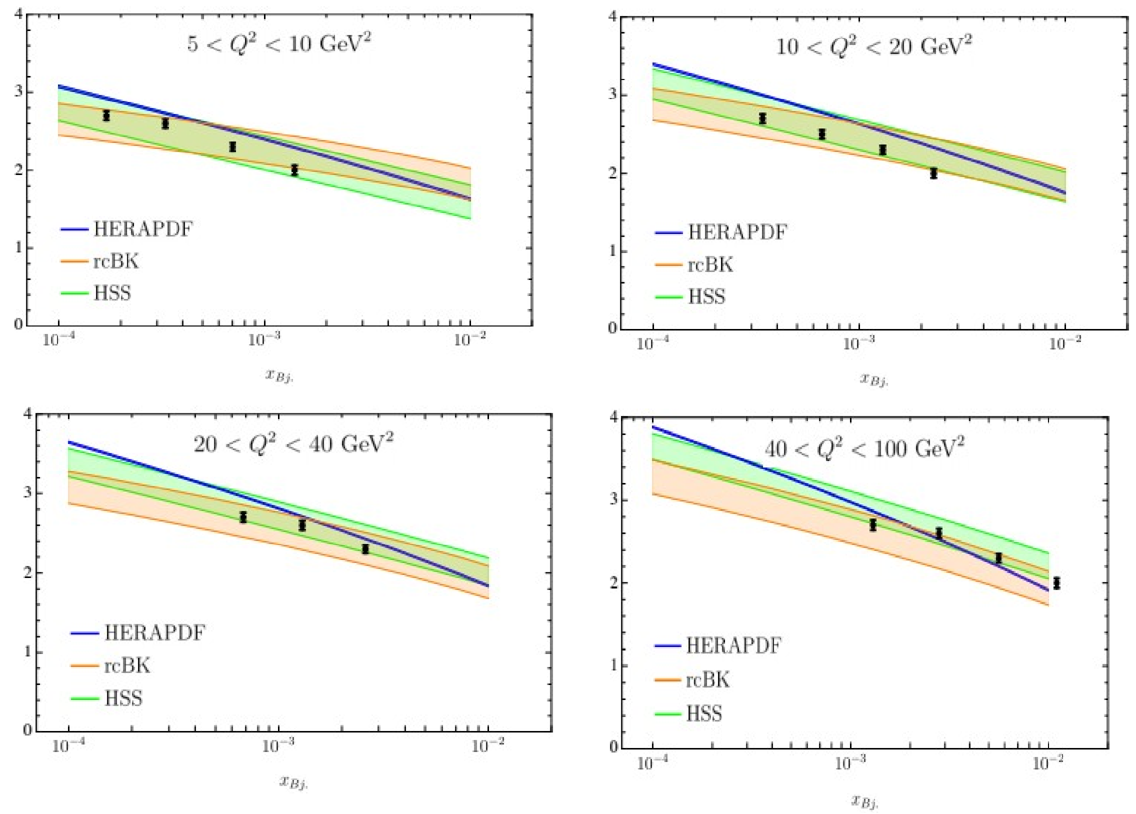
If we go to lower x we have more and more dipoles that cross the red line and entanglement grows

"Entanglement of predictions arises from the fact that the two bodies at some earlier time from in the true sense one system that is were interacting and have left behind choices on each other."

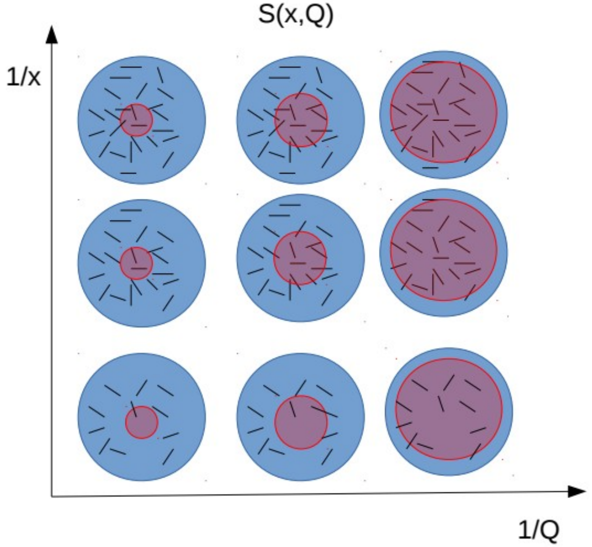
E. Schrodinger

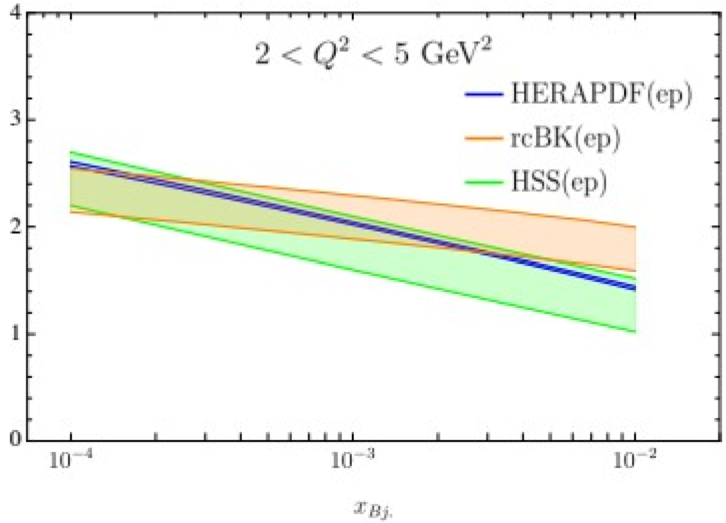
Large scales - description

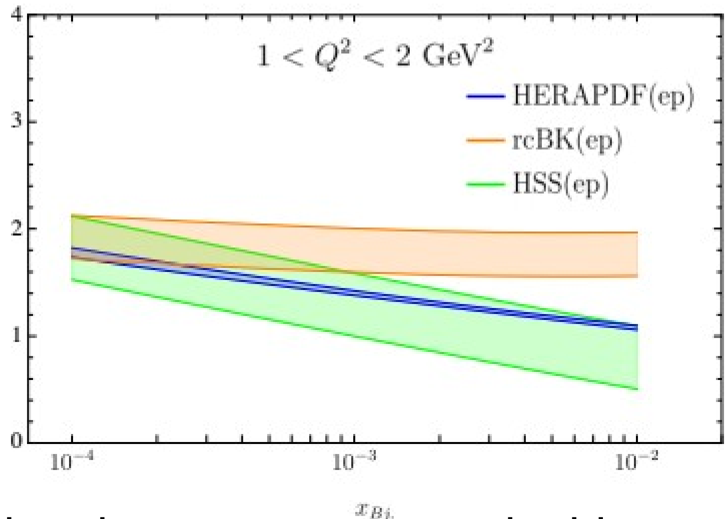




Small scales - prediction

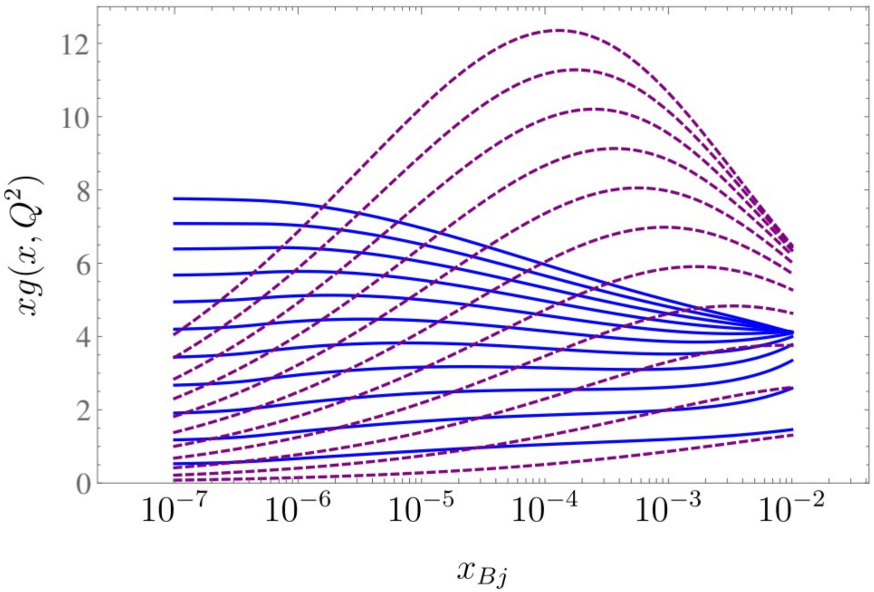


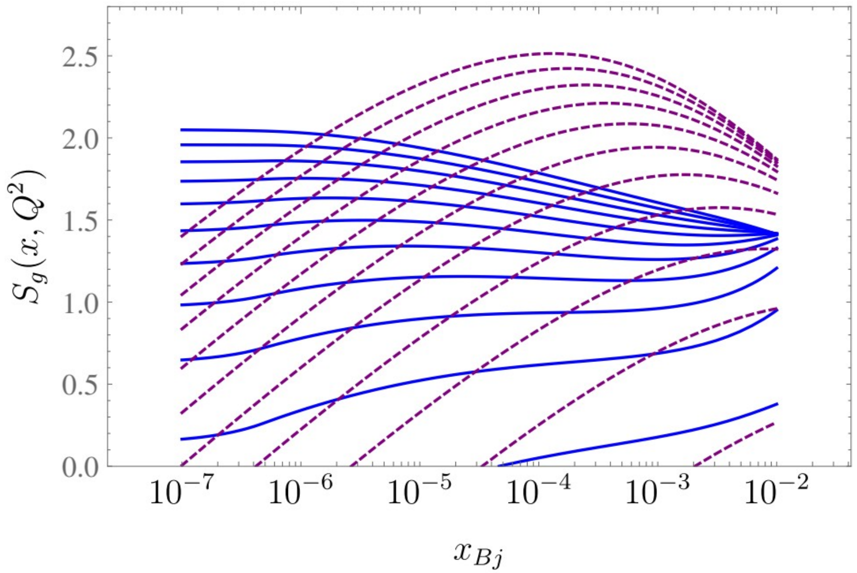




See also Hagivara, Hatta, Xiao ' 18
The genaralized KL model is used and entropy saturates in this approach
and Nowak, Liu, Zahed '22

Integrated gluon and entropy





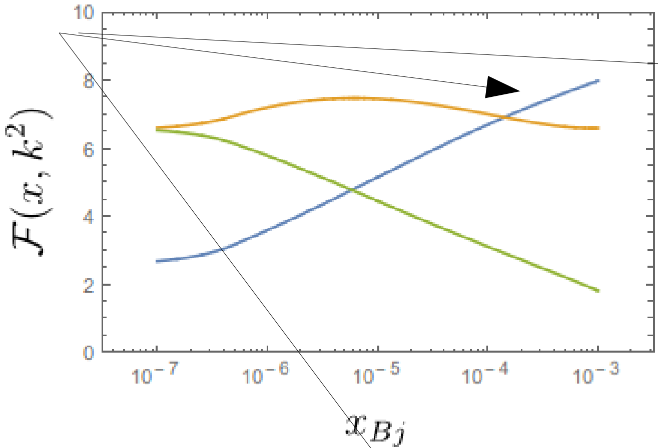
$$\lim_{Q^2 \gg Q_s^2} S(x,Q^2) = \ln\left(S_\perp Q_s^2(x)\right) + \ln\frac{N_c}{8\alpha_s\pi^2} = \lambda \ln\frac{1}{x} + \text{const}$$

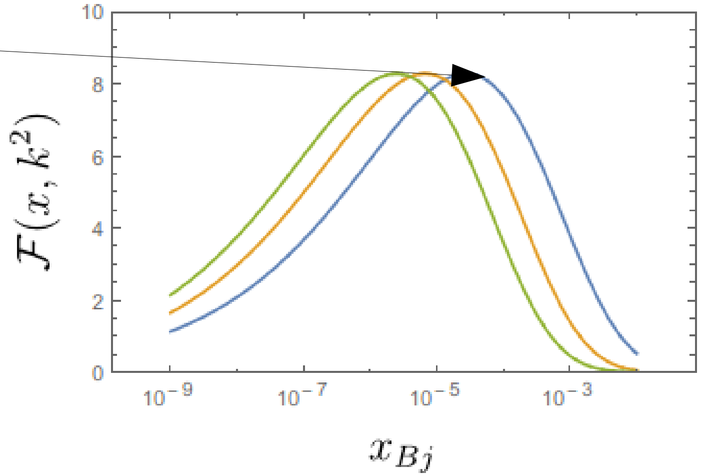
$$\lim_{Q^2 \ll Q_s^2} S(x,Q^2) = \ln\left(\frac{S_\perp Q^4}{Q_s^2(x)}\right) + \ln\frac{N_c}{16\alpha_s\pi^2}$$

Photon can not resolve proton anymore therefore the EE vanishes.
But it might be that the formalism breaks down for low scales.
There might be another source of entropy that keep the total entropy not vanishing → generalized second law Bekenstein

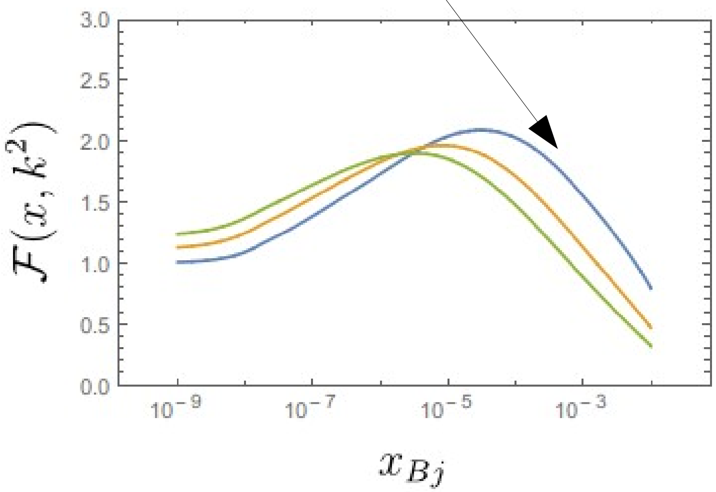
Dipole gluon density – x dependence, fixed k_T

small kT rcBK





GBW



$$\mathcal{F}(x,k^2) = \frac{N_c k^2 S_\perp}{8\pi^2\alpha_s}\int dr^2 (1 - N(r,x)) J_0(r^2k^2)$$

$$\mathcal{F}(x,k^2) = \frac{N_c S_\perp}{\alpha_s 8\pi^2}\frac{k^2}{Q_s^2}e^{-k^2/Q_s^2}$$

KS gluon i.e. BK + kinematical corrections

Similar plots in
hep-ph/0703068
KK PhD thesis

Entropy, area and gluon density

Process: inclusive gluon production in central rapidity.
2+1 case using GBW model.

K. Kutak '11

$$S = \frac{6C_F A_\perp}{\pi \alpha_s} Q_s^2(x) + S_0$$

$$N_G(x) \equiv \frac{dN}{du} = \frac{1}{S_\perp} \frac{d\sigma}{dy}$$

$$dE = TdS$$

$$M_G(x) = Q_s(x)$$

$$dM = TdS$$

$$T = \frac{Q_s(x)}{2\pi}$$

Kharzeev, Tuchin '05

$$n_G(x) \equiv \frac{1}{\pi} \int d^3r\, d^2k\, \Phi(x,k,r) = \frac{1}{\pi} \int d^2k\, \phi(x,k^2)$$

$$\frac{d\sigma}{dy} = \frac{2A_\perp^2 C_F Q_{s\,1}^2}{\pi^2 \alpha_s}$$

CGC review
Iancu, Venugopalan '03

$$n_G(x) = \frac{C_F A_\perp}{2\pi^2 \alpha_s} Q_s^2(x)$$

$$S = 12\pi\, n_G(x) + 3\pi\, N_{G\,0}$$

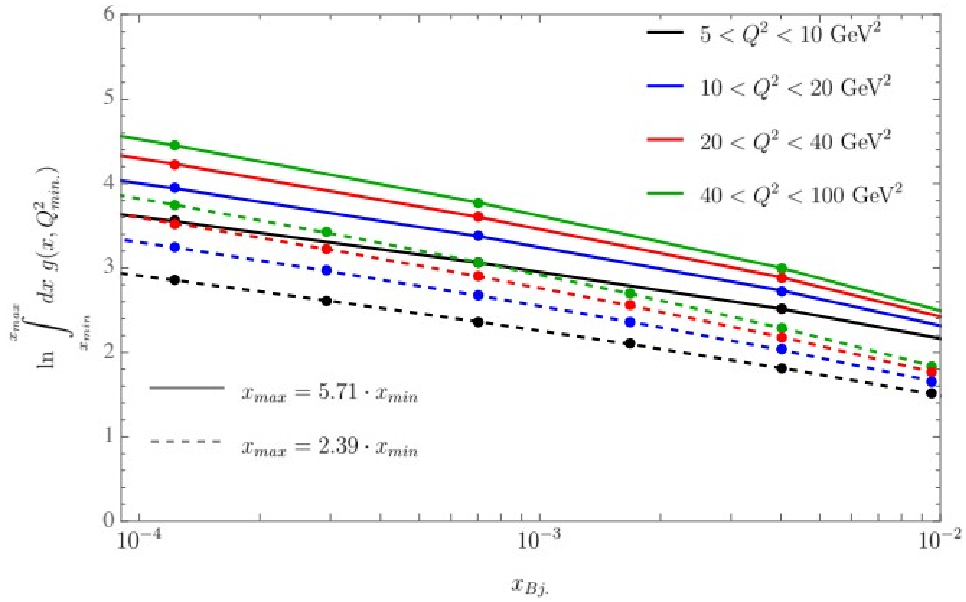
See also
Hagiwara, Hatta, Xiao, Yuan '18
Dvali, Venugopalan,21

Conclusions and outlook

- We show that the Kharzeev and Levin proposal for low x maximal entanglement entropy has a point.
- It can be systematically improved (quark contributions, NLO BFKL) and can describe successfully H1 data.
- We therefore provide phenomenological evidence which is essential for the further development of the field.
- We obtain saturation of entropy at small resolution scales.
- The hard scale dependence should be derived more directly.

Bacukp

Bining and KL formula



$$n_g(Q^2) = \int_0^1 dx g(x, Q^2)$$

$$n_g(\bar{x}) = \int_{x_{\min}}^{x_{\max}} dx g(x, Q^2)$$

$$\bar{n}_g(\bar{x}) = \frac{1}{y_{max} - y_{min}} \int_{y_{min}}^{y_{max}} dy \frac{dn_g}{dy} = \frac{n_g(y_{max}) - n_g(y_{min})}{y_{max} - y_{min}} \qquad y_{max,min} = \ln 1/x_{min,max}$$

$$\bar{n}_g(x, Q^2) = \frac{dn_g}{d\ln(1/x)} = xg(x, Q^2$$

$$\langle \bar{n}(x, Q^2) \rangle_{Q^2} = \frac{1}{Q^2_{\max} - Q^2_{\min}} \int_{Q^2_{\min}}^{Q^2_{\max}} dQ^2 \left[xg(x, Q^2) + x\Sigma(x, Q^2) \right]$$

$$\langle S(x, Q^2) \rangle_{Q^2} = \ln \langle \bar{n}(x, Q^2) \rangle_{Q^2}$$