Pairwise States vs. Dressed States and the Geometric Phase for Monopoles

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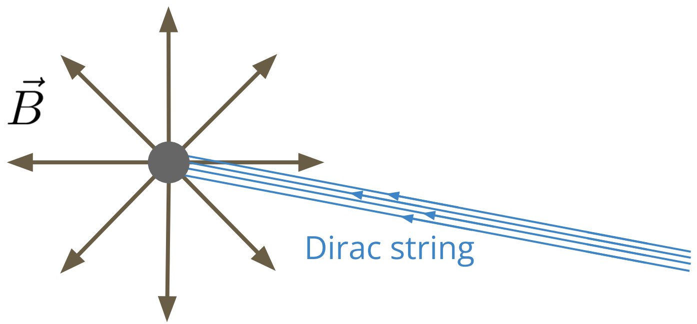
2209.03369 w/ C. Csáki, Z. Dong, J. Terning, S. Yankielowicz

Magnetic Monopoles

Sources of U(1) field with non-trivial winding number $\pi_1[\mathrm{U}(1)] = \mathbb{Z}$

$$\vec{B} = \frac{g}{4\pi r^2}\hat{r}$$



At $r \gg m^{-1}$ effectively abelian

Maxwell's eqs → Need Dirac string* Dirac '31

String unobservable → Dirac quantization Dirac '31, Wu & Yang '76

* Or nontrivial gauge bundle

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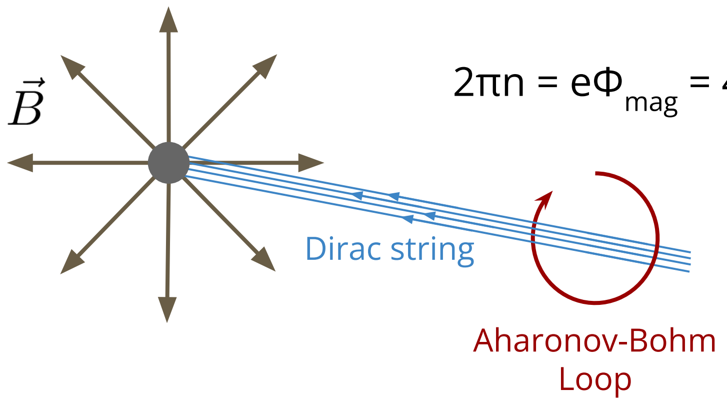

$2\pi n = e\Phi_{mag} = 4\pi q \qquad q \equiv eg/4\pi$

$$\boxed{q = n/2}$$

At $r \gg m^{-1}$ effectively abelian

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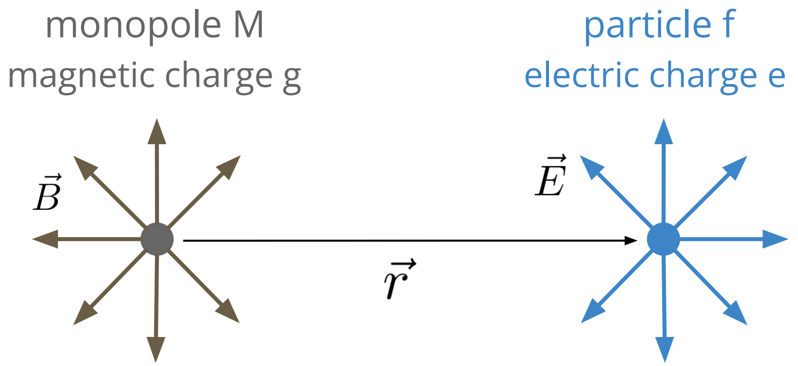
Monopoles and Charges: Angular Momentum in EM Field

Thomson 1904



$$\vec{J}_{\text{field}} = \frac{1}{4\pi}\int \vec{r}\,' \times \left(\vec{E}\times\vec{B}\right)\, d^3r\,' = -\frac{g}{4\pi}\int \left(\vec{\nabla}\,'\cdot\vec{E}\right)\hat{r}\,'\, d^3r' = -eg\hat{r}$$

Distance independent!

Some Overarching Themes and Questions

What is the action for electrodynamics + Monopoles ? How do we quantize it?

Weinberg '65; Schwinger '66; Zwanziger '71

What is the space of quantum multiparticle states with charges and monopoles?

Where does the extra angular momentum enter in the quantum (field) theory?

What is the role of Dirac quantization in the quantum field theory?

What is the role of the Dirac String in the quantum field theory?

Zwanziger '72; Csaki, Hong, Shirman, Terning, OT, Waterbury '20

Csaki, Hong, Shirman, Terning, OT '21 (PRL); Csaki, Shirman, Terning, OT '21; This work

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But also

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What Are the Quantum Multiparticle States for Charges and Monopoles?

Extra angular momentum in EM field, sourced by all *pairs* of charges and monopoles

$$\longrightarrow \quad |\text{charge}, \text{monopole}\rangle \neq |\text{charge}\rangle \otimes |\text{monopole}\rangle$$

Charges-monopole multiparticle states don't live in a *Fock Space*

How do we define charge-monopole multiparticle states?

The definition should reflect extra EM angular momentum in the EM field

Electric-Magnetic "Pairwise" States

Zwanziger '72

Csaki, Hong, Shirman, Terning, OT, Waterbury '20

Csaki, Hong, Shirman, Terning, OT, PRL '21

Electric-magnetic multiparticle states $\Big|\underbrace{p_1, \ldots, p_n}_{\text{momenta}};\ \underbrace{\sigma_1, \ldots, \sigma_n}_{\text{spins / helicities}};\ \underbrace{q_{12},\ q_{13}, \ldots, q_{n-1,n}}_{\text{pairwise helicities}}\Big\rangle$

The pairwise helicities are the Dirac-quantized $q_{ij} = \dfrac{e_i g_j - e_j g_i}{4\pi}$

They are the S-matrix manifestation of the extra angular momentum

$$\Delta \vec{J}_{ij} = -q_{ij}\hat{r}_{ij}$$

carried by the electromagnetic field that's sourced by the dyon (or charge-monopole) pair (i,j)

Electric-Magnetic "Pairwise" States

Zwanziger '72

Csaki, Hong, Shirman, Terning, OT, Waterbury '20

Csaki, Hong, Shirman, Terning, OT, PRL '21

Under a Lorentz transformation, transform with an extra pairwise Little Group phase:

$$U(\Lambda)\left|p_1,\ldots,p_n;\sigma_1,\ldots,\sigma_n;q_{12},q_{13},\ldots q_{f-1,f}\right\rangle =$$

$$\underbrace{e^{i\Phi_{LG}}}_{\text{pairwise phase}}\prod_{i=1}^{f}\mathcal{D}^{i}_{\sigma_i'\sigma_i}\left|\Lambda p_1,\ldots,\Lambda p_f;\sigma_1',\ldots,\sigma_f';q_{12},q_{13},\ldots,q_{f-1,f}\right\rangle$$

The pairwise phase $\Phi_{LG}\equiv-\sum_{l<m}q_{lm}\varphi_{LG}\left(p_l,p_m,\Lambda\right)$ leads to modified angular-momentum selection rules in the scattering of charges, monopoles and dyons

The Pairwise Little Group; A Short Dive-In

Consider the charge-monopole state $|p_1, p_2; q_{12}\rangle$

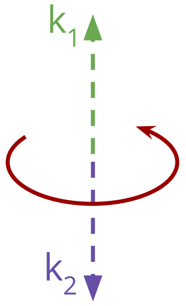
How does it transform under Lorentz?

1. Define the COM momenta $k^\mu_{1,2} = \left(E^c_{1,2}, 0, 0, \pm p_c\right)$

2. The pairwise Little Group (LG) is the subgroup of Lorentz which keeps k_1, k_2 invariant, i.e. U(1) rotations around the z-axis

3. The pairwise helicities q_{ij} label representations of each $U(1)_{ij}$

$$U\left[R_z(\varphi)\right]|k_1, k_2; q_{12}\rangle = e^{iq_{12}\varphi}\,|k_1, k_2; q_{12}\rangle$$



Pairwise Little Group = z-rotations

$$p_c = \sqrt{\frac{(p_1 \cdot p_2)^2 - m_1^2 m_2^2}{s}}$$

$$E^c_i = \sqrt{m_i^2 + p_c^2}$$

The Pairwise Little Group; A Short Dive-In

Zwanziger '72

Consider the charge-monopole state $|p_1, p_2; q_{12}\rangle$

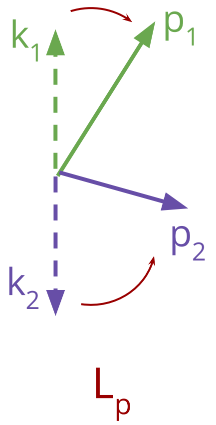
How does it transform under Lorentz?

4. Define L_p so that $p^{\mu}_{1,2} = [L_p]^{\mu}{}_{\nu}\, k^{\nu}_{1,2}$

5. Under any Lorentz transformation Λ

$$U[\Lambda]\,|p_1, p_2; q_{12}\rangle = e^{iq_{12}\varphi_{LG}(p_1,p_2,\Lambda)}\,|\Lambda p_1, \Lambda p_2; q_{12}\rangle$$

where $R_z\left[\varphi_{LG}\left(p_i, p_j, \Lambda\right)\right] = L^{-1}_{\Lambda p}\Lambda L_p$

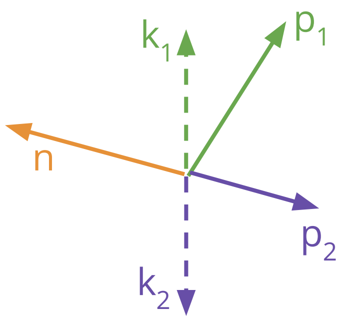


$\varphi_{LG}(p_1, p_2, \Lambda)$, Explicitly

Zwanziger '72

We fix the freedom in L_p by choosing arbitrary vector n^μ and let

$$[L_p]^\mu{}_2 = \hat{\epsilon}^\mu(p_1, p_2, n)$$



The pairwise phase is then

$$\boxed{\cos\left[\varphi_{LG}\left(p_1, p_2, \Lambda\right)\right] = \hat{\epsilon}\left(p_1, p_2, \Lambda^{-1} n\right) \cdot \hat{\epsilon}\left(p_1, p_2, n\right)}$$

n^μ is the pairwise LG analog of the Dirac string for the monopole

$$\hat{\epsilon}^\mu(abc) \equiv \frac{\epsilon^{\mu\nu\rho\sigma} a_\nu b_\rho c_\sigma}{|\epsilon^{\mu\nu\rho\sigma} a_\nu b_\rho c_\sigma|}$$

Pairwise States: Results

- Derived all 3pt amplitudes involving charges, monopoles, and dyons
- Reproduced the forced chirality flip in the lowest PW for fermion-monopole scattering
- Reconstructed monopole spherical harmonics from pairwise spinor-helicity variables

Csaki, Hong, Shirman, Terning, OT, Waterbury ‘20

- Derived geodesics in Taub-NUT background as the classical limit of “pairwise” amplitudes

Kol, O’Connell, OT, ‘21

- Defined pairwise states for mutually-non-local branes — Csaki, Terning, OT, upcoming
- Proposed an on-shell derivation for monopole catalysis of nucleon decay (the Rubakov-Callan effect)

Csaki, Shirman, Terning, OT, ‘21

A Lingering Question and Its Straightforward Answer

What is the actual origin of the “pairwise” charge-monopole states?

How does QED+monopoles “know” to create such complicated multiparticle states?

Today - a straightforward answer

$$|p_1, \ldots, p_f \,;\, q_{12}, q_{13}, \ldots, q_{n-1n}\rangle = |p_1, \ldots, p_f\rangle\!\rangle$$

pairwise
states

IR-dressed
charge-monopole states
of monopole-QED (QEMD)

Dressed States in QED

The QED S-matrix is IR-finite when taken between asymptotic states “dressed” by soft photons

$$S_{\mathrm{QED,finite}} = \langle\!\langle p_1^{out}, \ldots, p_m^{in} | p_1^{in}, \ldots, p_n^{in} \rangle\!\rangle$$

Faddeev-Kulish:
$$|p_1, \ldots, p_n\rangle\!\rangle = \exp\left[\mathcal{T}\int_{-\infty}^{\infty} dt\, V_I^{as}(t)\right] |p_1, \ldots, p_n\rangle$$

IR-dressed state

“bare” state

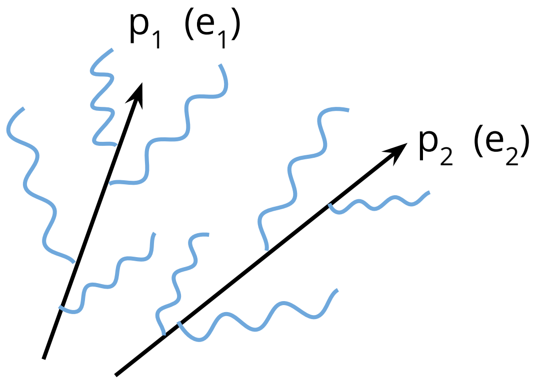
$$V_I^{as}(t) = -\lim_{t\to\pm\infty}\int d^3x\,[j^\mu A_\mu]$$

asymptotic potential:

generates retarded/advanced EM field associated with the charges moving with momenta $p_1,\ldots,p_n$

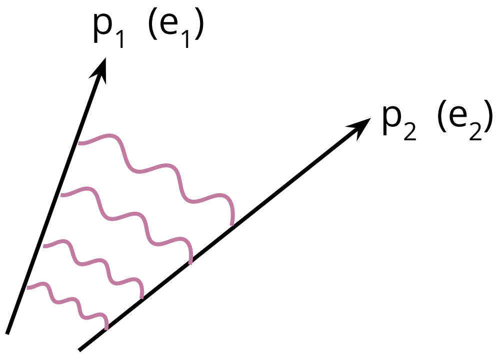
Dressed States in QED

$$\exp\left[\mathcal{T}\int_{-\infty}^{\infty} dt\, V_I^{as}(t)\right] = e^{R_{FK}}\, e^{i\Phi_{FK}}$$



$$R_{FK} = -i\int_{-\infty}^{\infty} dt\ V_I^{as}(t)$$

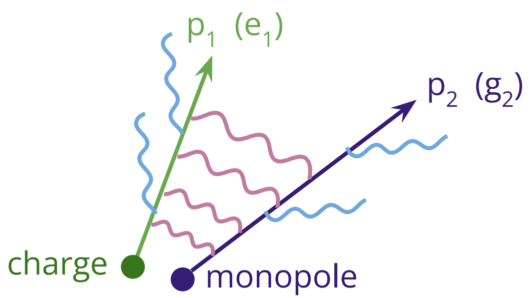
Real part of dressing (~soft photon creation operators)



$$\Phi_{FK} = \frac{i}{2}\int_{-\infty}^{\infty} dt_1 \int_{-\infty}^{t_1} dt_2\ \left[V_I^{as}(t_1)\,, V_I^{as}(t_2)\right]$$

Imaginary part of dressing (virtual photon exchange)

Dressed States in Monopole-QED (QEMD)



Add dual interaction: $V_{as}^I(t) = -\lim_{|t|\to\infty}\int d^3x\,[j_e^\mu A_\mu + j_g^\mu B_\mu]$

j_e^μ electric current density

j_g^μ magnetic current density

Gauge field $$A_\mu(x) = \sum_{a=\pm}\int \frac{d^3k}{(2\pi)^3}\frac{1}{2\omega_k}\left[\varepsilon_\mu^{*a}(\vec{k})a_a(\vec{k})e^{ik\cdot x} + \varepsilon_\mu^a(\vec{k})a_a^\dagger(\vec{k})e^{-ik\cdot x}\right]$$

Dual gauge field $$\widetilde{A}_\mu(x) = \sum_{a=\pm}\int \frac{d^3k}{(2\pi)^3}\frac{1}{2\omega_k}\underbrace{\left[\widetilde{\varepsilon}_\mu^{*a}(\vec{k})a_a(\vec{k})e^{ik\cdot x} + \widetilde{\varepsilon}_\mu^a(\vec{k})a_a^\dagger(\vec{k})e^{-ik\cdot x}\right]}_{\text{same creation/annihilation ops.}}$$

“One photon, two descriptions”

QEMD Dressed States = Pairwise States?

We want to show that the dressed state transforms the same as the pairwise state

$$U\left[\Lambda\right]|p_1,\ldots,p_f\rangle\!\rangle = e^{i\Phi_{LG}}\,|\Lambda p_1,\ldots,\Lambda p_f\rangle\!\rangle$$

For an infinitesimal $\Lambda^{\mu}_{\nu} = \exp\left(\delta\tau\omega^{\mu}_{\nu}\right)$ we have $U[\Lambda] = \exp\left[\frac{i}{2}\delta\tau M^{\mu\nu}\omega_{\nu\mu}\right]$

Where $M_{\mu\nu}$ is the Noether generator for Lorentz transformations in QEMD

From the definition of the dressed states:

$$\exp\left[\frac{i}{2}\delta\tau M^{\mu\nu}\omega_{\nu\mu}\right]e^{R_{FK}}e^{i\Phi_{FK}}\,|p_1,\ldots,p_f\rangle = e^{i\Phi_{LG}}e^{R_{FK}}e^{i\Phi_{FK}}\,|\Lambda p_1,\ldots,\Lambda p_f\rangle$$

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Using Baker-Campbell-Hausdorff, we need to show that

$$\Big\{ \underbrace{[M^{\mu\nu}, R_{FK}] + \frac{1}{2}\left[[M^{\mu\nu}, R_{FK}], R_{FK}\right]}_{\text{QEMD Lorentz generator acting on the real soft-photon cloud}} - \underbrace{\Delta\Phi^{\mu\nu}_{FK}}_{\text{shift of virtual soft photon phase under Lorentz}} \Big\} |p_1, \ldots, p_f\rangle = \underbrace{\Phi^{\mu\nu}_{LG}}_{\text{infinitesimal pairwise phase}} |p_1, \ldots, p_f\rangle$$

$$\Phi_{FK}\left|_{\Lambda p} - \Phi_{FK}\right|_{p} \equiv \frac{\delta\tau}{2}\omega_{\mu\nu}\Delta\Phi^{\mu\nu}_{FK} + \mathcal{O}\left(\delta\tau^2\right)$$

$$\Phi_{LG} \equiv \frac{\delta\tau}{2}\omega_{\mu\nu}\varphi^{\mu\nu}_{LG} + \mathcal{O}\left(\delta\tau^2\right)$$

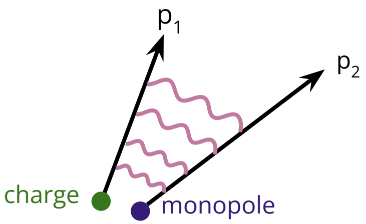
$$\Phi^{\mu\nu}_{LG} = -\sum_{l<m} q_{lm}\varphi^{\mu\nu}_{LG;lm}$$

$$\varphi^{\mu\nu}_{LG;lm} = \frac{\tau_{lm}}{\epsilon^2\left(p_l, p_m, n\right)} n^{[\mu}\epsilon^{\nu]}\left(p_l, p_m, n\right)$$

$$\tau_{lm} \equiv \sqrt{\left(p_l \cdot p_m\right)^2 - m_l^2 m_m^2}$$

we prove this via direct calculation

Calculating Φ_{FK} in QEMD



$$\Phi_{FK} \sim \frac{i}{2}\int_0^{\infty} dt_1 \int_0^{t_1} dt_2 \left[V_I^{as}(t_1), V_I^{as}(t_2)\right] - (t_1, t_2 \leftrightarrow -t_1, -t_2)$$

this has

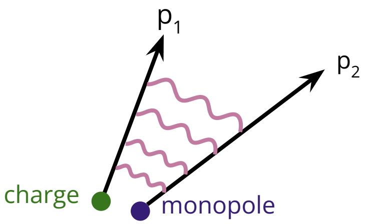
$$e\left[p_1 \cdot \varepsilon^{*a}(\vec{k})\right] a_a(\vec{k}) + \ldots$$

this has

$$g\left[p_2 \cdot \tilde{\varepsilon}^{a}(\vec{k})\right] a_a^{\dagger}(\vec{k}) + \ldots$$

$$\Phi_{FK} \sim \int d^3k \left[p_1 \varepsilon^a(\vec{k})\right]\left[p_2 \cdot \tilde{\varepsilon}^a(\vec{k})\right] \sim \underbrace{\int d^3k \frac{\epsilon(p_1, p_2, n, k)}{n \cdot k + i\epsilon} + cc.}_{\text{Integral over all soft photon exchanges between charge \& monopole}}$$

Calculating Φ_{FK} in QEMD

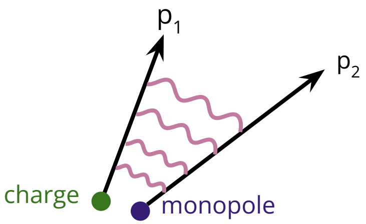


$$\Phi_{FK} = \sum_{l<m} q_{lm} \iint D_l p_a D_m p_b \left[\varphi_{FK}\left(p_a, p_b, n\right)\right]$$

$$\varphi_{FK}\left(p_1, p_2, p_3\right) = 4\pi \, \mathrm{Im}\left[\mathcal{I}\left(p_1, p_2, p_3\right) - \mathcal{I}\left(-p_1, p_2, p_3\right) - \mathcal{I}\left(p_1, -p_2, p_3\right) + \mathcal{I}\left(-p_1, -p_2, p_3\right)\right] \qquad p_3 \equiv n$$

$$D_l p \equiv d^3 p \left[b_l(\vec{p}) b_l^\dagger(\vec{p}) - d_l^\dagger(\vec{p}) d_l(\vec{p})\right] / \left[2\omega_l \cdot (2\pi)^3\right]$$

Calculating Φ_{FK} in QEMD



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where

$$\mathcal{I}\left(p_1, p_2, p_3\right) = \int \frac{d^4k}{(2\pi)^4} \frac{i\epsilon\left(p_1, p_2, p_3, k\right)}{\left(k^2 + i\epsilon\right)\left(p_1 \cdot k - i\epsilon\right)\left(p_2 \cdot k + i\epsilon\right)\left(p_3 \cdot k + i\epsilon\right)}$$

All that remains is to calculate this Feynman integral

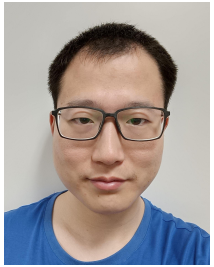
Alas! This integral is 0/0 and needs regularization...

$D_l p \equiv d^3p \left[b_l(\vec{p}) b_l^\dagger(\vec{p}) - d_l^\dagger(\vec{p}) d_l(\vec{p})\right] / \left[2\omega_l \cdot (2\pi)^3\right]$

A Slick Trick

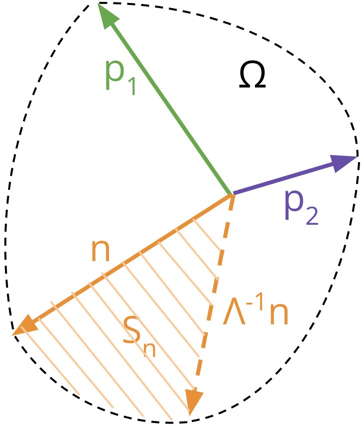
While $\mathcal{I}$ is ill-defined, what we need is actually $\Delta\Phi_{FK} \sim \Delta\mathcal{I}$

$$\Delta\mathcal{I} = \mathcal{I}\left(\Lambda p_1, \Lambda p_2, n\right) - \mathcal{I}\left(p_1, p_2, n\right)$$
$$= \mathcal{I}\left(p_1, p_2, \Lambda^{-1} n\right) - \mathcal{I}\left(p_1, p_2, n\right)$$



Ziyu Dong

which *is* well defined as can be shown in position space via Stokes' theorem



In momentum space, we showed using Schwinger parameters that

$$\Delta\mathcal{I} = -\frac{i}{2\pi}\Omega\left(-p_1, p_2, n, \Lambda^{-1} n\right)$$

So the integral really computes the 4D solid angle between $-p_1$, p_2, n, and Λ^{-1}n

Final Result for $\Delta\Phi_{FK}$

Substituting in $\varphi_{FK}(p_1,p_2,p_3) = 4\pi\,\mathrm{Im}\left[\mathcal{I}(p_1,p_2,p_3) - \mathcal{I}(-p_1,p_2,p_3) - \mathcal{I}(p_1,-p_2,p_3) + \mathcal{I}(-p_1,-p_2,p_3)\right]$

The 4D solid angle degenerates to a dihedral angle

$$\Delta\varphi_{FK}(p_1,p_2,n) = 2\arccos\left[\hat{\epsilon}\left(p_1,p_2,\Lambda^{-1}n\right)\cdot\hat{\epsilon}(p_1,p_2,n)\right] = -2\Phi_{LG}$$

shift in soft photon phase = -2 pairwise Little Group phase

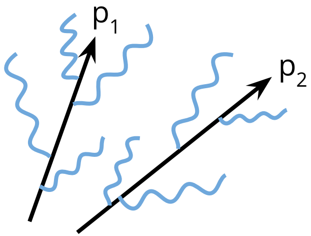
what about the contribution from real photons?

QEMD Dressed States = Pairwise States?

Using Baker-Campbell-Hausdorff, we need to show that

$$\left\{\underbrace{[M^{\mu\nu}, R_{FK}] + \frac{1}{2}\left[[M^{\mu\nu}, R_{FK}], R_{FK}\right]}_{\substack{\text{QEMD Lorentz generator acting}\\ \text{on the real soft-photon cloud}\\ \big\downarrow \\ ?}} - \underbrace{\Delta\Phi^{\mu\nu}_{FK}}_{\substack{\text{shift of virtual soft photon}\\ \text{phase under Lorentz}\\ \big\downarrow \\ 2\Phi^{\mu\nu}_{LG}}}\right\} |p_1, \ldots, p_f\rangle = \underbrace{\Phi^{\mu\nu}_{LG}}_{\substack{\text{infinitesimal}\\ \text{pairwise phase}}} |p_1, \ldots, p_f\rangle$$

Real Photon Contribution



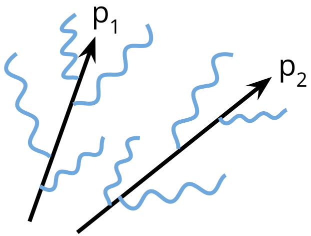
QEMD Energy-Momentum tensor: $\theta^{\mu\nu} = \theta^{\mu\nu}_{EM} + \theta^{\mu\nu}_{\varphi,A} + \theta^{\mu\nu}_{\varphi,B} + \dots$

$$\theta^{\mu\nu}_{EM} = \frac{1}{2}\left(F^{\mu}{}_{\alpha}F^{\alpha\nu} + \widetilde{F}^{\mu}_{\alpha}\widetilde{F}^{\alpha\nu}\right) \qquad \theta^{\mu\nu}_{\varphi,V} = \sum_l \frac{1}{2}\left(D^{\{\mu}_{V,l}\phi_l\right)\left(D^{\nu\}}_{V,l}\phi_l\right)^* - \frac{1}{2}\eta^{\mu\nu}\left[\eta_{\alpha\beta}\left(D^{\alpha}_{V,l}\phi_l\right)\left(D^{\beta}_{V,l}\phi_l\right)^* - m_l^2\phi_l\phi_l^*\right]$$

Noether generator for Lorentz: $M^{\mu\nu} \equiv M^{\mu\nu}_{EM} + M^{\mu\nu}_{\varphi,A} + M^{\mu\nu}_{\varphi,B}$ $\qquad M^{\mu\nu}_i = \int d^3x\; x^{[\mu}\theta^{0\nu]}_i$

commutators with R_{FK} ⟷ substituting “retarded EM field” in $M_{\mu\nu}$

Real Photon Contribution



Contribution from covariant derivative

$$[M_A^{\mu\nu} + M_B^{\mu\nu}, R_{FK}] = \sum_{l<m} q_{lm} \iint D_l p_a D_m p_b \left\{ \frac{\tau_{ab}\, n^{[\mu}\epsilon^{\nu]}\,(p_a, p_b, n)}{\epsilon^2\,(p_a, p_b, n)} - \frac{\epsilon^{\mu\nu}\,(p_a, p_b)}{\tau_{ab}} \right\}$$

Contribution from EM kinetic term

$$\frac{1}{2}\left[[M_{EM}^{\mu\nu}, R_{FK}], R_{FK}\right] = \sum_{l<m} q_{lm} \iint D_l p_a D_m p_b \left[\frac{\epsilon^{\mu\nu}\,(p_a, p_b)}{\tau_{ab}}\right]$$

Total contribution:

$$\left\{[M^{\mu\nu}, R_{FK}] + \frac{1}{2}\left[[M^{\mu\nu}, R_{FK}], R_{FK}\right]\right\} |p_1, \ldots, p_f\rangle = -\Phi_{LG}^{\mu\nu} |p_1, \ldots, p_f\rangle$$

QEMD Dressed States = Pairwise States!

Using Baker-Campbell-Hausdorff, we need to show that

$$\left\{ \underbrace{[M^{\mu\nu}, R_{FK}] + \frac{1}{2}\left[[M^{\mu\nu}, R_{FK}], R_{FK}\right]}_{\text{QEMD Lorentz generator acting on the real soft-photon cloud}} - \underbrace{\Delta\Phi^{\mu\nu}_{FK}}_{\text{shift of virtual soft photon phase under Lorentz}} \right\} |p_1, \ldots, p_f\rangle = \underbrace{\Phi^{\mu\nu}_{LG}}_{\text{infinitesimal pairwise phase}} |p_1, \ldots, p_f\rangle$$

QEMD Lorentz generator acting on the real soft-photon cloud $\longrightarrow -\Phi^{\mu\nu}_{LG}$

shift of virtual soft photon phase under Lorentz $\longrightarrow 2\Phi^{\mu\nu}_{LG}$

QEMD Dressed States = Pairwise States!

Using Baker-Campbell-Hausdorff, we need to show that

$$\Bigg\{\underbrace{[M^{\mu\nu}, R_{FK}] + \frac{1}{2}\left[[M^{\mu\nu}, R_{FK}], R_{FK}\right]}_{\substack{\text{QEMD Lorentz generator acting}\\ \text{on the real soft-photon cloud}\\ \downarrow \\ -\Phi^{\mu\nu}_{LG}}} - \underbrace{\Delta\Phi^{\mu\nu}_{FK}}_{\substack{\text{shift of virtual soft photon}\\ \text{phase under Lorentz}\\ \downarrow \\ 2\Phi^{\mu\nu}_{LG}}}\Bigg\} |p_1, \ldots, p_f\rangle = \underbrace{\Phi^{\mu\nu}_{LG}}_{\substack{\text{infinitesimal}\\ \text{pairwise phase}}} |p_1, \ldots, p_f\rangle$$

$$U[\Lambda]\,|p_1, \ldots, p_f\rangle\!\rangle = e^{i\Phi_{LG}}\,|\Lambda p_1, \ldots, \Lambda p_f\rangle\!\rangle$$ dressed states = pairwise states ✓

Bonus: A Nontrivial Geometric (Berry) Phase

We showed that the dressed states of monopole-QED transform as:

$$U[\Lambda]\,|p_1,\ldots,p_f\rangle\!\rangle = e^{i\Phi_{LG}}\,|\Lambda p_1,\ldots,\Lambda p_f\rangle\!\rangle$$

What happens if Λ is a 2π rotation?

$$\gamma_{\text{Berry}} \equiv \Phi_{FK}(2\pi\,\text{rotation}) = \pm 2\pi \sum_{l<m} q_{lm}$$

Allowing only fermionic or bosonic statistics, we have a fully relativistic derivation of Dirac quantization

If Σq_{lm} is a half-integer, we get a *minus sign* for a 2π rotation - like a fermion

For half-integer pairwise helicity, we can make fermions out of bosons! ✓

Conclusions

The multiparticle quantum states for charges and monopoles are not a Fock space;

They have pairwise helicities q_{ij} and transform with pairwise LG phases $\varphi_{LG}(p_i, p_j, \Lambda)$

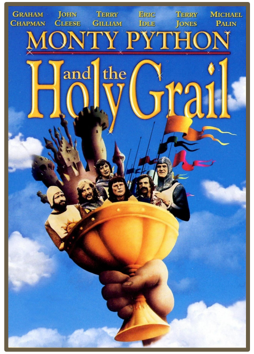
The pairwise LG provides modified angular momentum selection rules constraining the scattering amplitudes of monopoles and charges

The pairwise states are equivalent to the soft-photon dressed states of monopole-QED

The pairwise LG phase φ_{LG} is the shift of the soft photon phase φ_{FK} under a Lorentz transformation

The pairwise/dressed states have a geometric phase that can make fermions out of bosons!

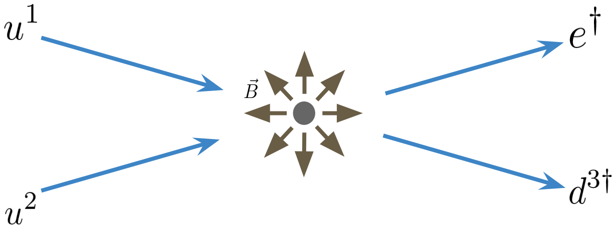
Future Directions



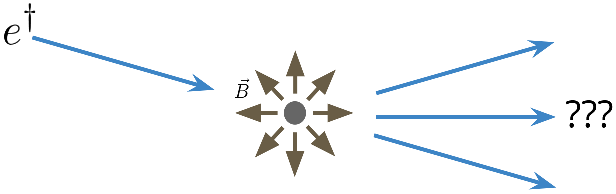
The holy grail:

A full 4D derivation of monopole catalysis of proton decay

And a solution to the monopole unitarity puzzle



monopole catalysis



monopole unitarity puzzle

We conjecture that the EM field sourced by monopoles and charges creates a (never before encountered) abelian instanton. This would-be instanton mediates proton decay at strong interaction rates.

Bonus: Topology!

Terning, Verhaaren '18

The soft photon phase φ_{FK} depends on the unphysical Dirac string

Can we measure it in an interference experiment?

For a closed path, we can apply Stokes' theorem

directly for φ_{FK} and not just $\Delta\varphi_{FK}$

However, the result is an unobservable $2\pi q_{12}$ x integer !

In fact, the φ_{FK} integral computes the topological linking number

between the charge worldline and the Dirac string worldsheet in 4D

This is the QFT generalization of the original Dirac quantization argument