CLUSTER OF EXCELLENCE
QUANTUM UNIVERSE

# Dark matter production from (p)reheating


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**Deutsches Elektronen-Synchrotron (DESY)**


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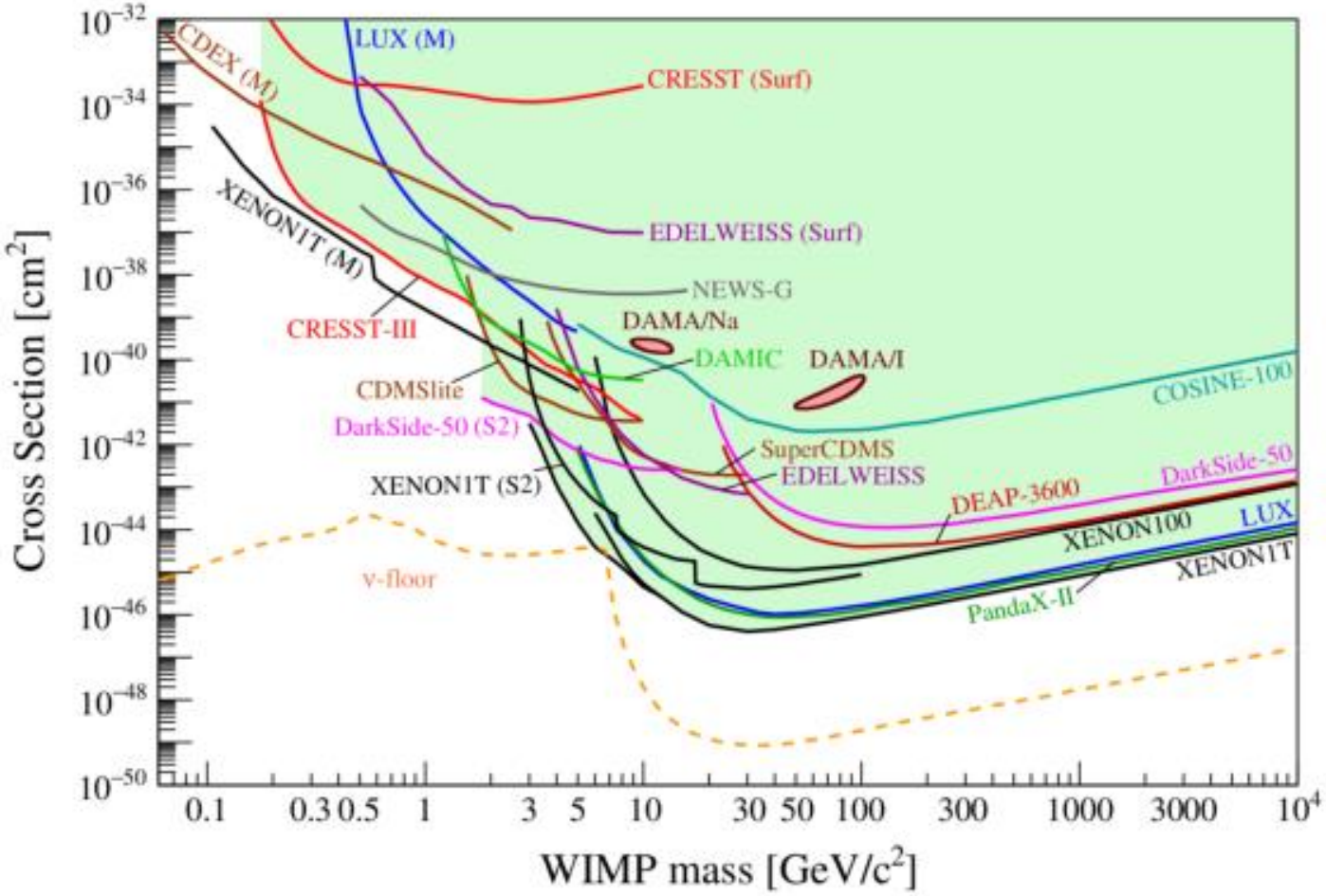
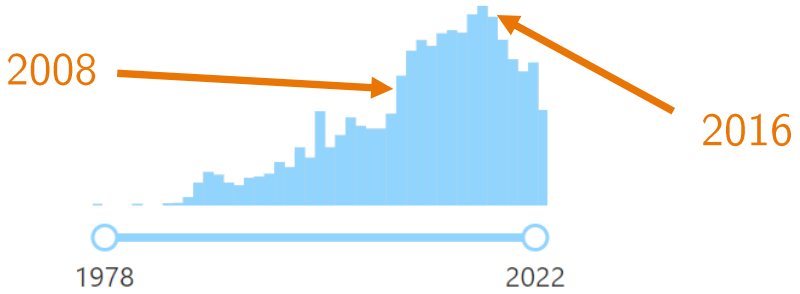
*Beyond the Standard Models: Particle Physics meets Cosmology*

Based on
[arXiv:2206.08940] with **M. A. G. Garcia** & **S. Verner**

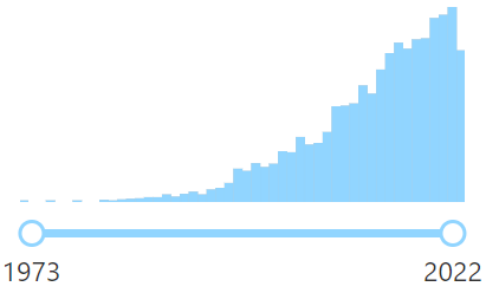
# Introduction



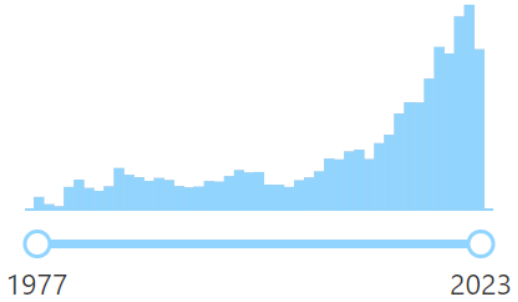

**No WIMP actually detected!**




Inspire-HEP papers "WIMP"



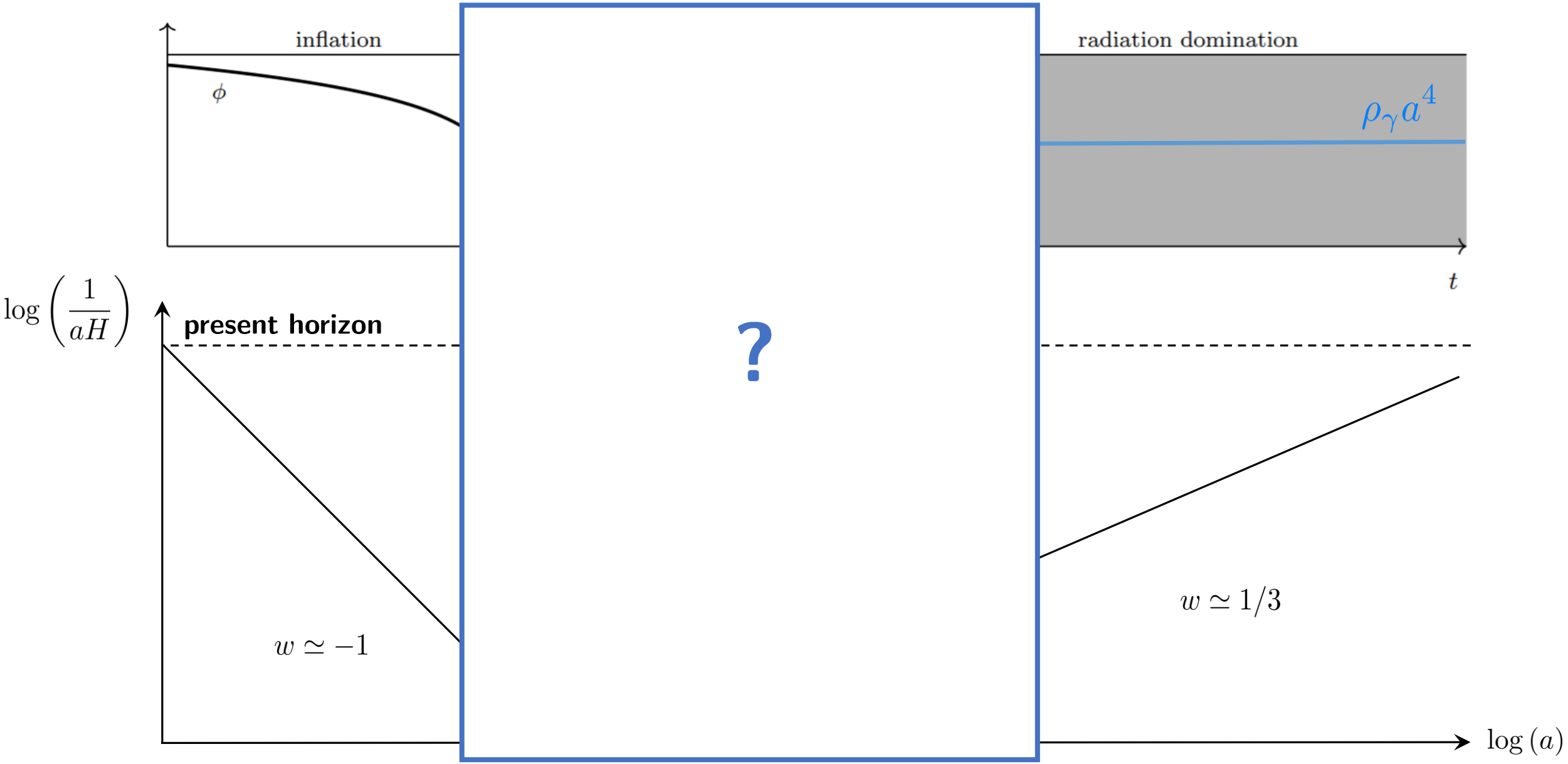

Inspire-HEP papers "freeze-in"




Inspire-HEP papers "axion"

**What could source dark matter production in the early universe?**

# The early universe



# End of inflation

For concreteness, **consider**

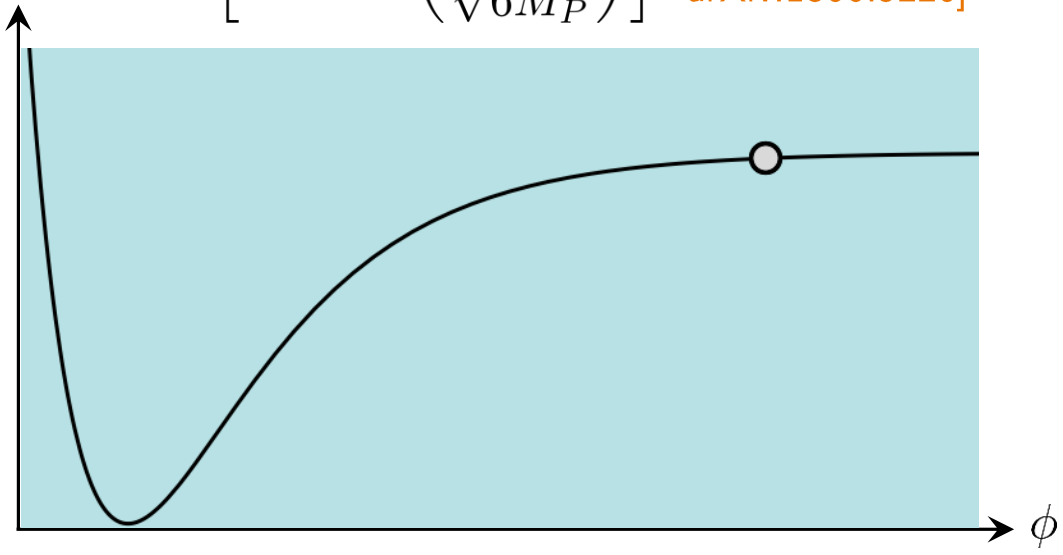
[J. Ellis, M. A. G. Garcia, D. V. Nanopoulos, K. A. Olive, and S. Verner - arXiv:2112.04466]

$$\mathcal{S} = \int \mathrm{d}^4x \sqrt{-g}\left(\frac{1}{2}(\partial_\mu\phi)^2 - V(\phi)\right)$$

$$\ddot{\phi} + 3H\dot{\phi} + V_\phi = 0$$

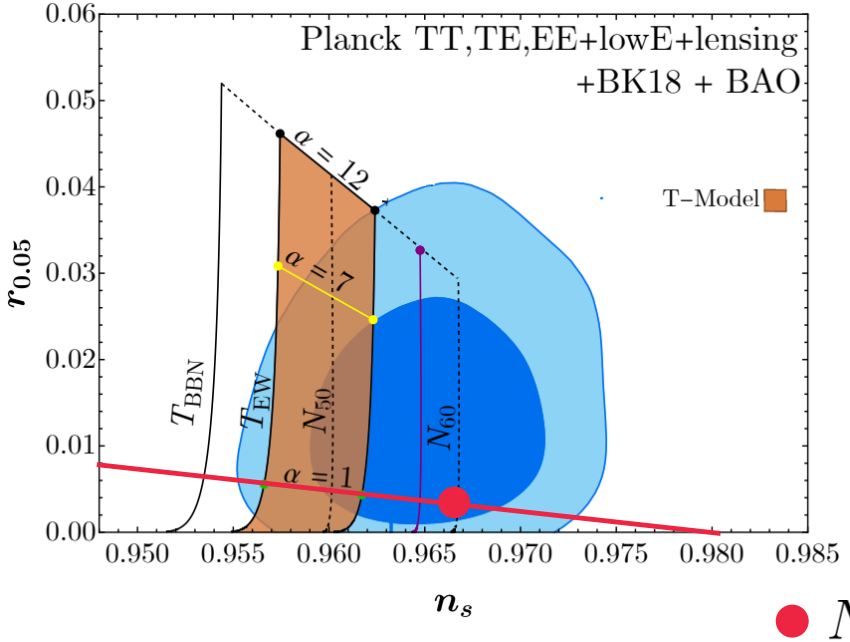
$$V(\phi) = \lambda M_P^4 \left[\sqrt{6}\tanh\left(\frac{\phi}{\sqrt{6}M_P}\right)\right]^2$$

[Kallosh & Linde arXiv:1306.5220]




$$\lambda \simeq \frac{18\pi^2 A_s(k_\star)}{6N_*^2} \sim 10^{-11}$$

$k_* = 0.05\,\mathrm{Mpc}^{-1}$




● $N_\star = 60$

$$r \simeq 16\epsilon_\star \simeq \frac{12}{N_\star^2} \qquad n_s \simeq 1 - 6\epsilon_\star + 2\eta_\star \simeq 1 - \frac{2}{N_\star}$$

$A_s(k_\star) \simeq 2.1 \times 10^{-9}$ [Planck 18']

$\phi$ **inflaton**

# End of inflation

For concreteness, **consider**

$$\mathcal{S} = \int \mathrm{d}^4x \sqrt{-g}\Big(\frac{1}{2}(\partial_\mu\phi)^2 - V(\phi)\Big)$$

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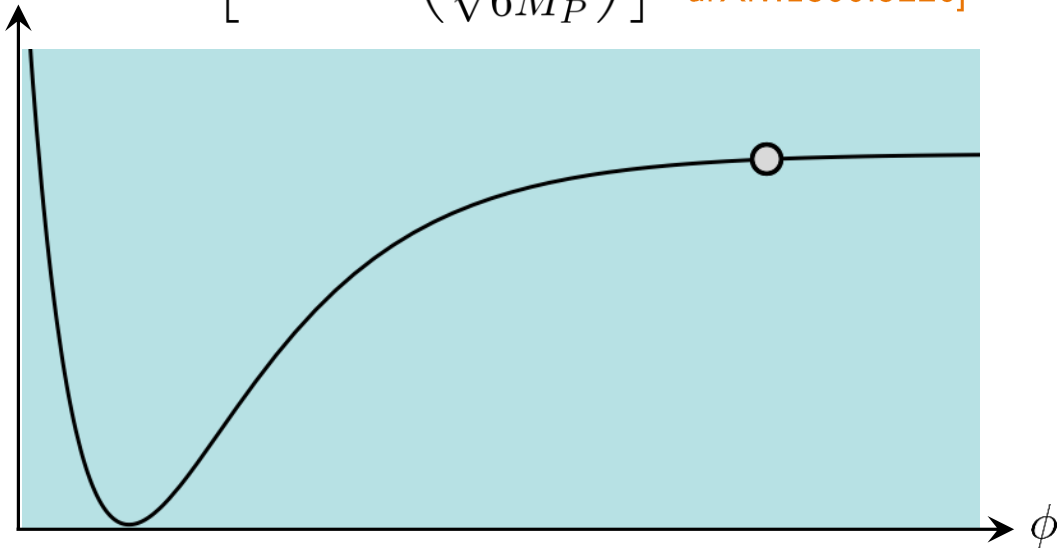
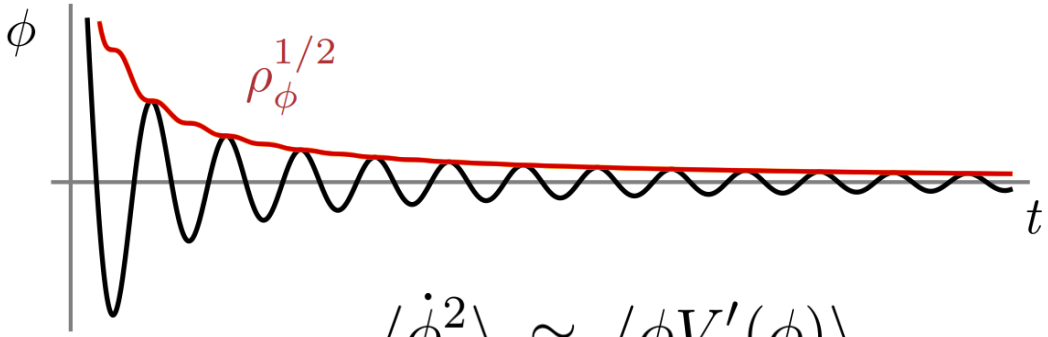



$$\lambda \simeq \frac{18\pi^2 A_s(k_\star)}{6N_*^2} \sim 10^{-11}$$

**Close to the minimum**

$$V(\phi) \simeq \frac{1}{2}m_\phi^2\phi^2 = \lambda\phi^2 M_P^2 \quad (\phi \ll M_P)$$




$$\langle\dot{\phi}^2\rangle \simeq \langle\phi V'(\phi)\rangle$$

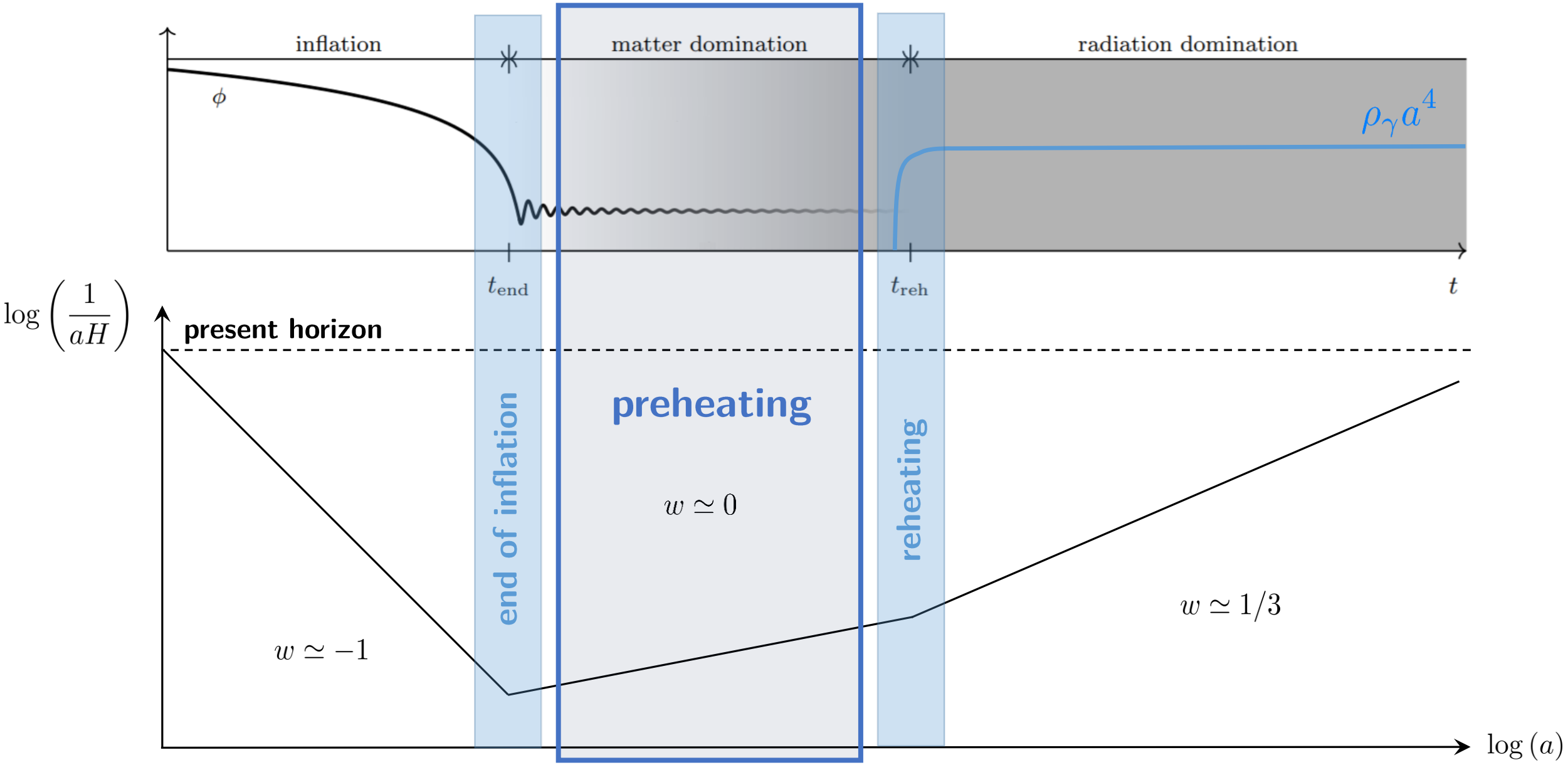
$$\Rightarrow \quad \phi(t) \simeq \phi_0(t)\cos(m_\phi t) \qquad \phi_0(t) \sim a(t)^{-3/2}$$

$$\langle P_\phi\rangle = \frac{1}{2}\langle\dot{\phi}^2\rangle - \langle V(\phi)\rangle \simeq 0$$

$$\langle\rho_\phi\rangle = \frac{1}{2}\langle\dot{\phi}^2\rangle + \langle V(\phi)\rangle \simeq V(\phi_0)$$

$$\langle w_\phi\rangle \simeq 0$$

# Preheating


inflation
matter domination
radiation domination
$\phi$
$\rho_\gamma a^4$
$t_{\rm end}$
$t_{\rm reh}$
$t$
$\log\left(\frac{1}{aH}\right)$
present horizon
end of inflation
preheating
reheating
$w \simeq 0$
$w \simeq -1$
$w \simeq 1/3$
$\log(a)$

# Dark matter production from preheating

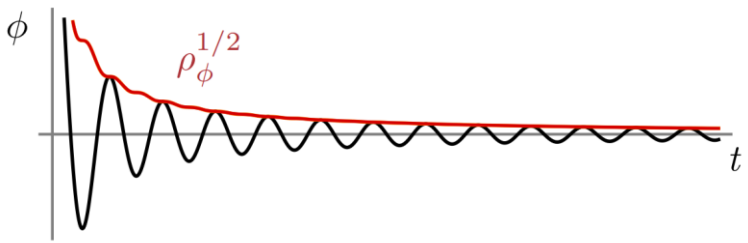
# Particle production during preheating

- Consider **coupling** to **scalar dark matter**

$$\mathcal{S} = \int \mathrm{d}^4x \sqrt{-g}\Big( R + \frac{1}{2}(\partial_\mu\phi)^2 - \lambda\phi^2 M_P^2 \quad \text{inflaton}$$

$$+\frac{1}{2}(\partial_\mu\chi)^2 - \frac{1}{2}\sigma\phi^2\chi^2 - \frac{1}{2}m_\chi^2\chi^2 \Big) \quad \text{scalar}$$

minimal coupling to gravity




$$\Rightarrow \quad m^2_{\chi,\mathrm{eff}}(t) = m_\chi^2 + \sigma\phi^2(t)$$

**Goal:** Estimate DM production for all regimes of $\sigma/\lambda$

$$n_\chi(t) = \frac{g_\chi}{(2\pi)^3}\int \mathrm{d}^3\boldsymbol{P}\, f_\chi(P_0, t)$$

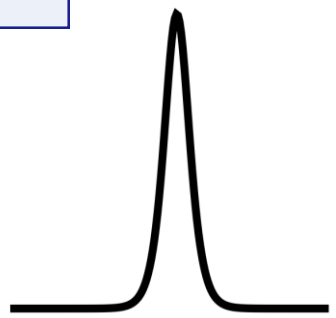
# Scalar preheating: fluid approach

**First approach**: fluid picture – approximate energy density by envelope

- **Phase space distribution** from

$$\frac{\partial f_\chi}{\partial t} - H|\boldsymbol{P}|\frac{\partial f_\chi}{\partial |\boldsymbol{P}|} = \mathcal{C}[f_\chi(|\mathbf{P}|,t)]$$

$$\mathcal{C}[f_\chi(|\mathbf{P}|,t)] = \frac{1}{P^0}\int \frac{\mathrm{d}^3\boldsymbol{k}}{(2\pi)^3 n_\phi}\frac{\mathrm{d}^3\boldsymbol{P}'}{(2\pi)^3 2P'^0}(2\pi)^4\delta^{(4)}(k-P-P')|\overline{\mathcal{M}}|^2_{\phi\phi\to\chi\chi}$$
$$\times\left[f_\phi(k)(1+f_\chi(P))(1+f_\chi(P')) - f_\chi(P)f_\chi(P')(1+f_\phi(k))\right]$$



$$f_\phi(\boldsymbol{k},t) = (2\pi)^3 n_\phi(t)\delta^{(3)}(\boldsymbol{k})$$

- **Collision term** given by:

$$\frac{\partial f_\chi}{\partial t} - H|\boldsymbol{P}|\frac{\partial f_\chi}{\partial |\boldsymbol{P}|} = \frac{\pi^2}{\beta^2 m_\phi^3}\rho_\phi\Gamma_{\phi\phi\to\chi\chi}\delta\left(|\boldsymbol{P}| - m_\phi\beta(t)\right)(1+2f_\chi(|\boldsymbol{P}|))$$

Bose enhancement

$$\beta(t) \equiv \sqrt{1-\frac{m_{\mathrm{eff}}^2(t)}{m_\phi^2}}$$

: kinematic blocking

# Scalar preheating: fluid approach

- Treat **inflaton** as **coherent oscillating condensate**

$$\phi(t) \simeq \phi_0(t) \sum_{n=-\infty}^{\infty} \mathcal{P}_n e^{-in\omega_\phi t} \qquad E_n = n\omega_\phi \;\Rightarrow\; \Gamma_{\phi\phi\to\chi\chi} = \frac{1}{8\pi(1+w_\phi)\rho_\phi} \sum_{n=1}^{\infty} E_n \beta_n |\mathcal{M}_n|^2$$

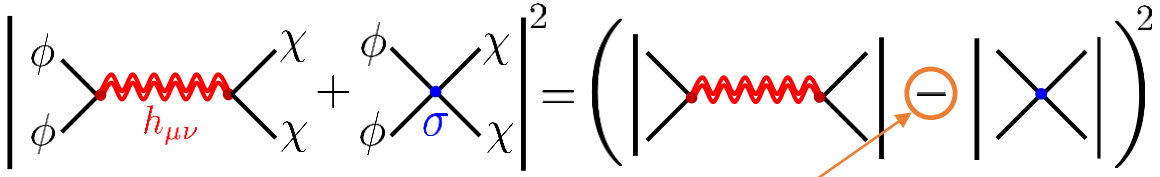
$$\langle \chi(p_1)\chi(p_2)| i \int \mathrm{d}^4x \mathcal{L}_{\rm int} |0\rangle = i(2\pi)^4 \sum_{n=-\infty}^{\infty} \mathcal{M}_n \delta^{(4)}(p_n - p_1 - p_2) \qquad \beta_n \equiv \sqrt{1 - \frac{m_{\rm eff}^2(t)}{E_n^2}}$$

[M. A. G. Garcia, K. Kaneta, Y. Mambrini, K. A. Olive, S. Verner, arXiv:2109.13280]

- For **quadratic potential** $\omega_\phi = m_\phi$ and equivalent of treating inflaton as collection of **particles** in **Minkowski space-time**!

$$g_{\mu\nu} \simeq \eta_{\mu\nu} + 2\frac{h_{\mu\nu}}{M_P}$$

$|\mathcal{M}|^2_{\phi\phi\to\chi\chi} =$




$$\mathcal{L}_{\rm int} = -\frac{1}{M_P} h_{\mu\nu}\left(T^{\mu\nu}_\phi + T^{\mu\nu}_\chi\right) - \frac{\sigma}{2}\phi^2\chi^2 \;\Rightarrow\; \Gamma_{\phi\phi\to\chi\chi} = \frac{1}{32\pi}\frac{\rho_\phi^2(t)}{m_\phi^3}\left[\sigma - \lambda\left(1 + \frac{m_\chi^2}{2m_\phi^2}\right)\right]^2 \beta_2$$

# Scalar preheating: the field picture

- **Treat dark matter as quantum field in curved space-time**

Equation of motion:
$$\left(\frac{\mathrm{d}^2}{\mathrm{d}t^2} - \frac{\nabla^2}{a^2} + 3H\frac{\mathrm{d}}{\mathrm{d}t} + m_\chi^2 + \sigma\phi^2\right)\chi = 0$$

- **Quantize** the (rescaled) field
$$X(\tau, \boldsymbol{x}) \equiv a\chi = \int \frac{\mathrm{d}^3\boldsymbol{p}}{(2\pi)^{3/2}} e^{-i\boldsymbol{p}\cdot\boldsymbol{x}} \left[X_p(\tau)\hat{a}_{\boldsymbol{p}} + X_p^*(\tau)\hat{a}^\dagger_{-\boldsymbol{p}}\right]$$

- **Harmonic oscillator** with **time-dependent** frequency

Gravity!

$$X_p'' + \omega_p^2 X_p = 0 \qquad \omega_p^2(t) = p^2 + a^2(t)\hat{m}_{\mathrm{eff}}^2(t) \qquad \hat{m}_{\mathrm{eff}}^2(t) = m_\chi^2 + \sigma\phi^2 + \frac{R}{6}$$

- **Distribution function** from **occupation number**

$$n_p = \frac{1}{2\omega_p}\left|\omega_p X_p - iX_p'\right|^2 \quad \Rightarrow \quad f_\chi(P,t) = n_{aP}(t)$$

$$' \equiv \frac{\mathrm{d}}{\mathrm{d}\tau}$$
$$\mathrm{d}t = a\,\mathrm{d}\tau$$

[L. Kofman, A. Linde, A. Starobinsky - arXiv:9704452 – arXiv:9405187]
[M. A. G. Garcia, K. Kaneta, K. Olive, Y. Mambrini, S. Verner - arXiv: 2109.13280]

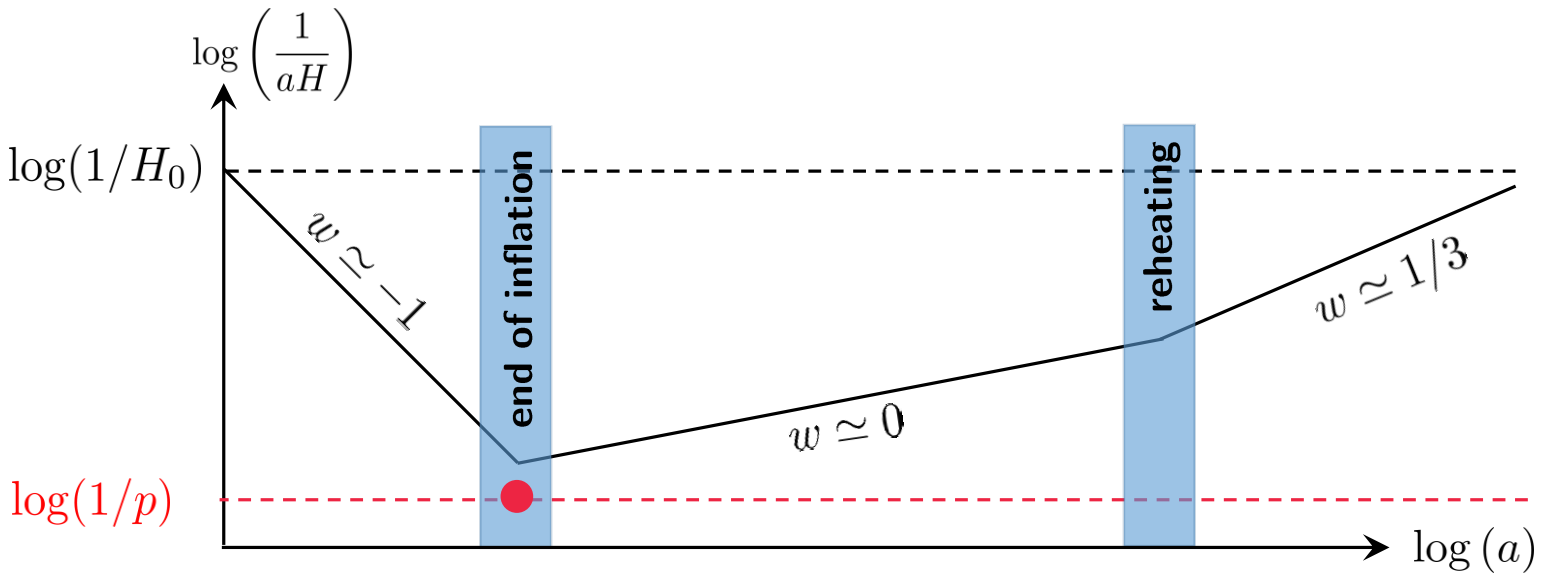
# Scalar preheating: the field picture

- Set **initial condition** for **mode functions** in **Bunch-Davies** vacuum

$$X_p(\tau_0) = \frac{1}{\sqrt{2\omega_p}} \qquad X_p'(\tau_0) = -\frac{i\omega_p}{\sqrt{2\omega_p}}$$

$$\boxed{\omega_p^2(t_{\rm end}) = p^2 + a_{\rm end}^2\left(m_\chi^2 + \sigma\phi^2 - H_{\rm end}^2\right)}$$

- **For small physical scales** **modes always inside horizon** $\omega_p^2 > 0$ ● $\tau_0 = \tau_{\rm end}$



# Scalar preheating: the field picture

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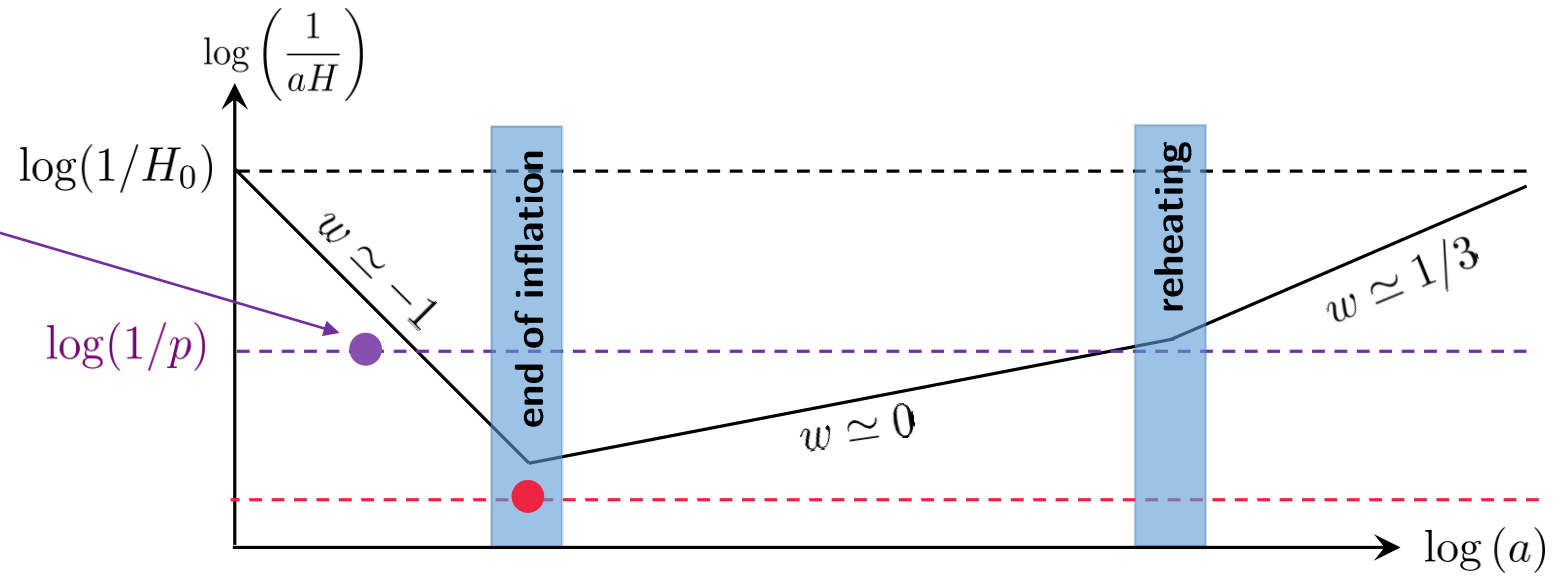
- **For small physical scales** **modes always inside horizon** $\omega_p^2 > 0$ ● $\tau_0 = \tau_{\rm end}$
- **For** $m_\chi^2 + \sigma\phi^2 < H_{\rm end}^2$ **modes** with $p^2 < a_{\rm end}^2\left(H_{\rm end}^2 - m_\chi^2 - \sigma\phi^2\right)$ $\omega_p^2(t_{\rm end}) < 0$

● $\tau_0 < \tau_{\rm end}$

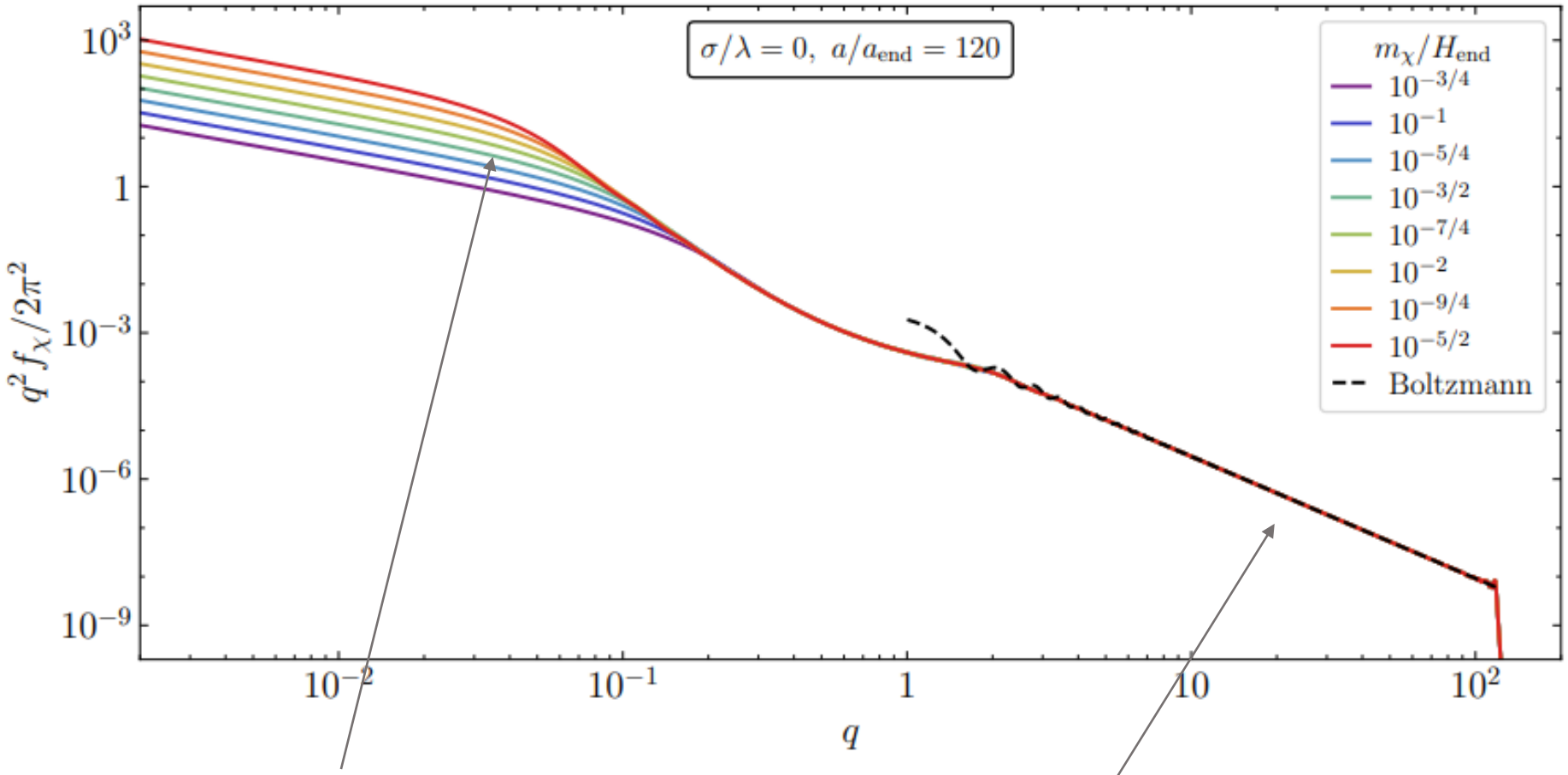
$p^2 \gg a(\tau_0)^2 H(\tau_0)^2$

➡ **Particle production!**
**Red-tilt of the spectrum!**



# Gravitational production $\sigma/\lambda \ll 1$

[N. Herring, D. Boyanovsky & A. R. Zentner - arXiv:1912.10859]
[S. Ling & A. J. Long - arXiv:2101.11621]




$$q \equiv \frac{P}{T_\star}\left(\frac{a}{a_0}\right)$$

$$T_\star \equiv m_\phi\left(\frac{a_{\rm end}}{a_0}\right)$$

- **Excitation of light field in quasi De-Sitter space:** increases as mass decreases
- Recover fluid **(Boltzmann) regime** at **large *q*** $f_\chi \sim q^{-9/2}$

# Gravitational production $\sigma/\lambda \ll 1$

[N. Herring, D. Boyanovsky & A. R. Zentner - arXiv:1912.10859]
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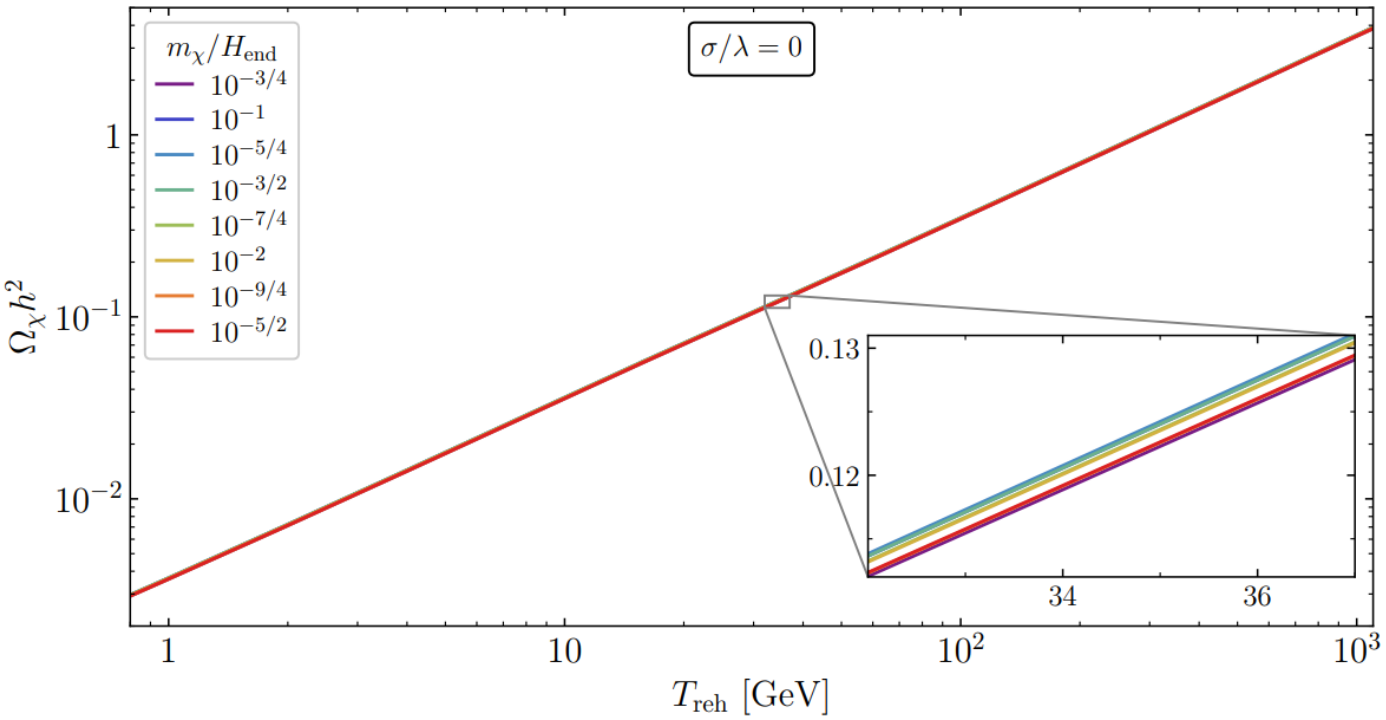
➡ **Take IR cutoff** as **present horizon**

$$q_{\rm IR} = q_0 = \left(\frac{90}{\pi^2}\right)^{1/4} \left(\frac{11}{43}\right)^{1/3} g_{\rm reh}^{1/12} \left(\frac{H_{\rm end} M_P}{m_\phi^2}\right)^{1/2} \frac{H_0}{T_0} \left(\frac{a_{\rm reh}}{a_{\rm end}}\right)^{1/4} \quad \Leftrightarrow \quad p_0 = a_0 H_0$$



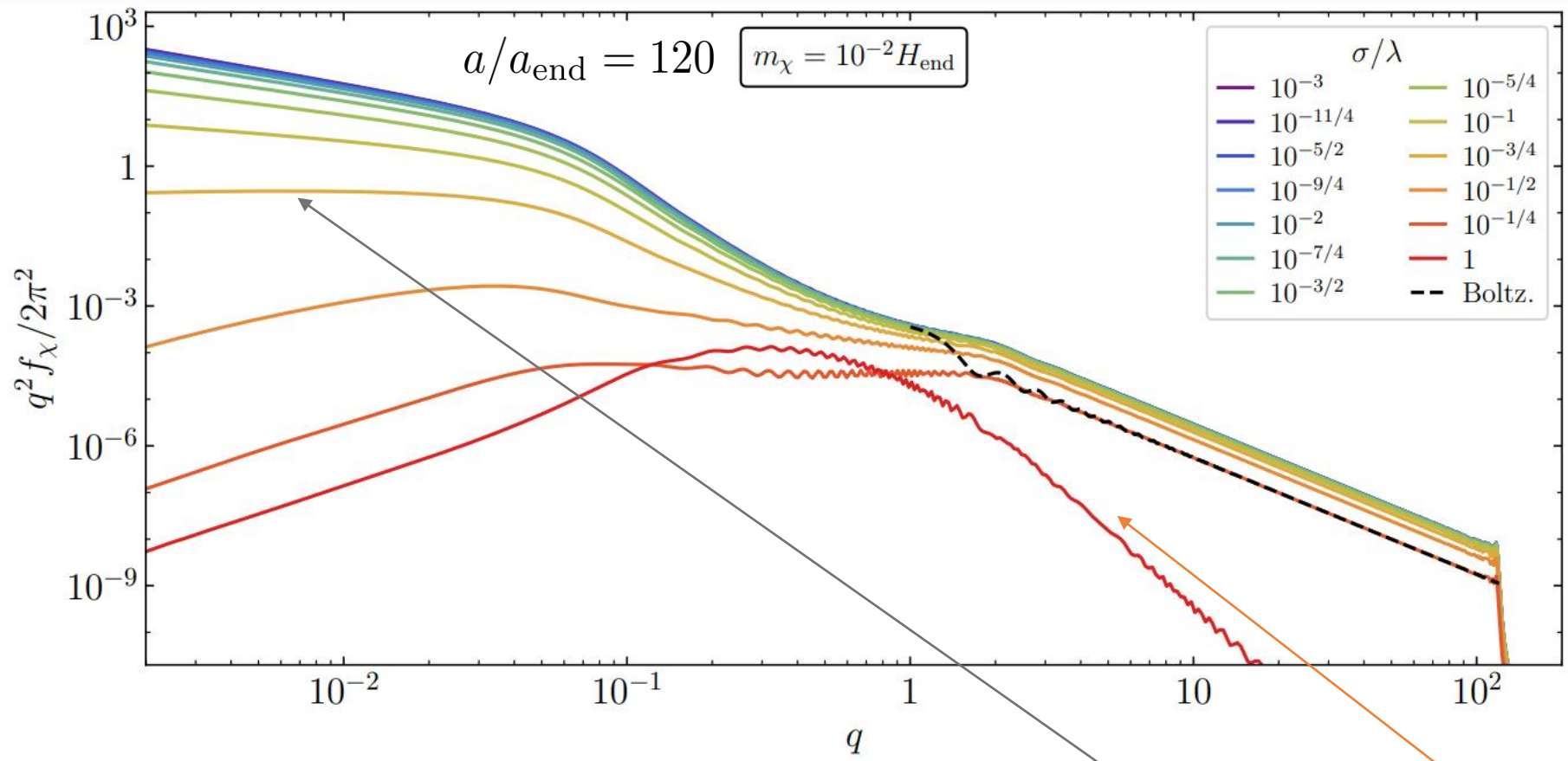

**For small DM mass**

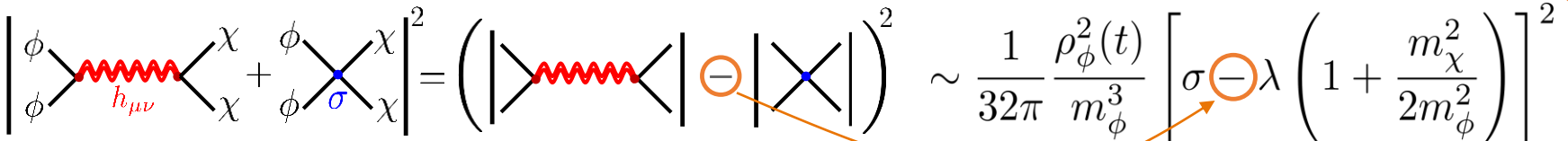
**For** $T_{\rm reh} > 34\ {\rm GeV}$

**Overclose the universe!**

**+ isocurvature constraints !?**

# Gravitational interferences $0 < \sigma/\lambda < 1$




- **Effective mass** behaves a **IR modes regulator** and **suppresses density production!**
- $\Gamma_{\phi\phi\to\chi\chi} \sim$ 

$\sim \frac{1}{32\pi} \frac{\rho_\phi^2(t)}{m_\phi^3} \left[ \sigma - \lambda \left( 1 + \frac{m_\chi^2}{2m_\phi^2} \right) \right]^2$ **Interferences!**

# Resonance effects $\sigma/\lambda \gg 1$

[M. A. G. Garcia, K. Kaneta, Y. Mambrini, K. A. Olive, S. Verner, arXiv:2109.13280]

- **Changing variables**

$$x_p \equiv a^{1/2} X_p \qquad z \equiv m_\phi t + \frac{\pi}{2}$$

$$A_p = \frac{p^2}{m_\phi^2 a^2} + 2\hat{q}$$
(“energy²”)

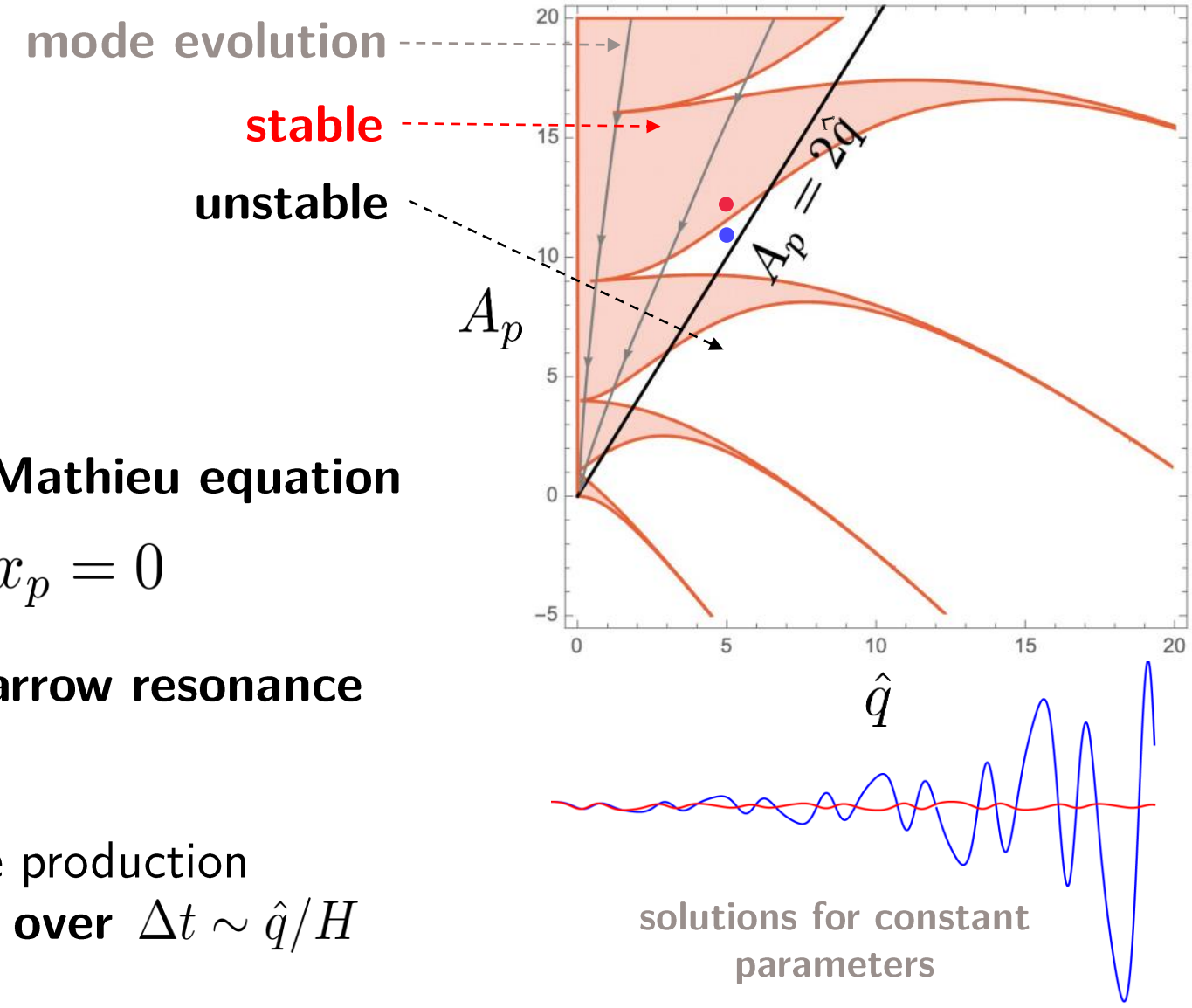
$$\hat{q} = \frac{\sigma \phi_0^2}{4 m_\phi^2}$$
(“mass²”)

- **Equation** for **mode functions** $\Longleftrightarrow$ **Mathieu equation**

$$\ddot{x}_p + (A_p - 2\hat{q}\cos(2z))x_p = 0$$

- For **large** momentum, i.e. $A_p \gg \hat{q}$ **narrow resonance**
  $A_p \sim 1 \pm \hat{q}$ ➡ **stable**

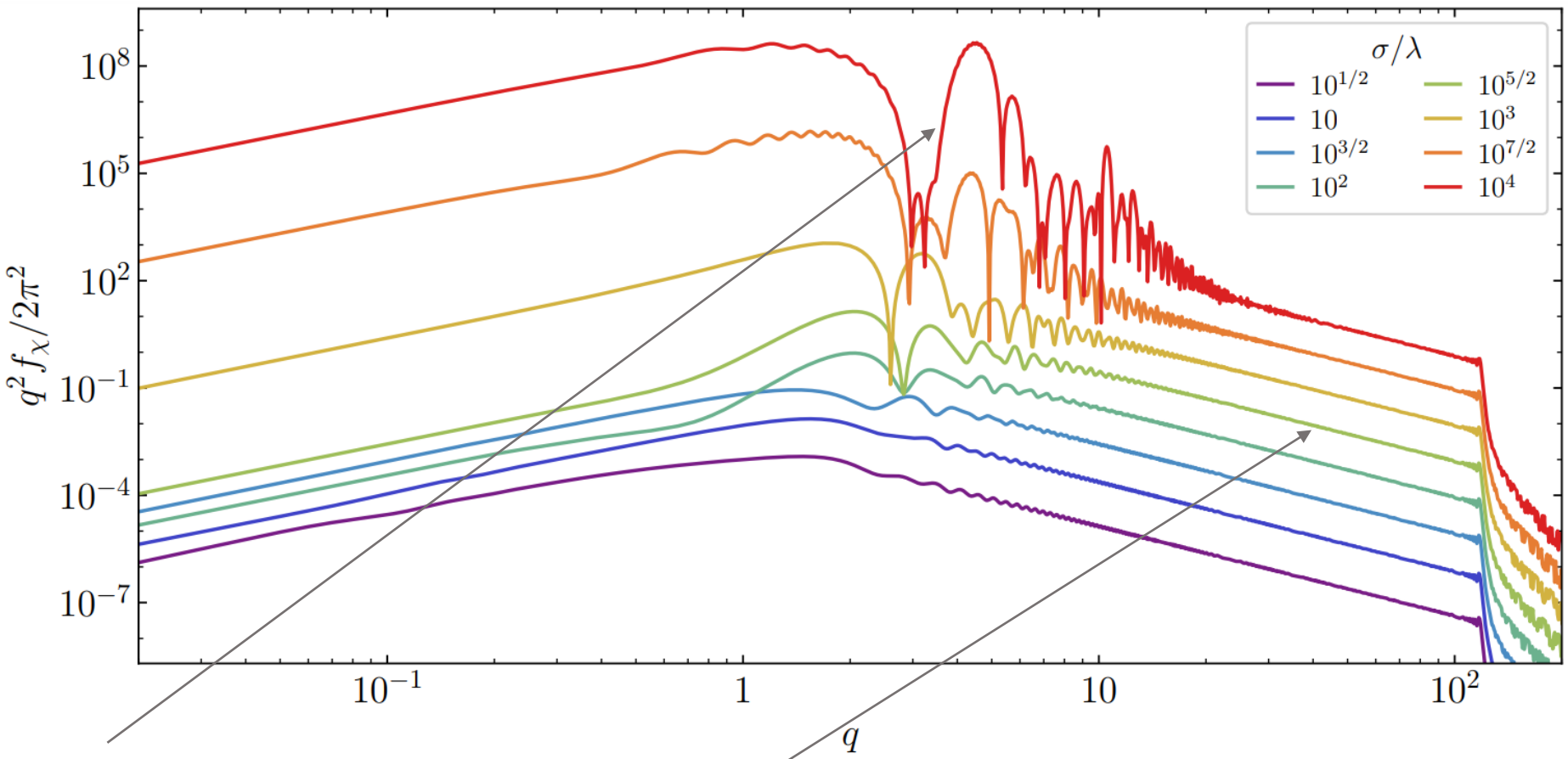
- Close to $A_p \sim 2\hat{q}$, **efficient** particle production
  as **crossing** several **instability bands over** $\Delta t \sim \hat{q}/H$




[L. Kofman, A. Linde, A. Starobinsky - arXiv:9704452]

# Small couplings $1 < \sigma/\lambda < 10^4$

$$a/a_{\rm end} = 120$$




- **Broad** resonances at **large** coupling from instabilities!
- Recover **Boltzmann regime** at **large *q*** $f_\chi \sim q^{-9/2}$

[L. Kofman, A. Linde, A. Starobinsky arXiv:9704452 – arXiv:9405187]

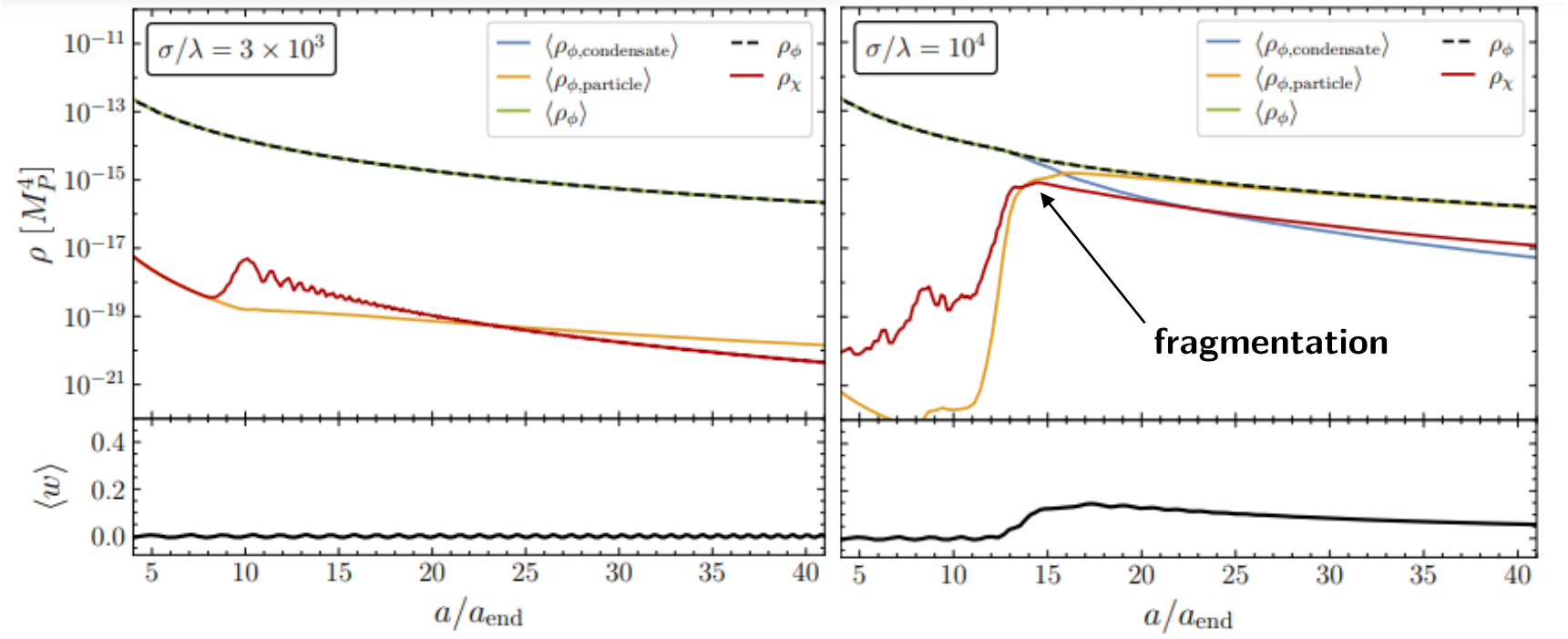
# Large couplings $\sigma/\lambda > 10^4$

Copiously produced **dark matter disrupts inflaton condensate**

- **Hartree approximation** $\ddot{\phi} + 3H\dot{\phi} + V_\phi + \sigma\phi^2\langle\chi^2\rangle = 0$

- **Real space lattice simulations** CosmoLattice — *A modern code for lattice simulations of scalar and gauge field dynamics in an expanding universe*

[D. G. Figueroa, A. Florio, F. Torrenti, W. Valkenburg, arXiv:2102.01031]

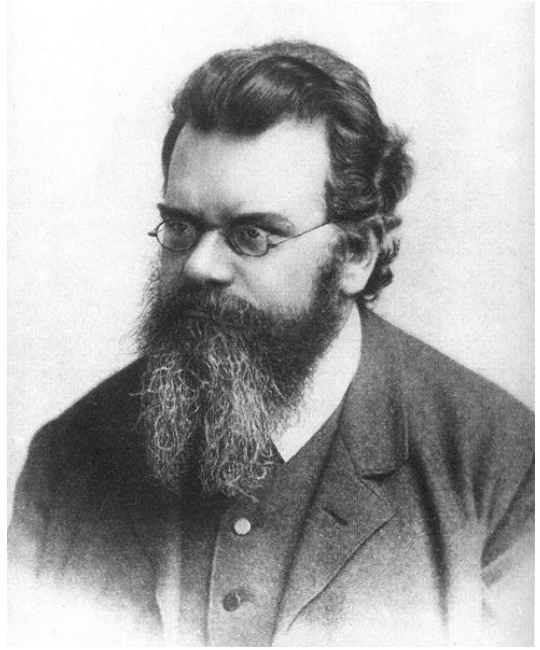



$$\rho_{\phi,\mathrm{condensate}} \equiv \frac{1}{2}\dot{\bar{\phi}}^2 + V(\bar{\phi})$$

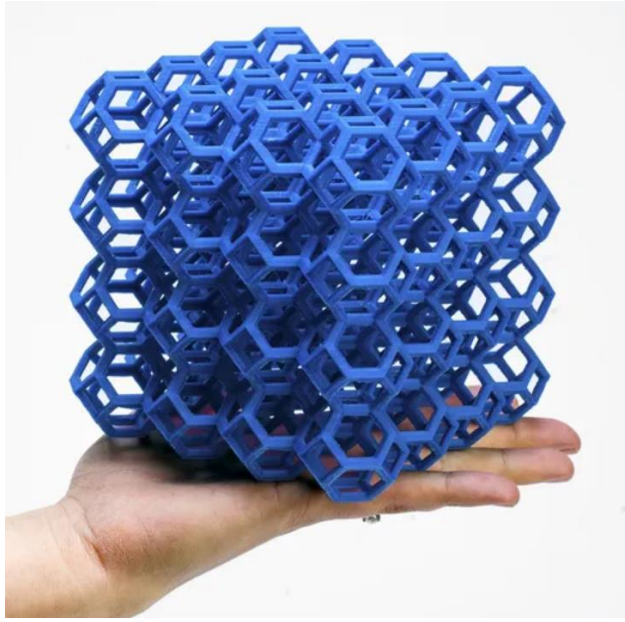
$$\rho_{\phi,\mathrm{particle}} \equiv \rho_\phi - \rho_{\phi,\mathrm{condensate}}$$

# Large couplings $\sigma/\lambda > 10^4$

**Hartree**

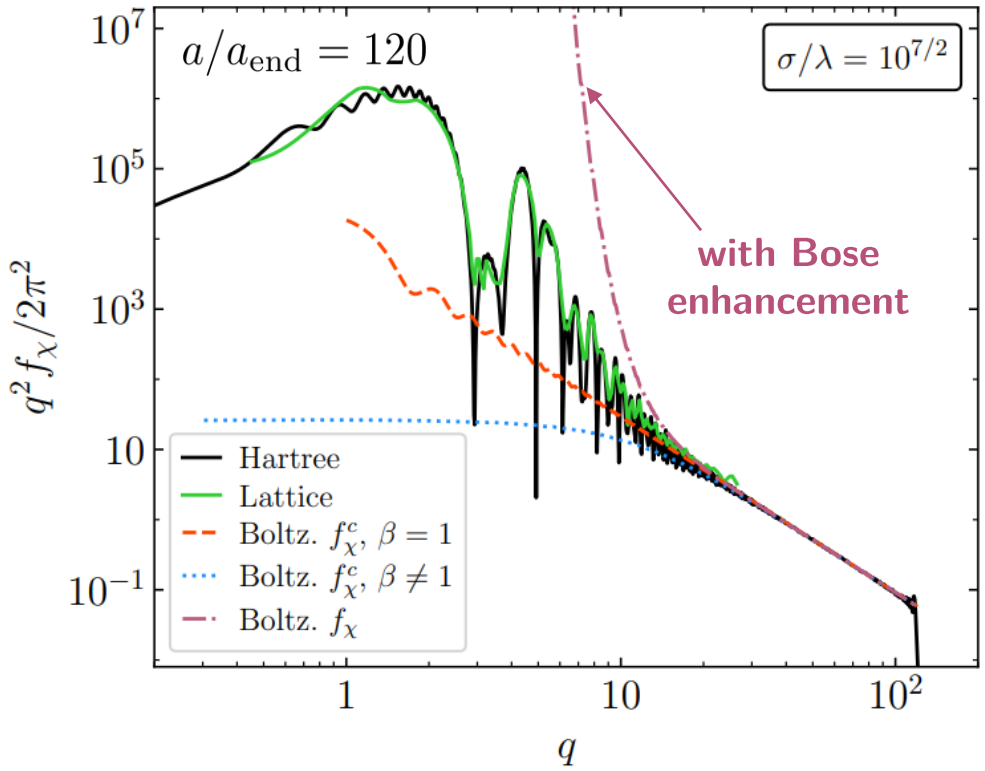


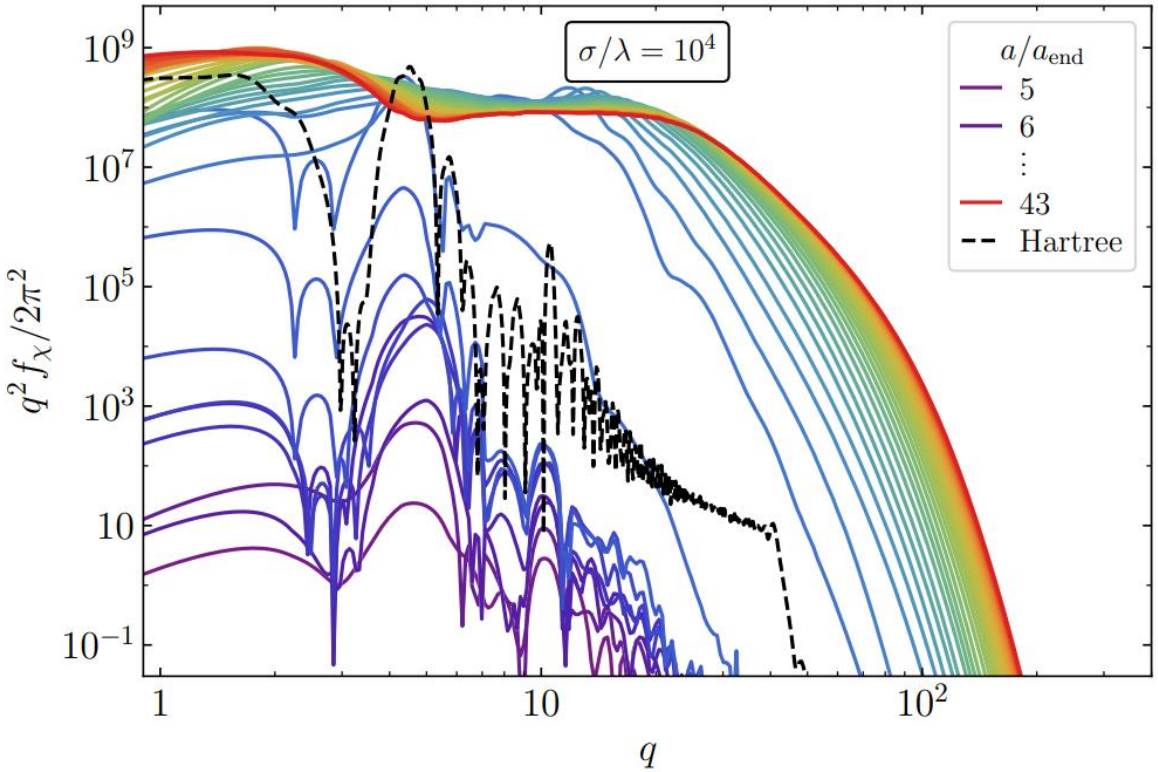
**Boltzmann**



**or lattice?**

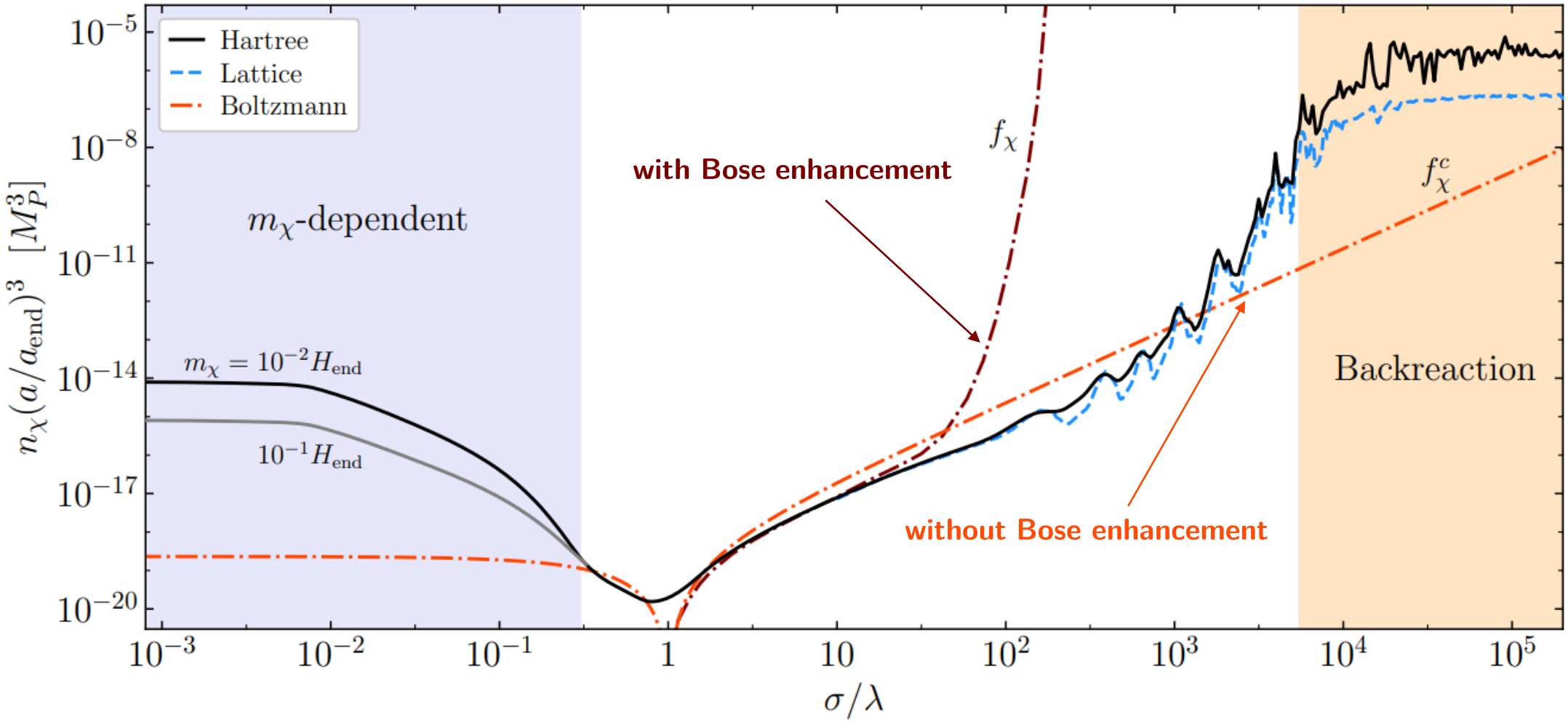
# Large couplings $\sigma/\lambda > 10^4$




- **Hartree** and **lattice consistent**
- **Boltzmann fails** at capturing IR w±o Bose or kinematic $\beta$ factors
- **Quasi-termal** distribution from lattice
- **Hartree fails** if backreaction too large

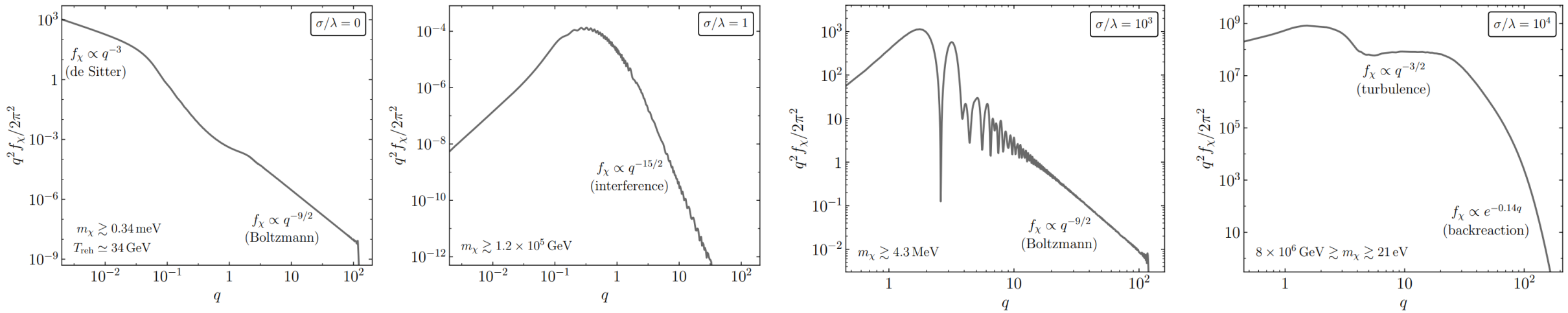
# Scalar preheating phases




➡ Applicable to **generic light scalar**, not just **dark matter**

# Take home message

$\sigma/\lambda$

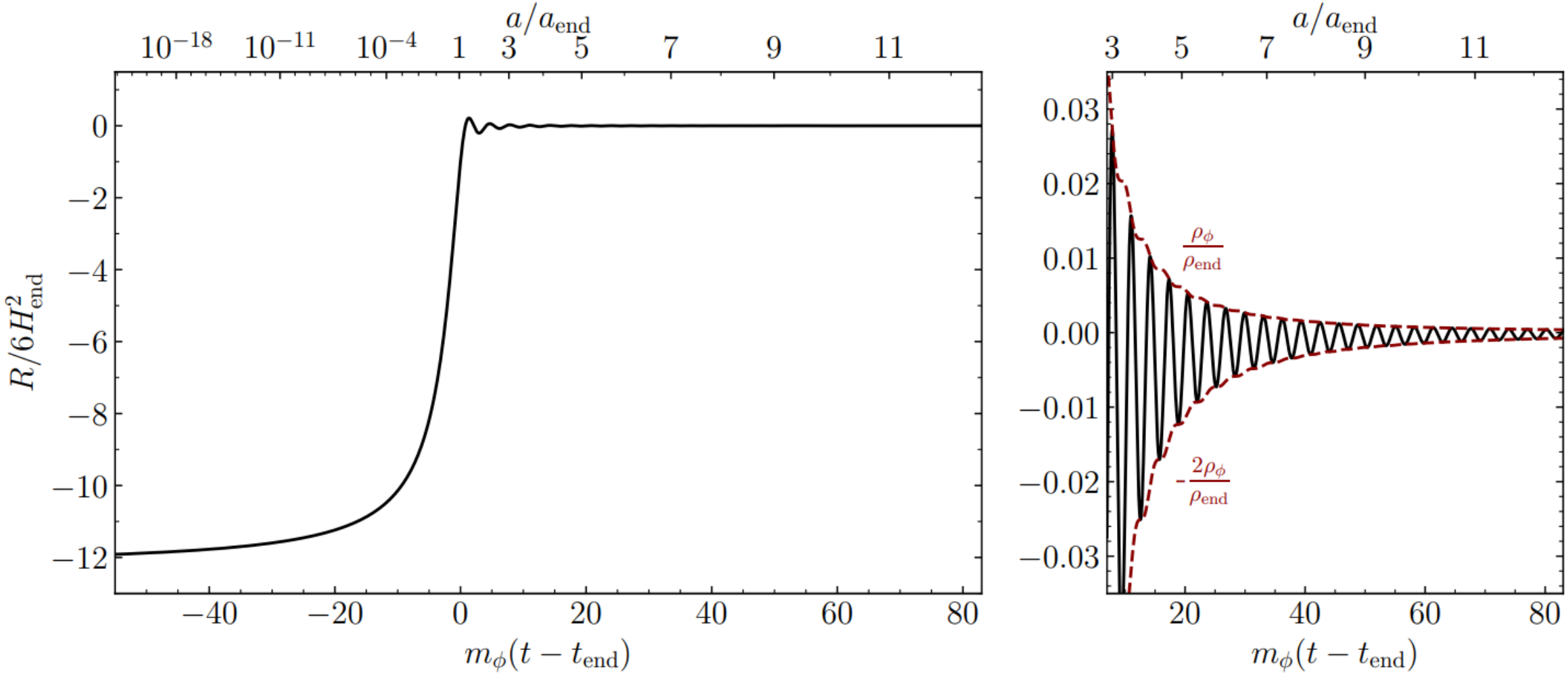



- Characterized **scalar preheating** over **large spectrum** of **couplings**
- Checked **consistency** of **several approaches**

## Thank you for your attention

# Back-up slides

# Gravitational contribution to the effective mass



# Reheating

- In **fluid picture**: transition to radiation era via **dissipation** term $\equiv \Gamma_\phi \rho_\phi (1 + w_\phi)$

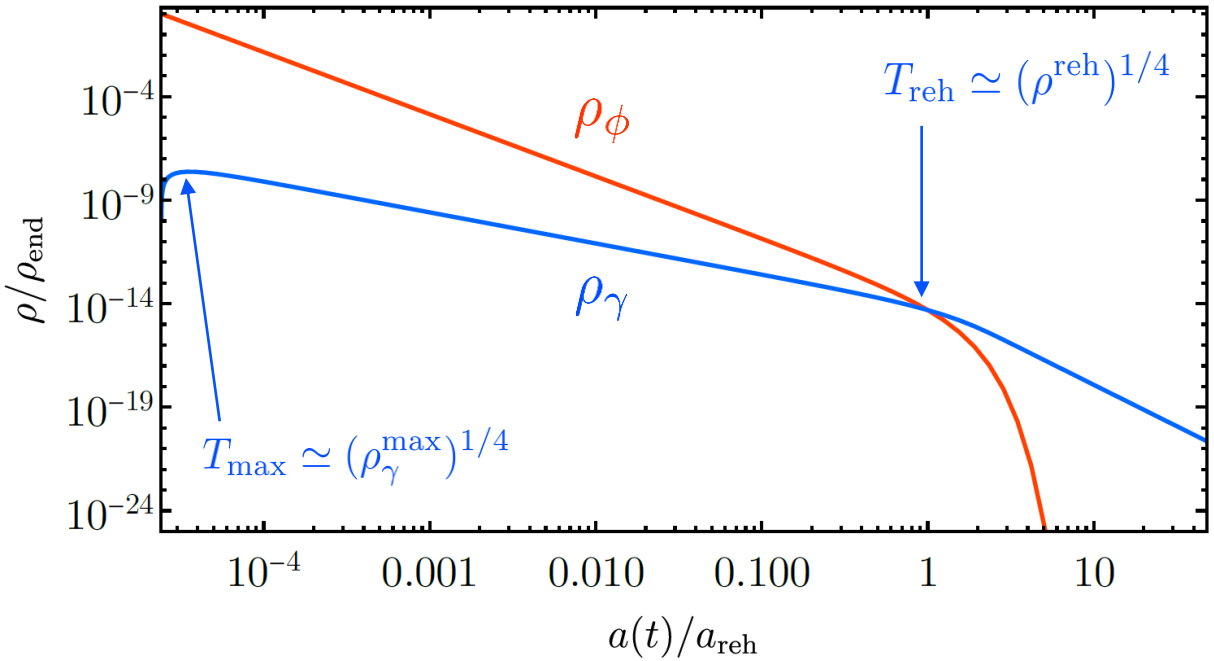
$$T^{\mu\nu}_{\rm tot} = T^{\mu\nu}_\phi + T^{\mu\nu}_\gamma \qquad \nabla_\mu T^{\mu\nu}_{\rm tot} = 0 \qquad \nabla_\mu T^{\mu\nu}_\phi = -\nabla_\mu T^{\mu\nu}_\gamma$$

- System of **Friedmann-Boltzmann equations**

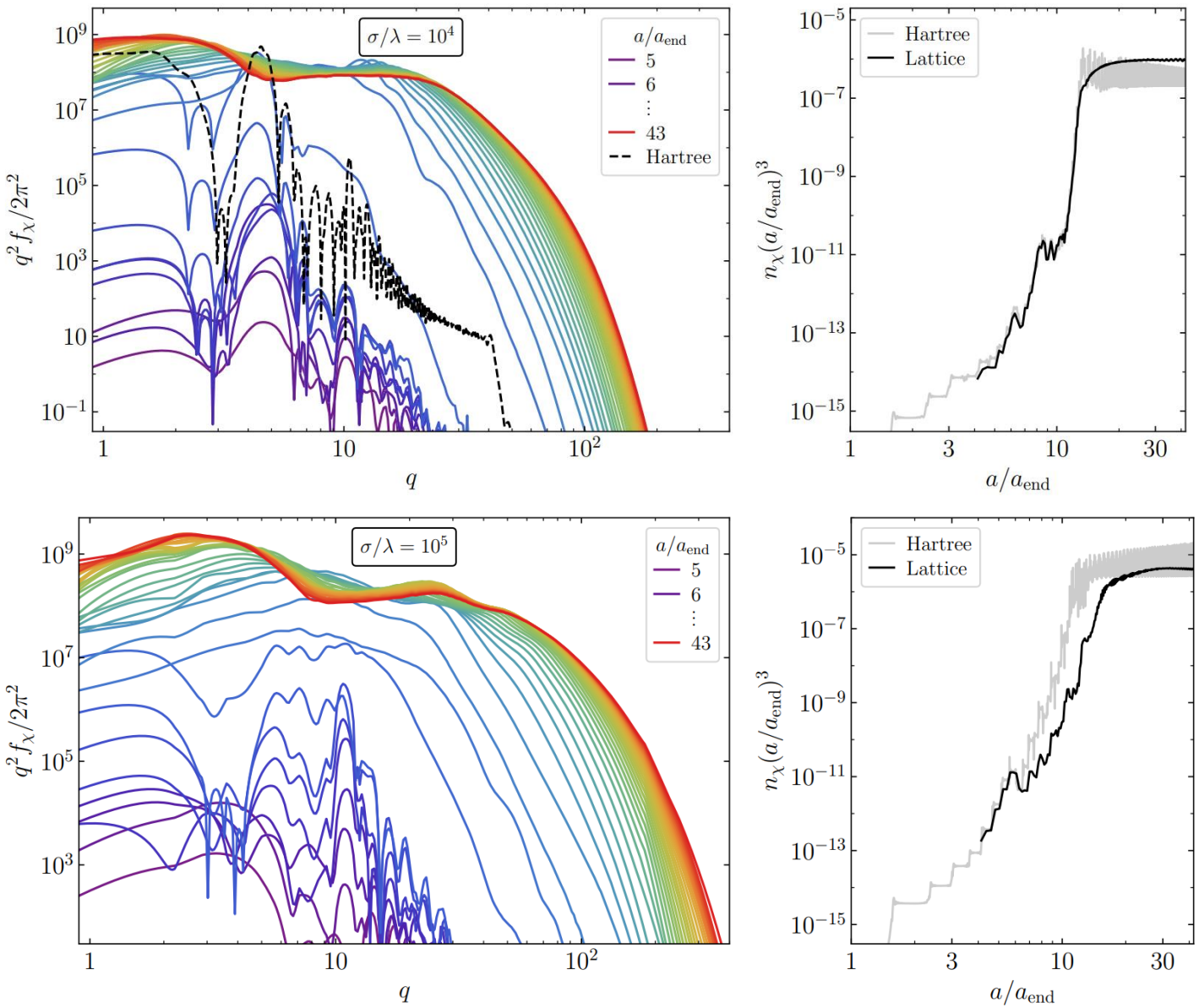
$$\begin{aligned} \dot\rho_\phi + 3H(1+w_\phi)\rho_\phi &= -\Gamma_\phi \rho_\phi (1+w_\phi) \\ \dot\rho_\gamma + 4H\rho_\gamma &= \Gamma_\phi \rho_\phi (1+w_\phi) \\ H^2 &= \frac{1}{3M_{\rm Pl}}(\rho_\phi + \rho_\gamma) \end{aligned}$$

$$\Rightarrow \quad \rho_\phi(t) \simeq \rho_{\rm end} \left(\frac{a}{a_{\rm end}}\right)^{-3} e^{-\Gamma_\phi (t - t_{\rm end})}$$

$\Gamma_\phi \ll m_\phi$



# Lattice simulations: phase space distribution

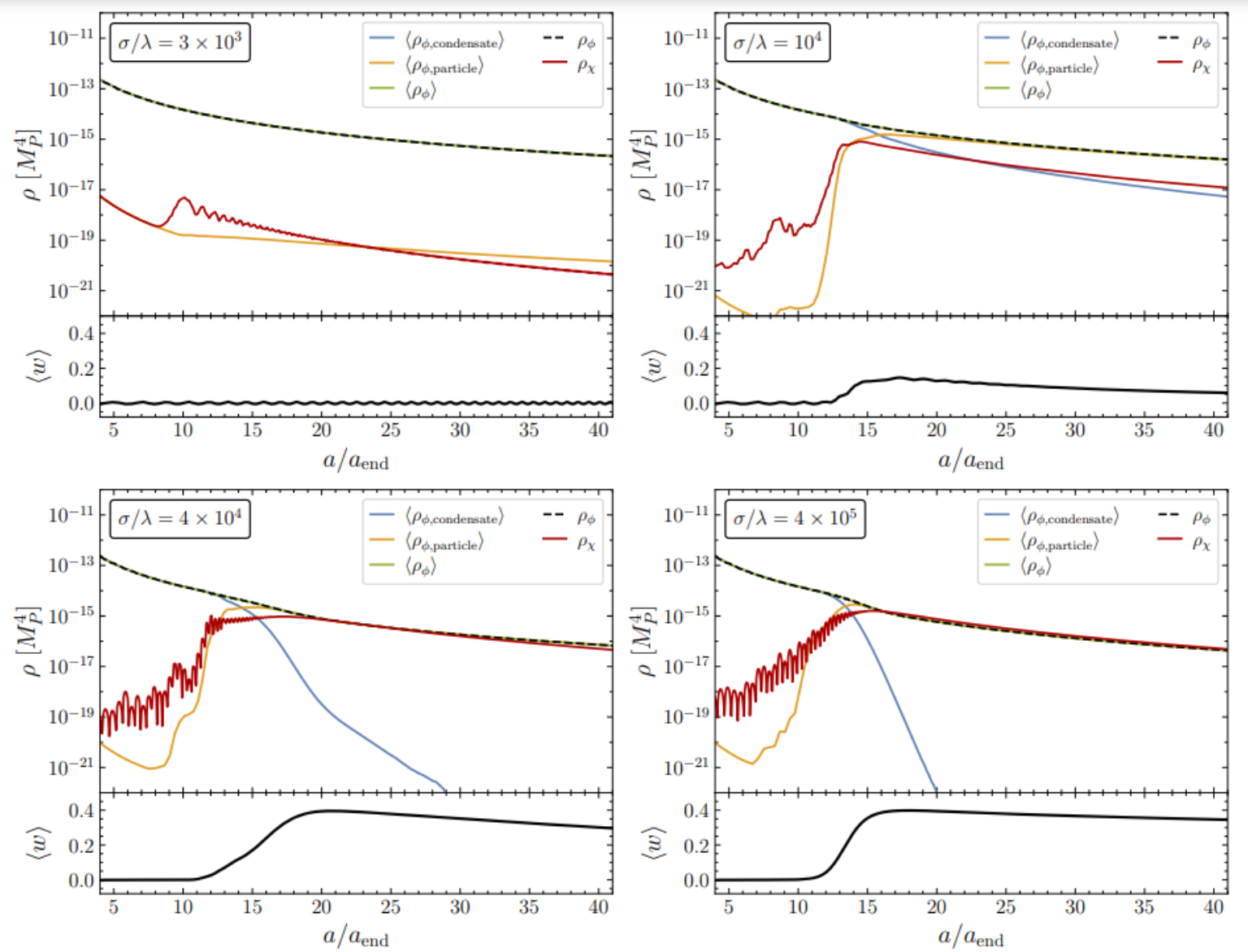



CosmoLattice

*A modern code for lattice simulations of scalar and gauge field dynamics in an expanding universe*

[D. G. Figueroa, A. Florio, F. Torrenti, W. Valkenburg, arXiv:2102.01031]

# Lattice simulations: energy density and fragmentation




$$\rho_{\phi,\mathrm{condensate}} \equiv \frac{1}{2}\dot{\bar{\phi}}^2 + V(\bar{\phi})$$

$$\rho_{\phi,\mathrm{particle}} \equiv \rho_\phi - \rho_{\phi,\mathrm{condensate}}$$

$$\langle w(a)\rangle \equiv \frac{1}{\Delta a}\int_a^{a+\Delta a} w(\tilde{a})\,\mathrm{d}\tilde{a}$$

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# Constraints on parameter space