Quantum Computing for Particle Theory and Cosmology

New lamp-posts shedding light on problems in particle physics

Jay Hubisz - Syracuse University - October 20, 2022

Featuring work in collaboration with:
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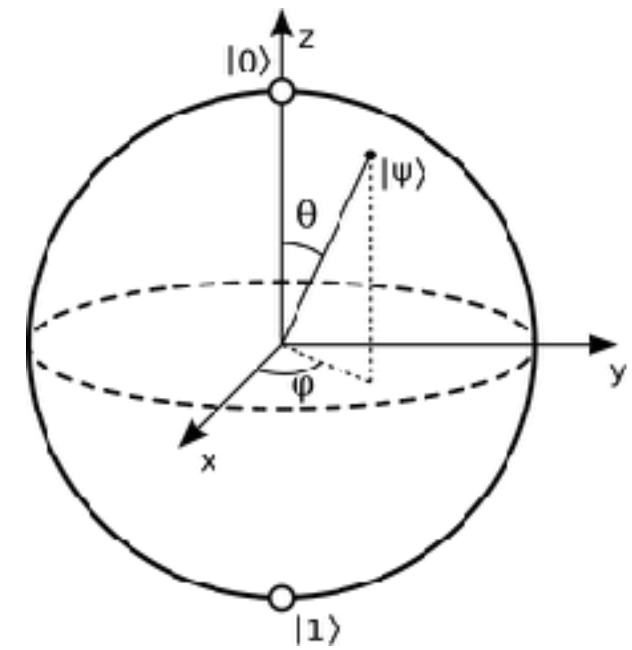
ift Beyond the Standard Models: Particle Physics meets Cosmology

Motivation

- Quantum computing is a new tool in our arsenal for studying field theories in regimes difficult to access with standard classical hardware
  - Quantum time dynamics (QCD jets)
  - Theories with chiral fermions (the lattice sign problem)
  - Strongly coupled physics at finite density (the lattice sign problem)
- In its infancy, with potential realization of particle theory goals In ~10-30ish? years
- Ripe for exploration (parallels dev. leading up to other international-scale projects):
  - What are the things we can do with it?
  - Solutions of problems “on paper” will help guide development of hardware
    - Experimentalists excited to know “how should we build them?”
  - There are 10’s of giga-$’s flowing into development of hardware/infrastructure for quantum technologies

Hilbert space is big

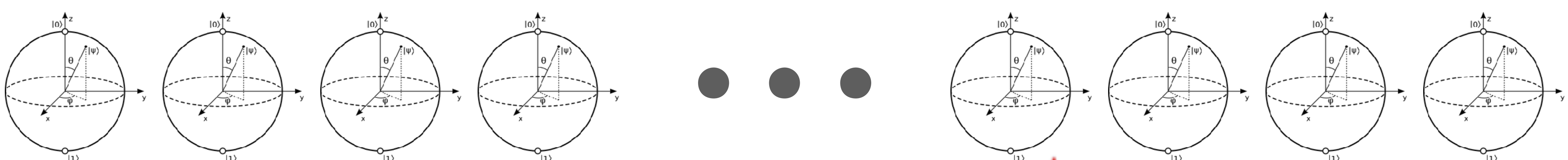
Consider the Simplest Field Theory

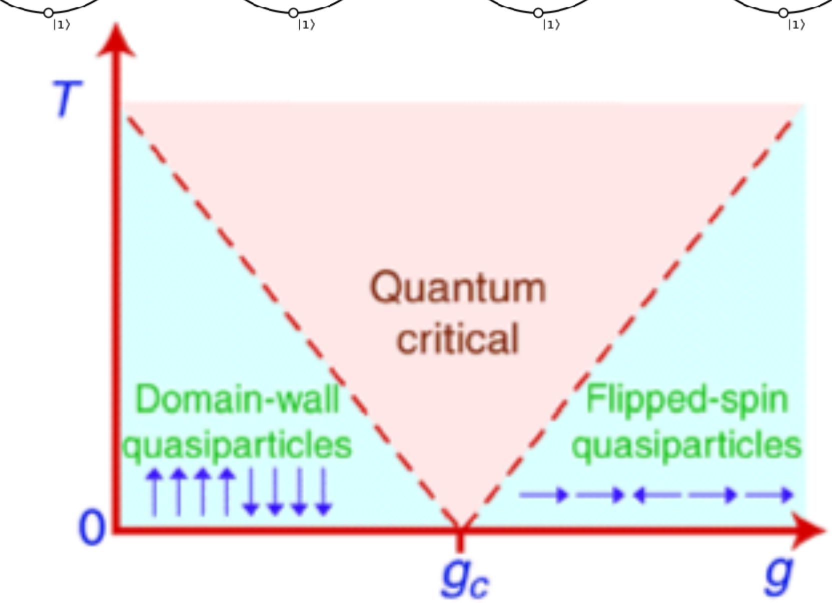


1+1D quantum Ising Model

A string of coupled 2-state systems (qubits)

$$\hat{H}_{\text{Ising}} = -\sum_{\langle ij \rangle} \hat{\sigma}_i^z \hat{\sigma}_j^z - \frac{h_x}{\lambda} \sum_i \hat{\sigma}_i^x$$





Matrices parametrizing evolution are 2^Nx2^N

Exponential growth of system requirements

Point of interest: T=0 “quantum” phase transition at $h_x/\lambda = g_c = 1$ as $N \to \infty$

Correlations span entire volume: truncation limits study

Lattice Sign Problem

Complex weights

Lattice Field Theorists study analog statistical physics problems in 4D Euclidean space:

$$Z = \int D\phi e^{\frac{i}{\hbar}S[\phi]} \longrightarrow \int D\phi e^{-\beta H[\phi]}$$

This fixes the "original" sign problem…quantum path integral oscillates wildly, no convergence of integral

Fermions: There is no type "grassman" to declare in c++ — Integrate out the fermions

$$\int D\phi D\Psi e^{\frac{i}{\hbar}S[\phi,\Psi]} \longrightarrow \int D\phi D\Psi e^{-\beta H[\phi,\Psi]} \longrightarrow \int D\phi \, \mathrm{Pf}\mathcal{O}[\phi] e^{-\beta H_{\mathrm{eff}}[\phi]}$$

This is typically complex-valued

If a theory is vector-like, Pfaffian is real-valued and weights are positive since eigenvalues appear in CC pairs (so there is no problem doing QCD)

Chemical potential

e.g. Finite baryon density

This also has a sign problem (even for bosons)

$$Z = \int D\phi D\phi^* e^{-\beta H + \mu Q}$$

But Q is the Noether charge, which we all remember has an i in it:

$$Q = \int d^d x J^0 = i \int d^d x \left(\phi^* \partial^0 \phi - \phi \partial^0 \phi^* \right)$$

Again, the answer for the partition function in the end is real, but individual weights will be complex.

For fermions, generally know if there is a sign problem by checking for "γ_5 hermiticity"

$$\not{D}^\dagger = \gamma_5 \not{D} \gamma_5$$

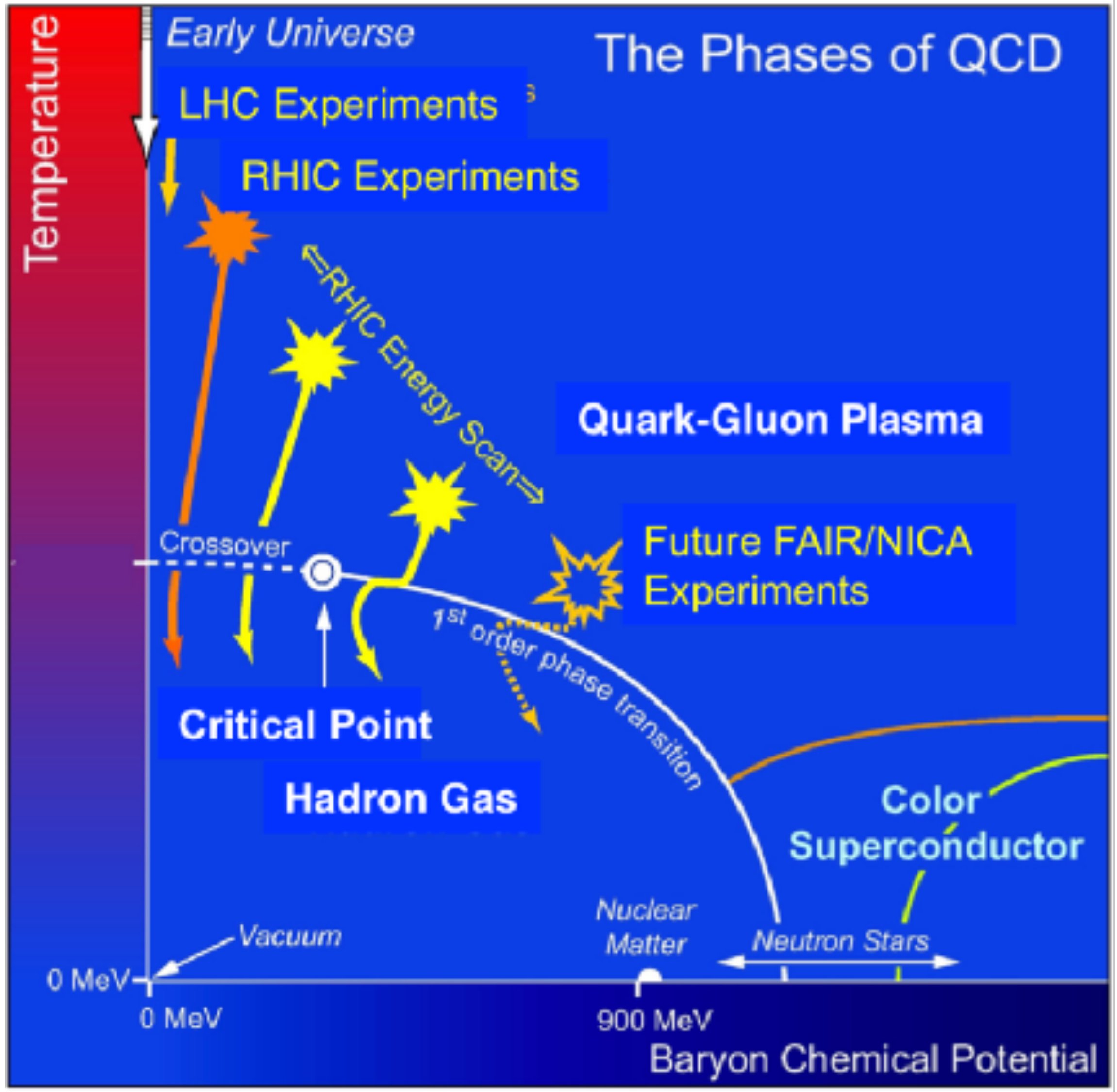
Finite μ: $$[\not{D}(\mu)]^\dagger = \gamma_5 \not{D}(-\mu) \gamma_5$$

QCD at finite density

- The phase diagram of strongly coupled vector like theories along the $\mu = 0$ axis is straightforwardly accessed using traditional lattice methods

- Minkowski path integral $\rightarrow$euclidean thermal partition function accesses finite temperature QCD-like theories

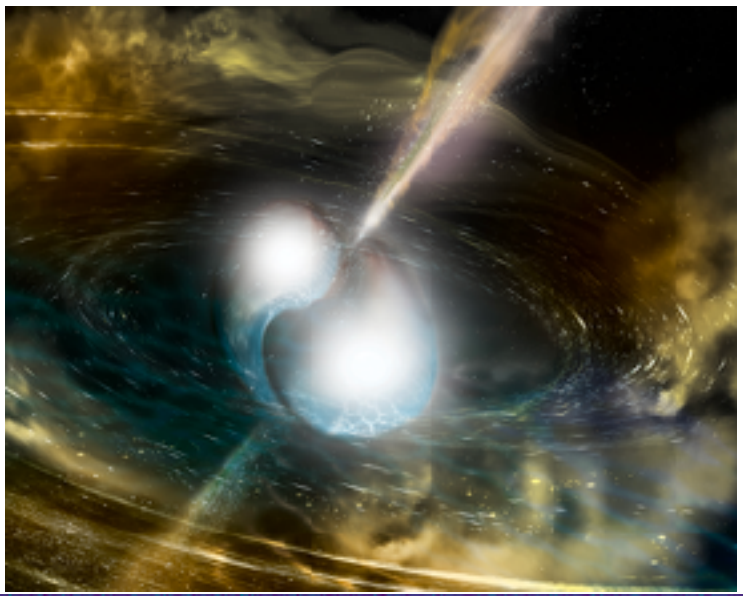
- Finite density QCD has a sign problem

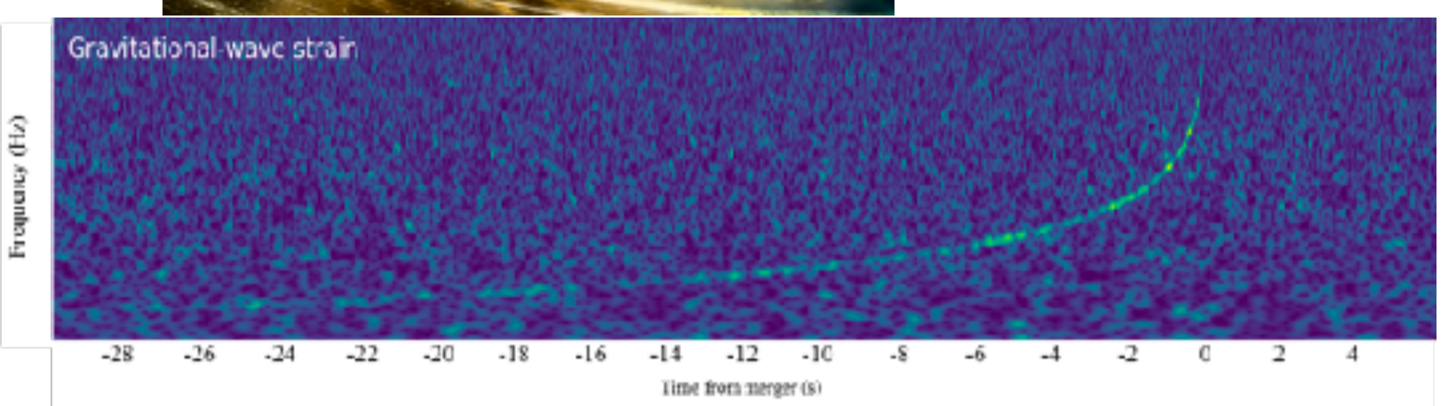


Only physics near the μ=0 axis is accessible with traditional lattice methods

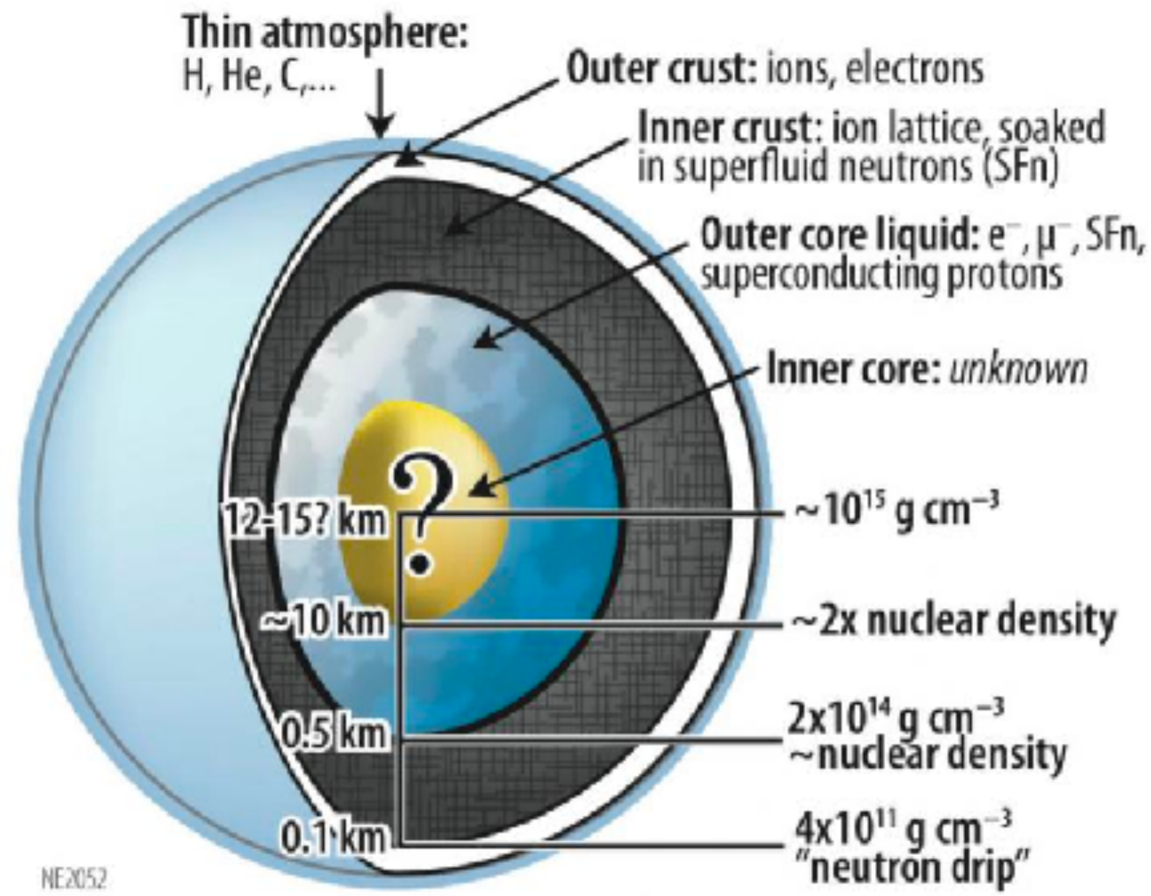
Why should people here care?

Neutron stars are laboratories for new physics





Omega Scan of LIGO's First Neutron Star Merger Detection



Shape of the chirp contains information about tidal deformability, which in turn depends on QCD equation of state

Without theoretical control over the EOS, cannot separate new physics from SM

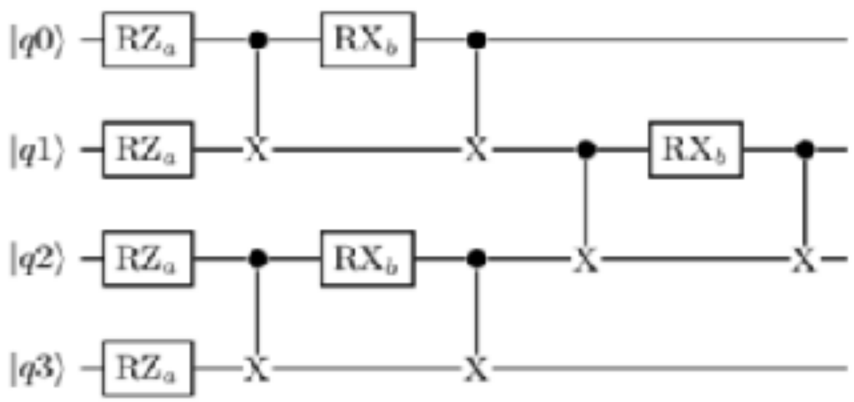
Dark matter, ALP's, dark energy

Quantum Computing

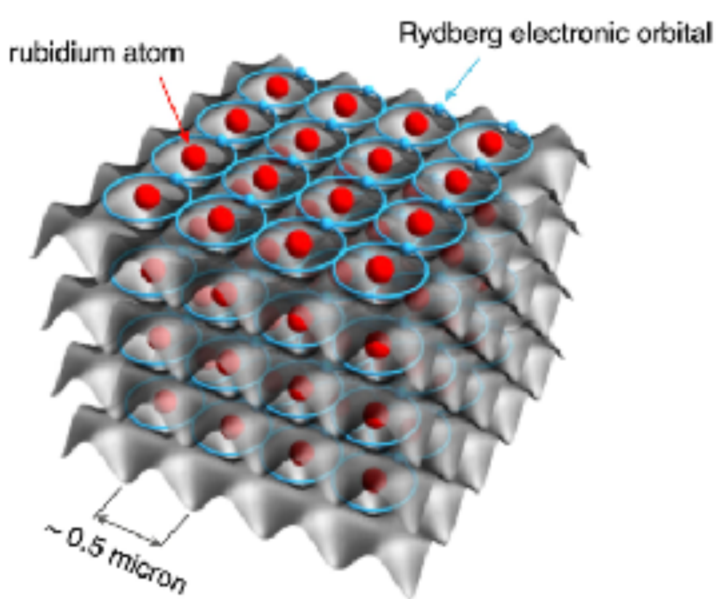
Quantum mechanics handles the i's for you

And inherently manages the large 2^N's

- Gate based machines:
  - Map your system of interest onto qubits
  - you come up with a quantum operation that you want to perform
  - You break that up into quantum logic gates (+measurement) acting on 1 or 2 qubits at a time



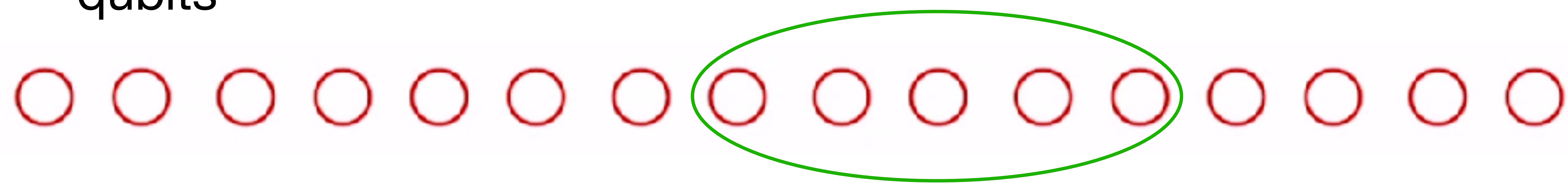
- Analog quantum simulation:
  - You find a way to build a system in analogy to the physics you want to study, and build it



What is currently possible?

NISQ (noisy intermediate-scale quantum) era hardware

- Current chips have upwards of 100 qubits, but these are **very** noisy - coherent entanglement still only over a handful of nearby qubits



IBM is building these things like crazy, with a new machine every couple months
Through partnership with BNL scientist Ilya Drosdov, we have access to ~30 qubit machine

- Coherence time is pretty bad - can get a handful of discrete time-steps before it falls apart

- No error correction

- Interesting developments in manipulable arrays of long-coherence time 2-state systems (Rydberg atom arrays) "Programmable quantum simulators"

Long way from QCD on a chip

But we are familiar with focusing on the long-game

- The LHC being decades away didn’t stop people from working on what-if scenarios that fed back on detector design, trigger criteria, etc.
- What can we in principle achieve?
- How does the QC field need to evolve in order to meet these goals?

Outline

- Quantum evolution on quantum hardware
  - Trotterized unitary/non-unitary quantum evolution
- Open quantum systems
  - The sign problem and non-hermitian evolution
- Discrete AdS and lattice AdS/CFT
  - Strongly coupled physics in the bulk
- Future work

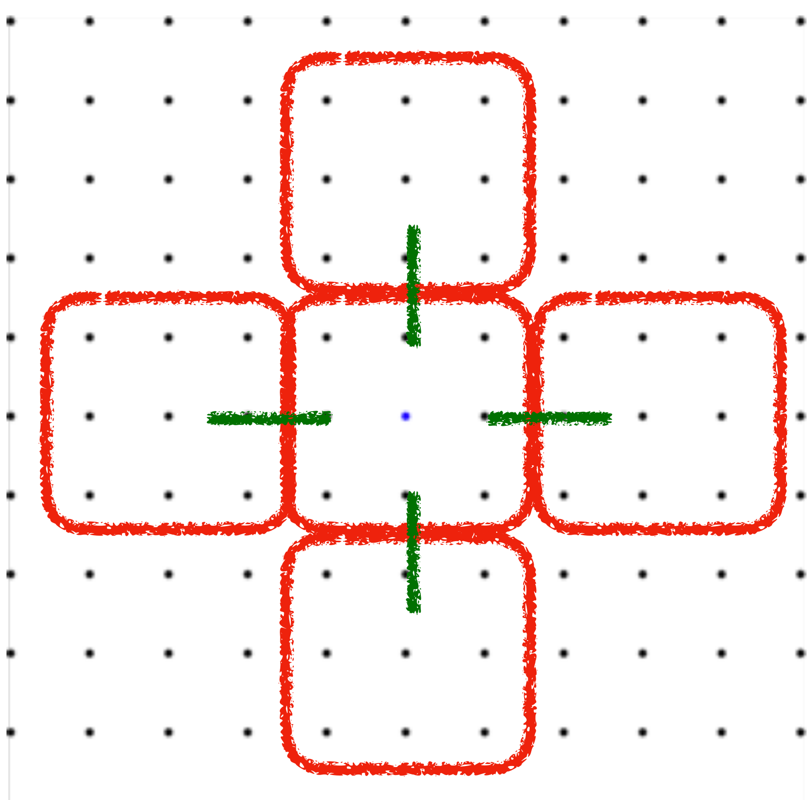
Approximating e^{-iHt}

Suzuki-Trotter

In field theory, interactions are local

Hamiltonian splits into pieces only involving a few degrees of freedom (local + neighbors)

$$H = \sum_i \boxed{H_L^i} + \sum_{<ij>} \overline{H_{NN}^{ij}}$$



Discretize time into small steps: $t = N\delta t$

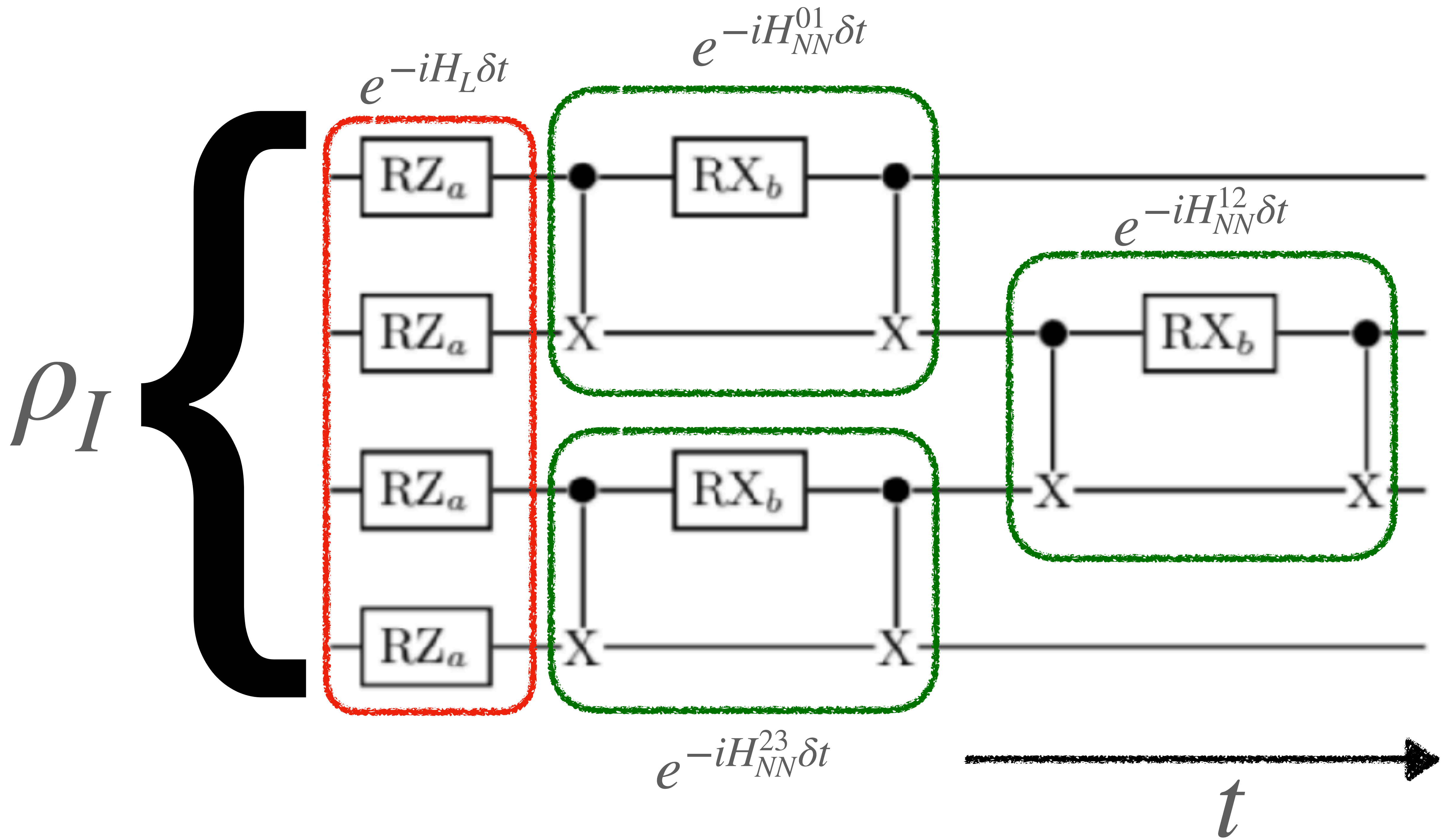
$$e^{-iHt} = \left[\prod_i e^{-iH_L^i \delta t} \prod_{<ij>} e^{-iH_{NN}^{ij}\delta t} \right]^N (1 + \mathcal{O}(t\delta t))$$

Each of these is a relatively simple unitary operator

From this point - not difficult to break up into gates

Quantum Ising Model

4 Sites, open boundary conditions

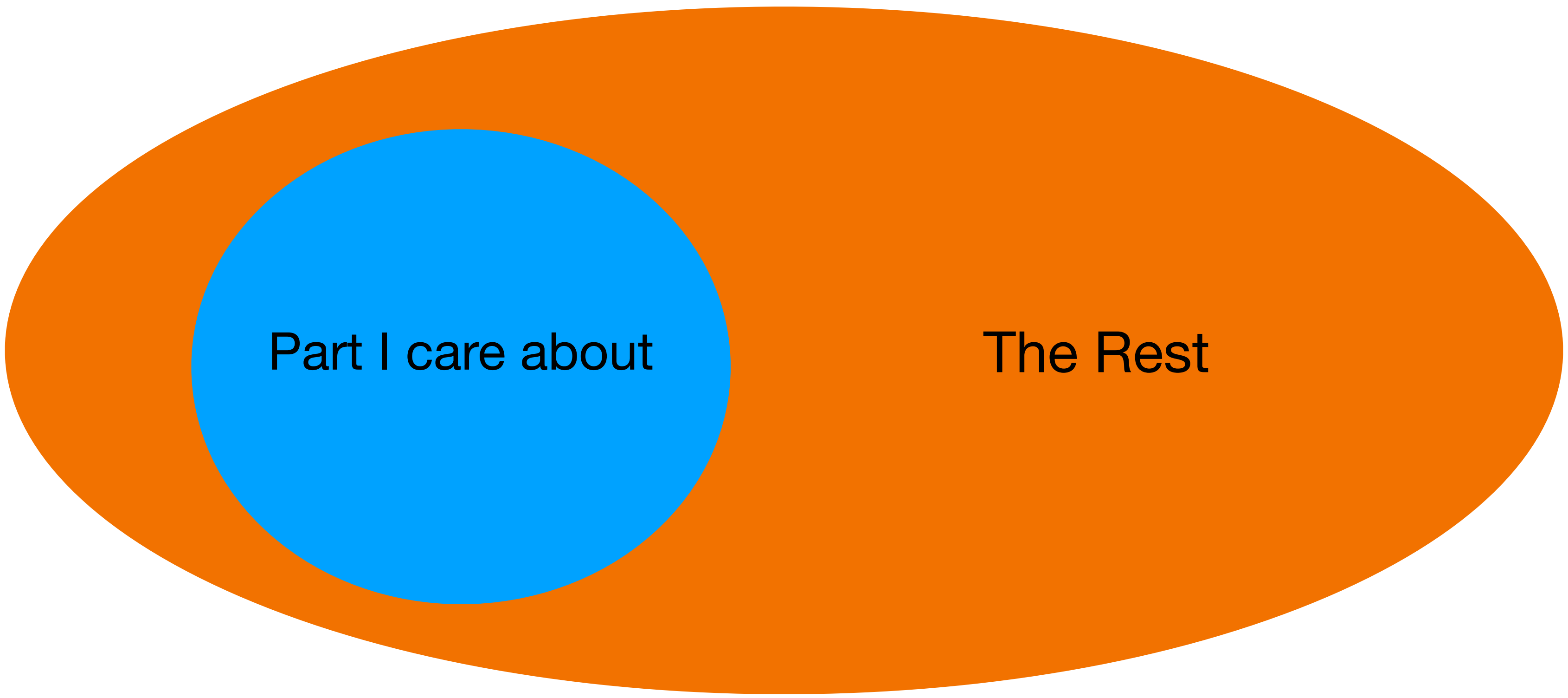


Repeat N times

Open quantum systems

Finite Density/Temp, Cosmology, Generalized EFT’s

In many cases, it is beneficial to trace out some part of a quantum mechanical system



QCD fields ↔ Baryonic bath

My observable universe ↔ super horizon modes

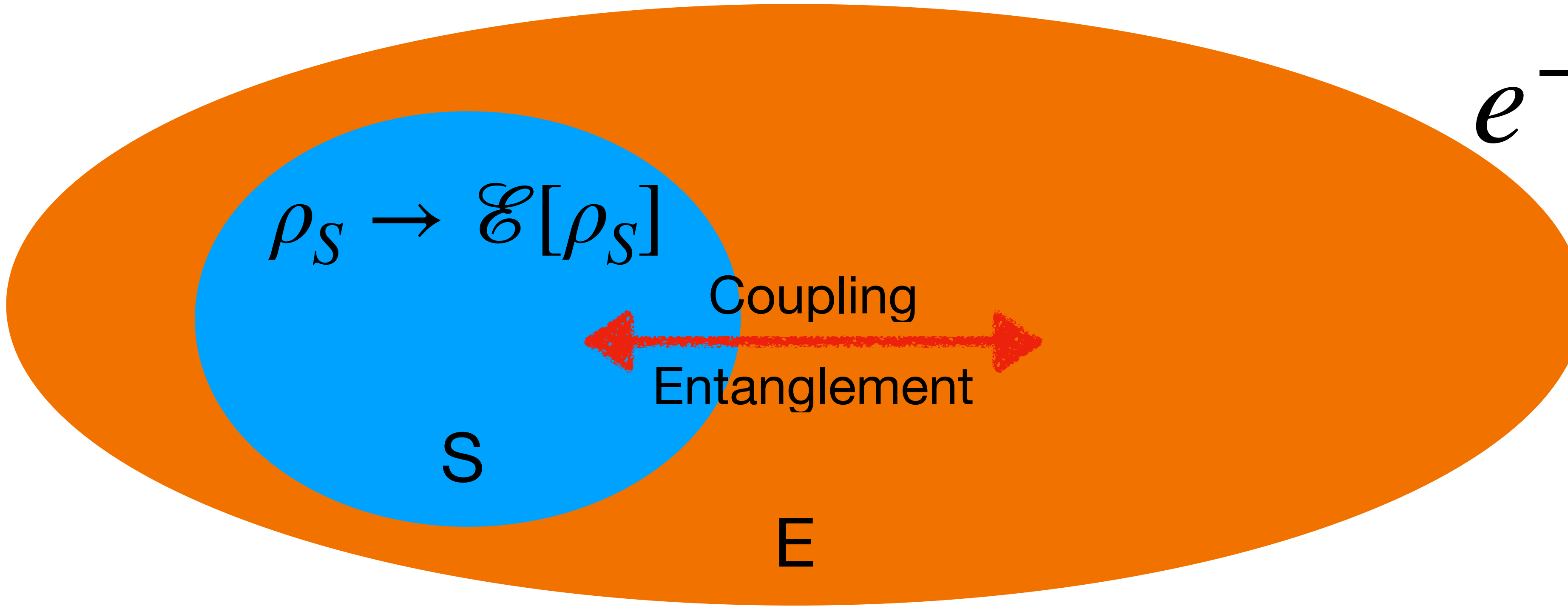
Stuff that hits my detector ↔ very weakly coupled fields

Open quantum systems

Non-unitary evolution

$$e^{-iH_{\text{full}}t}$$

$$\rho_S = \mathrm{Tr}_E \rho_{SE}$$



- We are perhaps most familiar with integrating out UV degrees of freedom to produce low energy EFT
    - In this case, evolution remains unitary unless you probe theory at $E > \Lambda$
- Generally, it is the case that tracing out the environment yields a non-unitary evolution of ρ_S
    - pure states typically evolve into mixed states

Formalism for Open Systems

Kraus operator decomposition

What is $\mathscr{E}[\rho_S]$?

For simplicity, assume system and environment begin in factorizable state $\rho_{SE} = \rho_S \otimes |e_0\rangle\langle e_0|$

$$\rho_S \rightarrow \mathrm{Tr}_E[e^{-iHt}\rho_{SE}e^{iHt}] = \sum_i \langle e_i | e^{-iHt}(\rho_S \otimes |e_0\rangle\langle e_0|)e^{iHt}|e_i\rangle$$

$$= \sum_i E_i \rho_S E_i^\dagger \qquad E_i = \langle e_i | e^{-iHt} | e_0 \rangle$$

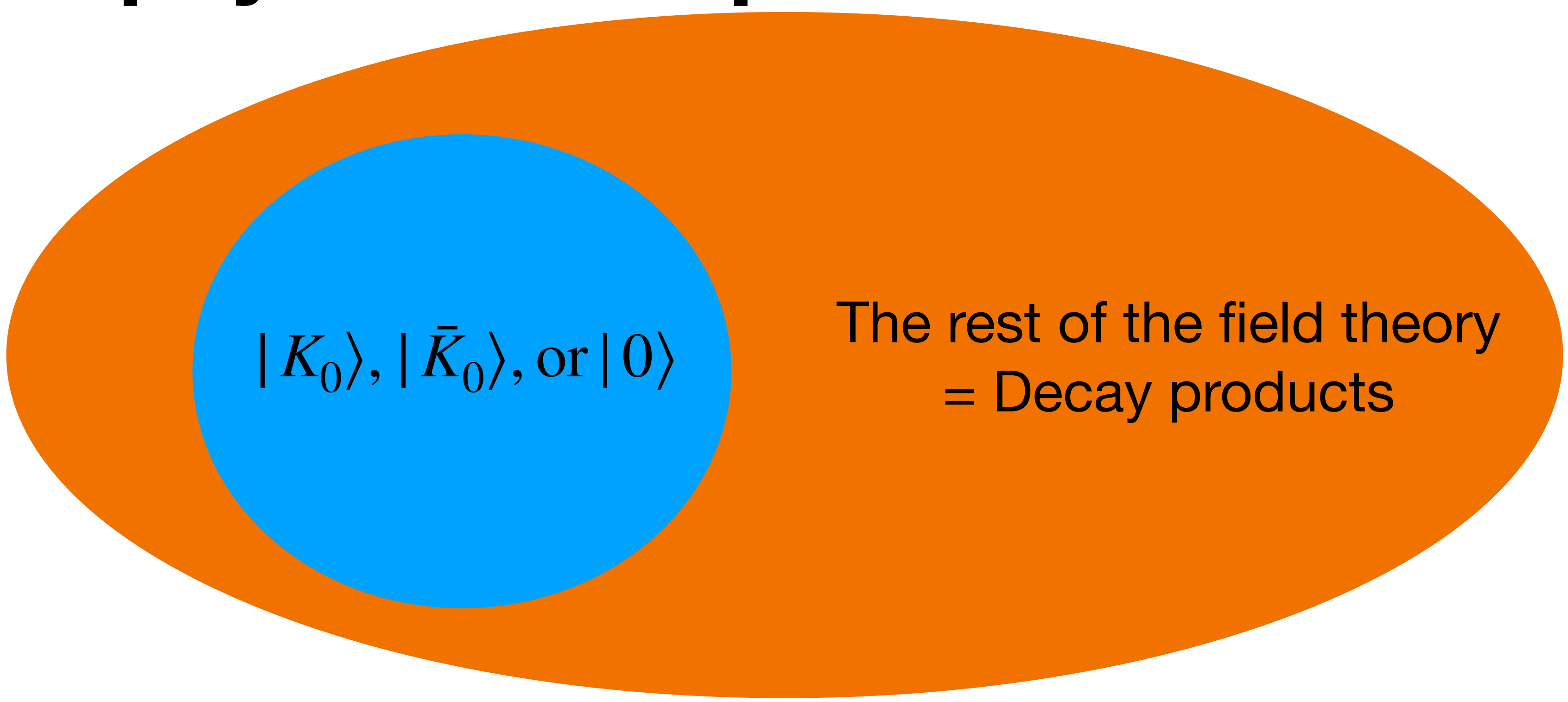
Remnant of unitarity of complete system: $\sum_i E_i E_i^\dagger = I$

Preserve $\mathrm{Tr}\rho_S$

In any realistic system where experimentalists have limited measurement capability, this is the actual definition of unitary evolution

The effective Hamiltonian for $K_0 - \bar{K}_0$

A particle physics example



Commonly refer to only the 2-state system $|K_0\rangle, |\bar{K}_0\rangle$

$$H_{\text{eff}} = G - \frac{i}{2}\Gamma$$

Non-hermitian 2x2 hamiltonian parametrizes oscillation and decay

$$\rho_S = \begin{pmatrix} \rho_{K\bar{K}} & 0 \\ 0 & 0 \end{pmatrix} \to \begin{pmatrix} e^{-iH_{\text{eff}}}\rho_{K\bar{K}}e^{iH^\dagger_{\text{eff}}} & 0 \\ 0 & 1 - \text{Tr}\rho'_{K\bar{K}} \end{pmatrix}$$

$$E_0 = \begin{pmatrix} e^{-iH_{\text{eff}}t} & 0 \\ 0 & 0 \end{pmatrix}$$

$E_{1,2}$ populate the $|0\rangle\langle 0|$ state

The Minimal Environment

Completion of the generalized EFT

- The partial trace operation is not invertible
- General theorem:
  - If your EFT is dimension d, the most general environment can be modeled with a system of (at most) dimension d^2
- Good because if you want to simulate an open system, can always represent environment without huge additional resource requirements

For Kaon example: The kaon system had three states.
There were 3 Kraus operators required to meet unitarity requirements,
$\Longrightarrow$Environment must have at least 3 states

Kaons on qubits

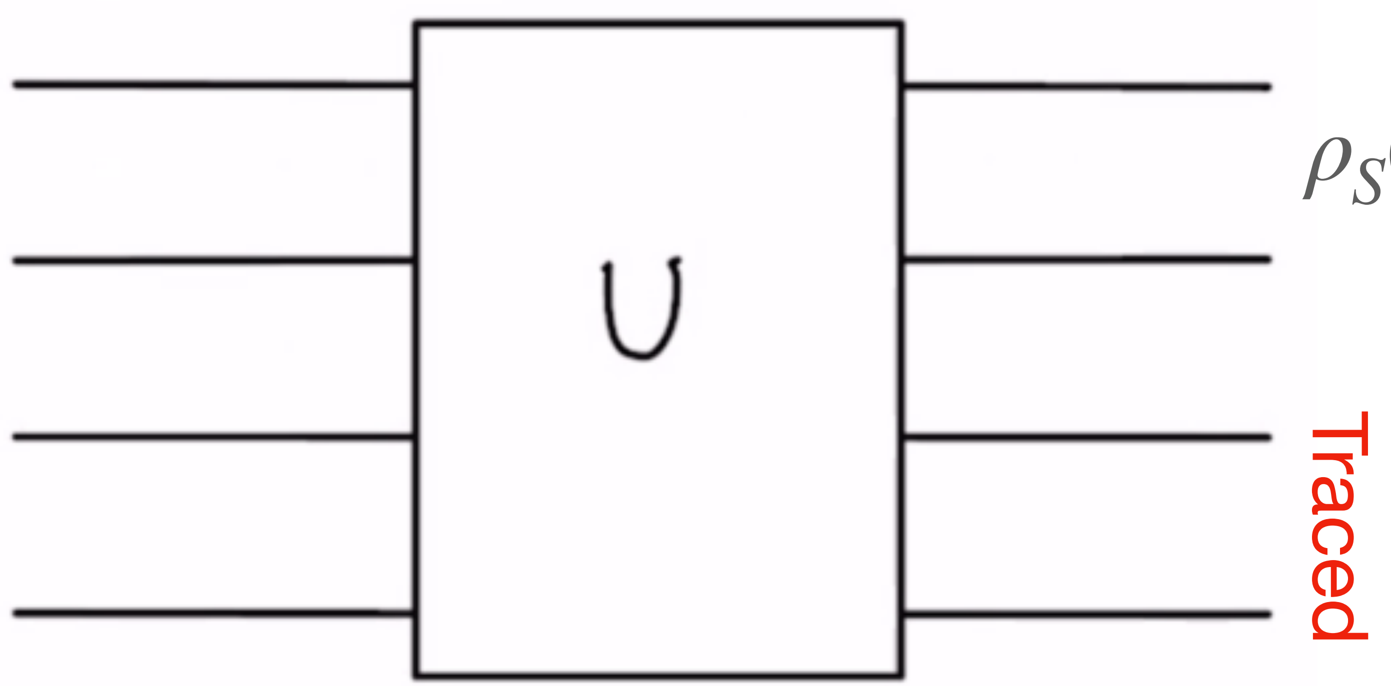
Ancillaries simulate environment

$$\rho_S = \begin{pmatrix} \rho_{K\bar{K}} & 0 \\ 0 & 0 \end{pmatrix}$$

Ancilaries = Environment

$|0\rangle$

$|0\rangle$



$$\rho_S(t) = \sum_{i=0}^{2} E_i \rho_S E_i^\dagger$$

If you measure the ancillaries, then you know which of the E's happened:

$$[\rho_S(t)]_i = \frac{E_i \rho_S E_i^\dagger}{\mathrm{Tr}[E_i \rho_S E_i^\dagger]}$$

Outcome 0 means you still have kaons…
they haven't decayed

A Non-unitary Ising model

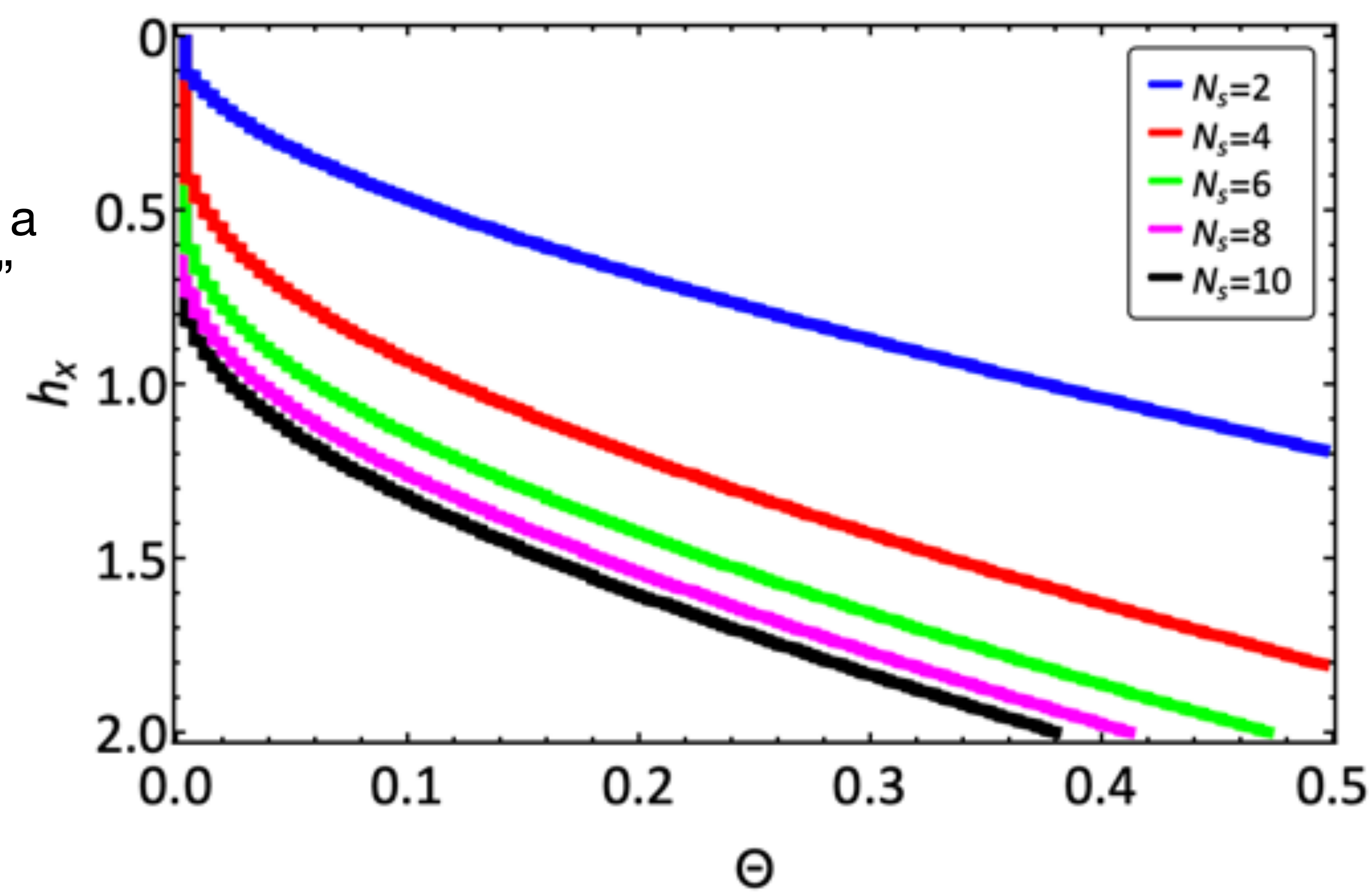
A toy example for finite μ

$$H_{\text{eff}} = \sum_{<ij>} \sigma_i^z \otimes \sigma_j^z + h_x \sum_i \sigma_i^x - \overset{\text{Dissipative term}}{\boxed{i\Theta \sum_i (1 - \sigma_i^z)}}$$

This is well-studied in the case of statistical mechanics - zeros in the partition function in the $h_x - \Theta$ plane
The Lee-Yang edge singularity - a non-unitary critical point

In finite size quantum hamiltonian this corresponds to a merger of lowest 2 eigenvectors - "exceptional point"

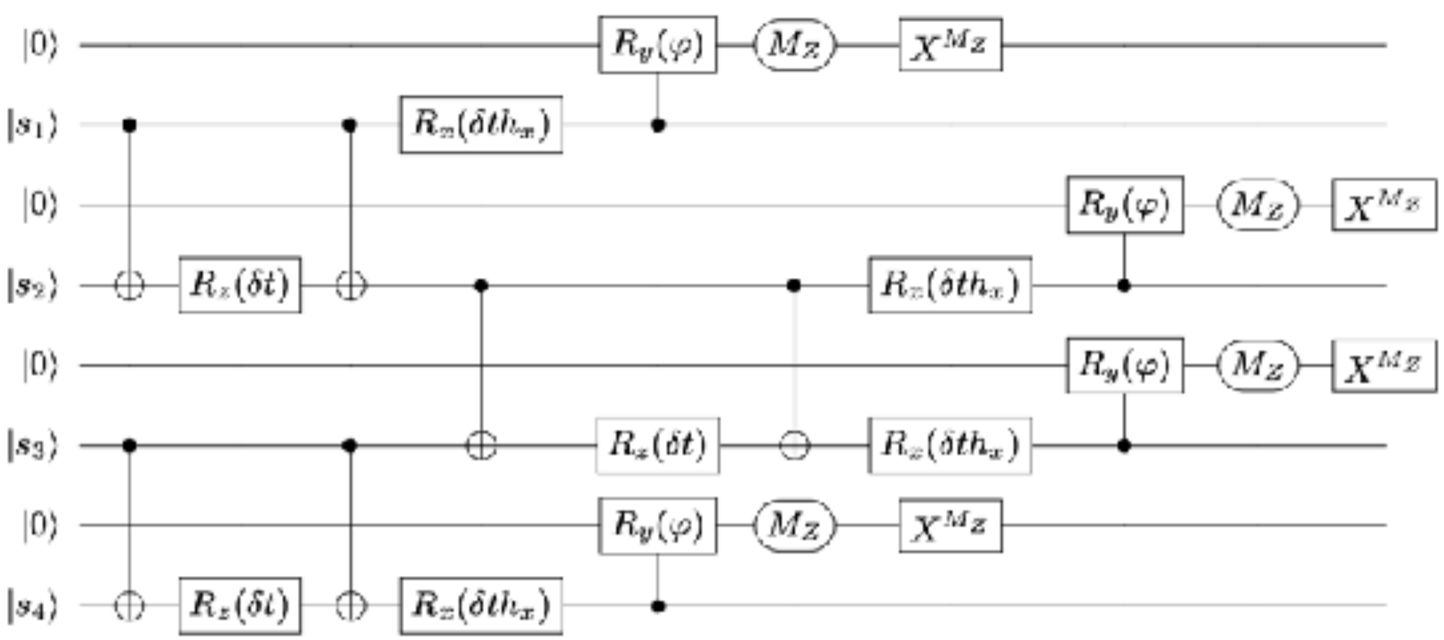
Critical lines analogous to phase transition at finite density



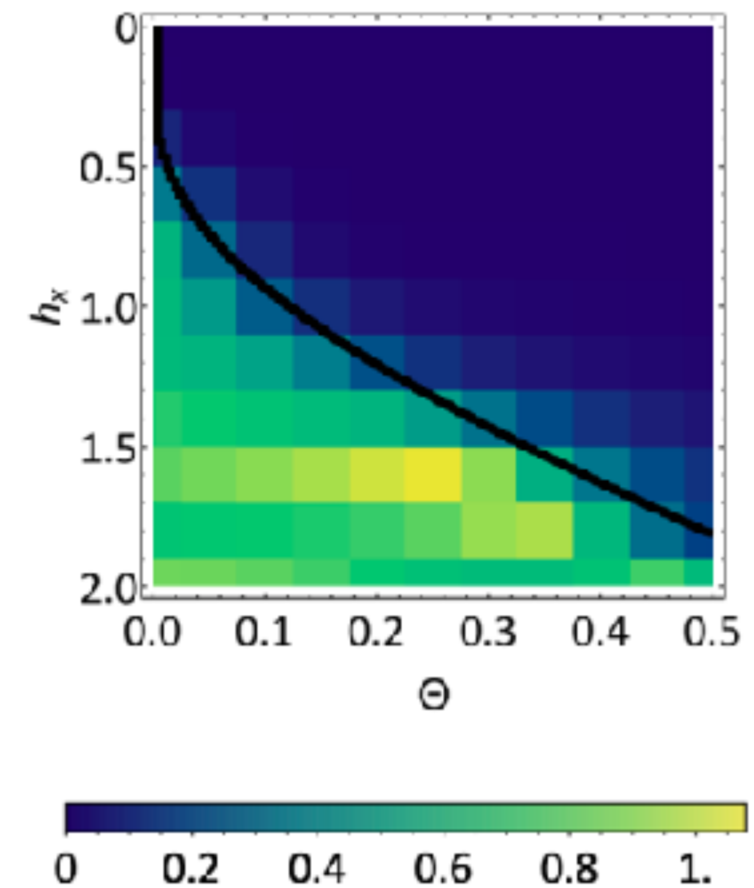
JH, Bharath Sambasivam, Judah Unmuth-Yockey 2012.05257

Non-unitary Ising time-step

Example - An ancillary for each Ising site to implement the anti-unitary dissipation



2'nd Renyi Entropy of final state

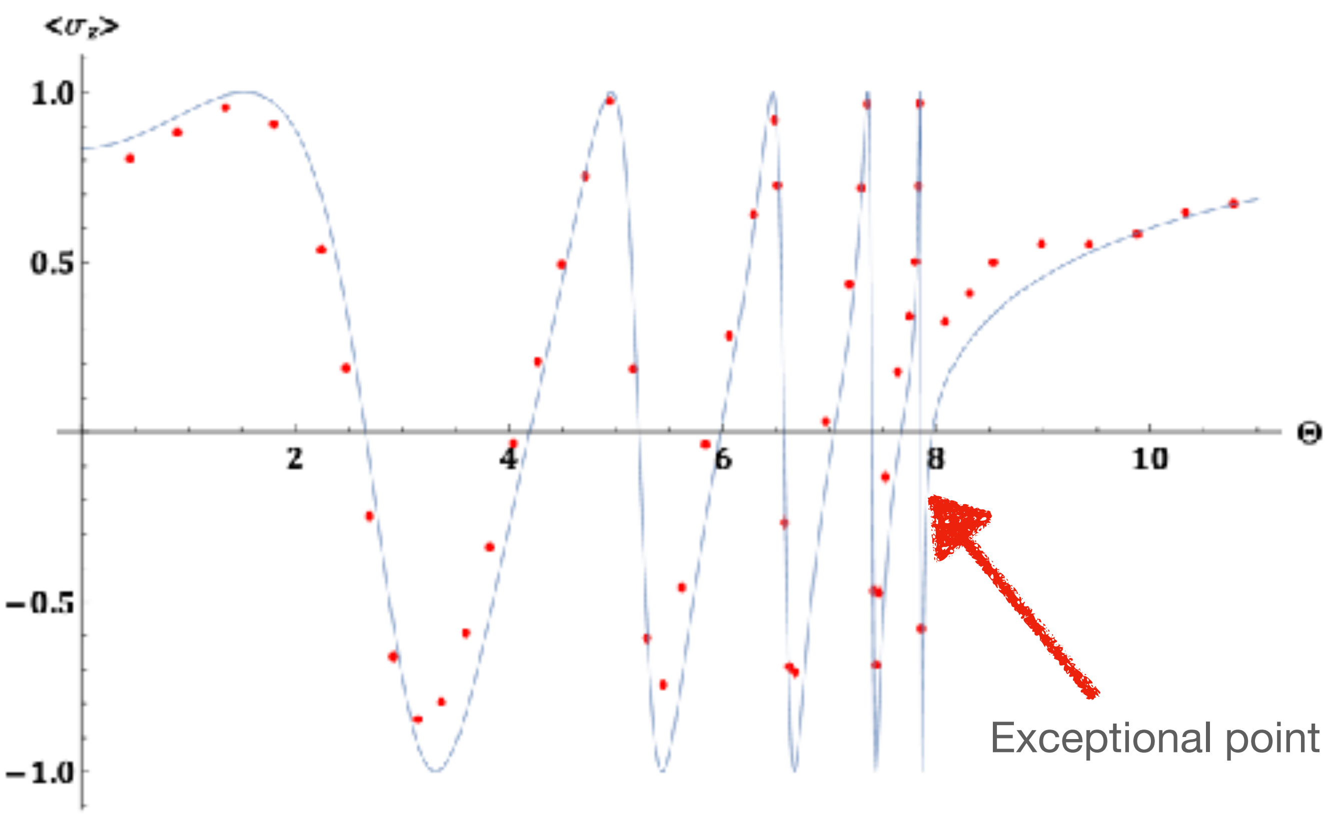


Results on right are classical calculation - not done on lossy quantum computer

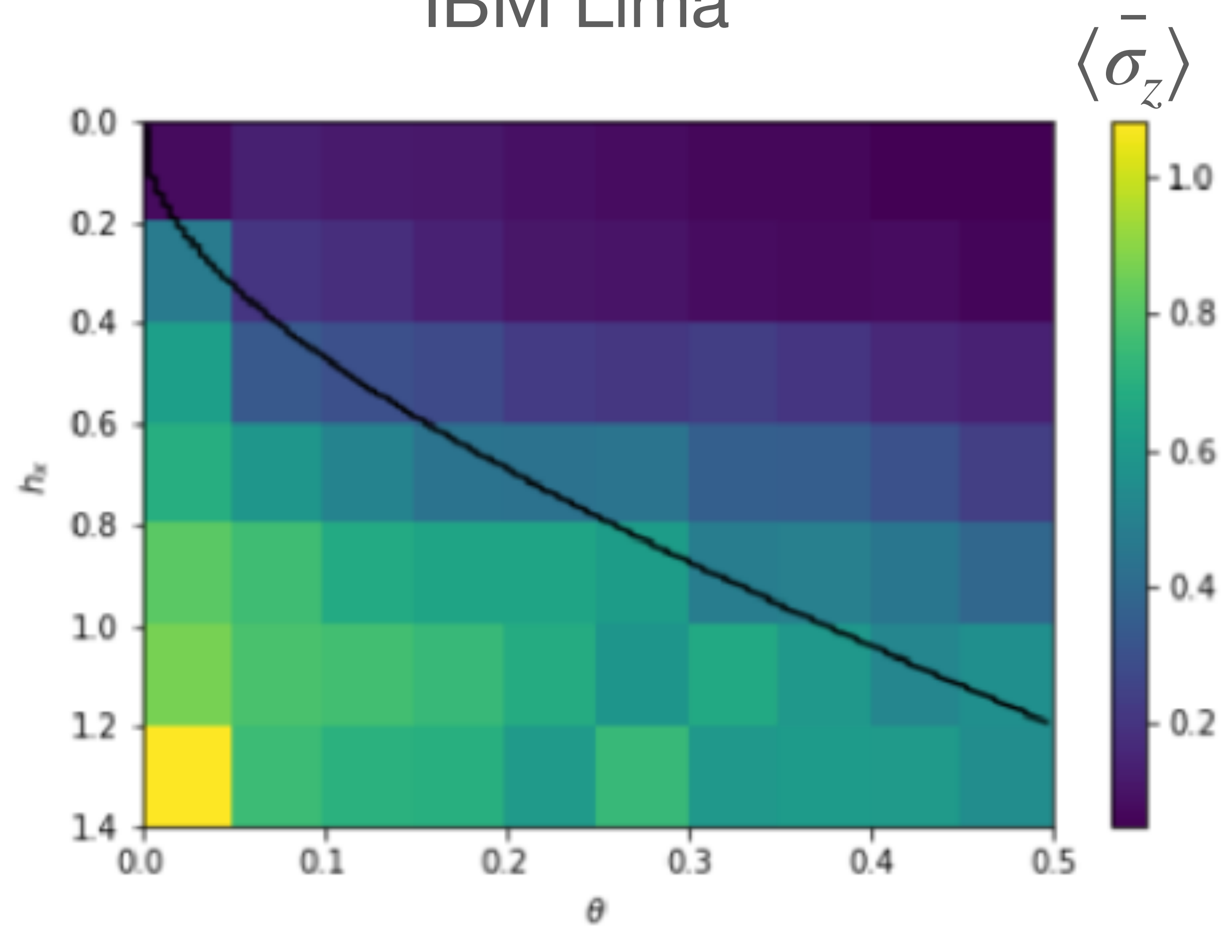
Quantum Simulation

Followup in progress with Michael Hite (Iowa), Erik Gustavson (FNAL)

IBM Yorktown



IBM Lima



Just one system qubit
no NN interactions, $h_x = 1$

Two system qubits
One NN interaction

Example Goal: central charge on critical line should be c = -22/5, which can be extracted from behavior near critical line

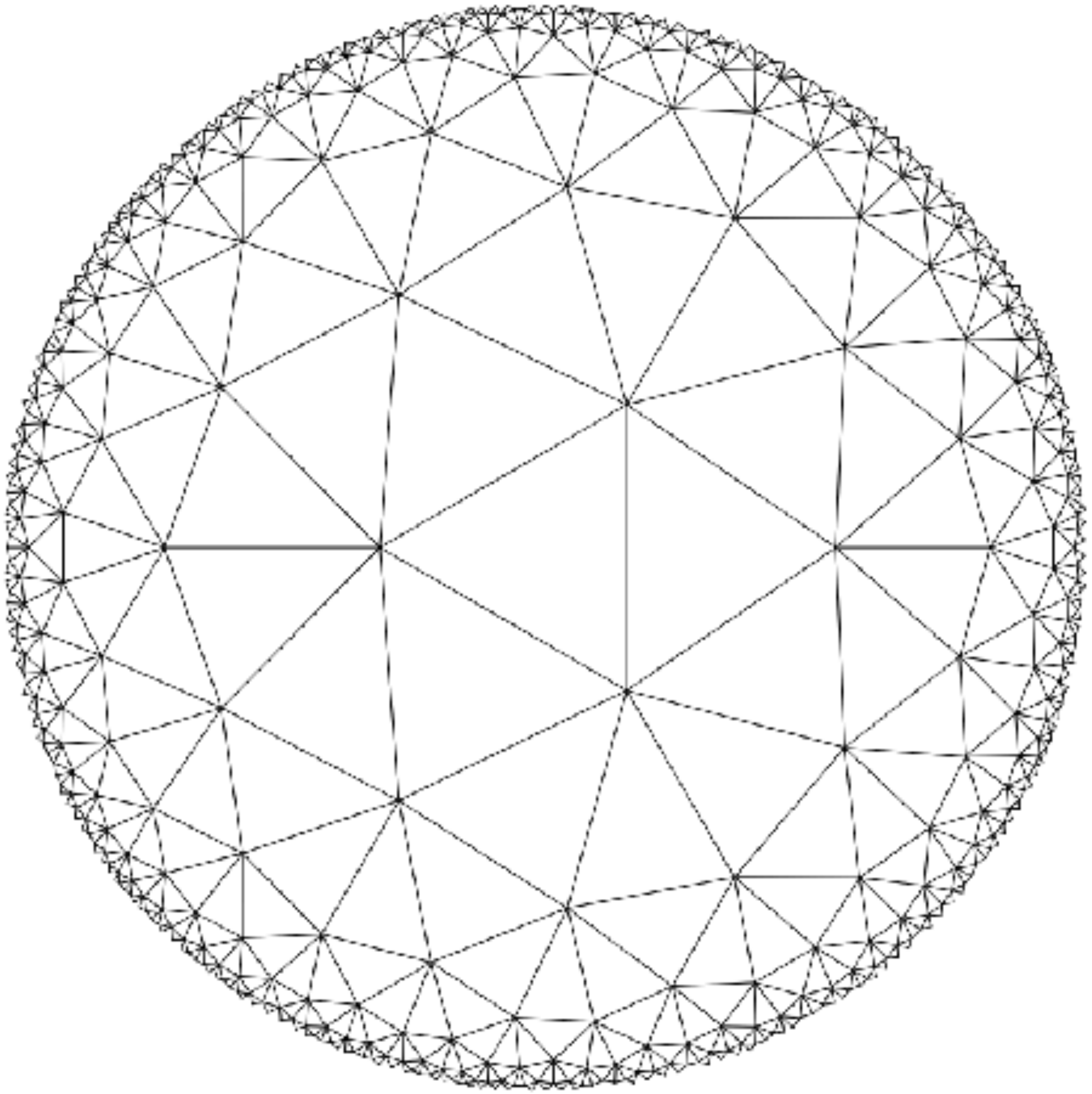
Future open system work

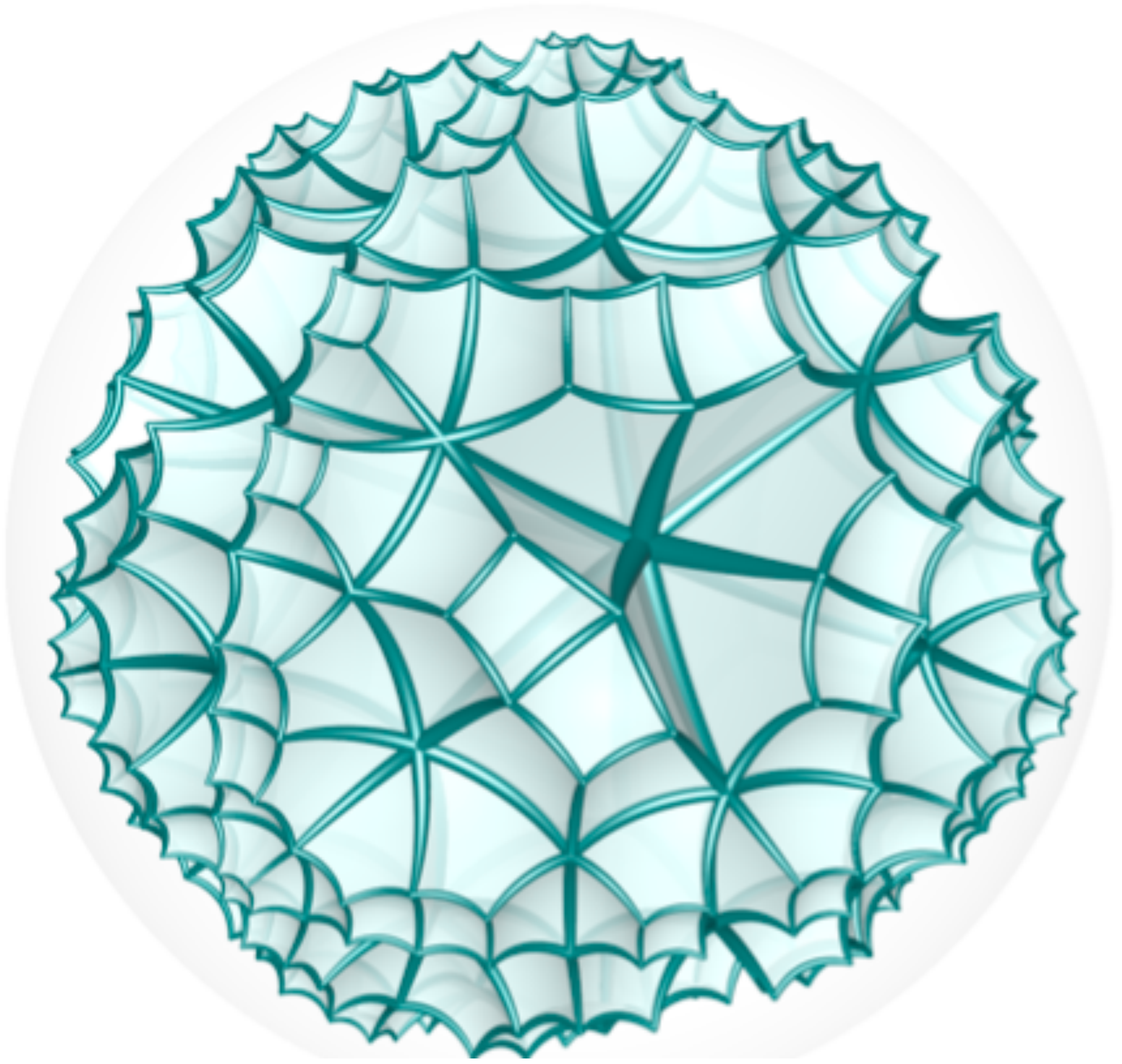
- Extract more interesting observables
  - Central charge of non-unitary critical point?
- Study more interesting models
  - O(N) models exhibit richer structure (topological order, confinement like dynamics)
  - Phase diagram at finite chemical potential is not known due to sign problem

Lattices with Curvature

Discrete “AdS/CFT” (Thermal - not quantum yet)

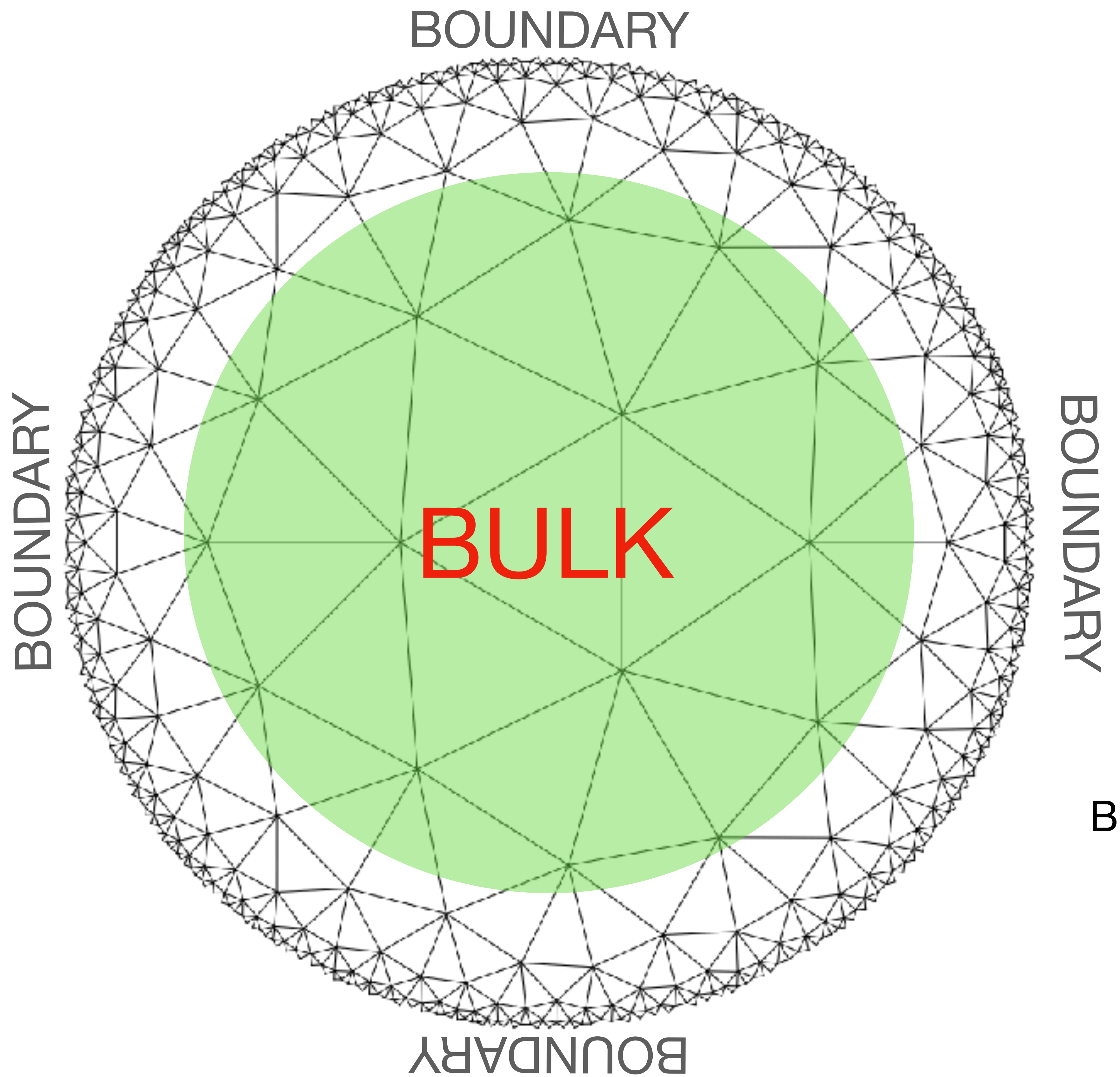
With Asad Asaduzzaman, Simon Catterall, Roice Nelson, Judah Unmuth-Yockey





Regular or semi-regular Hyperbolic lattices of various dimension are not difficult to generate
Standard lattice techniques can be used to study field theories on these curved spaces

Build the lattice from the inside and work outwards



UV scales:
lattice spacing, warping

IR scales:
Boundary topology, bulk depth

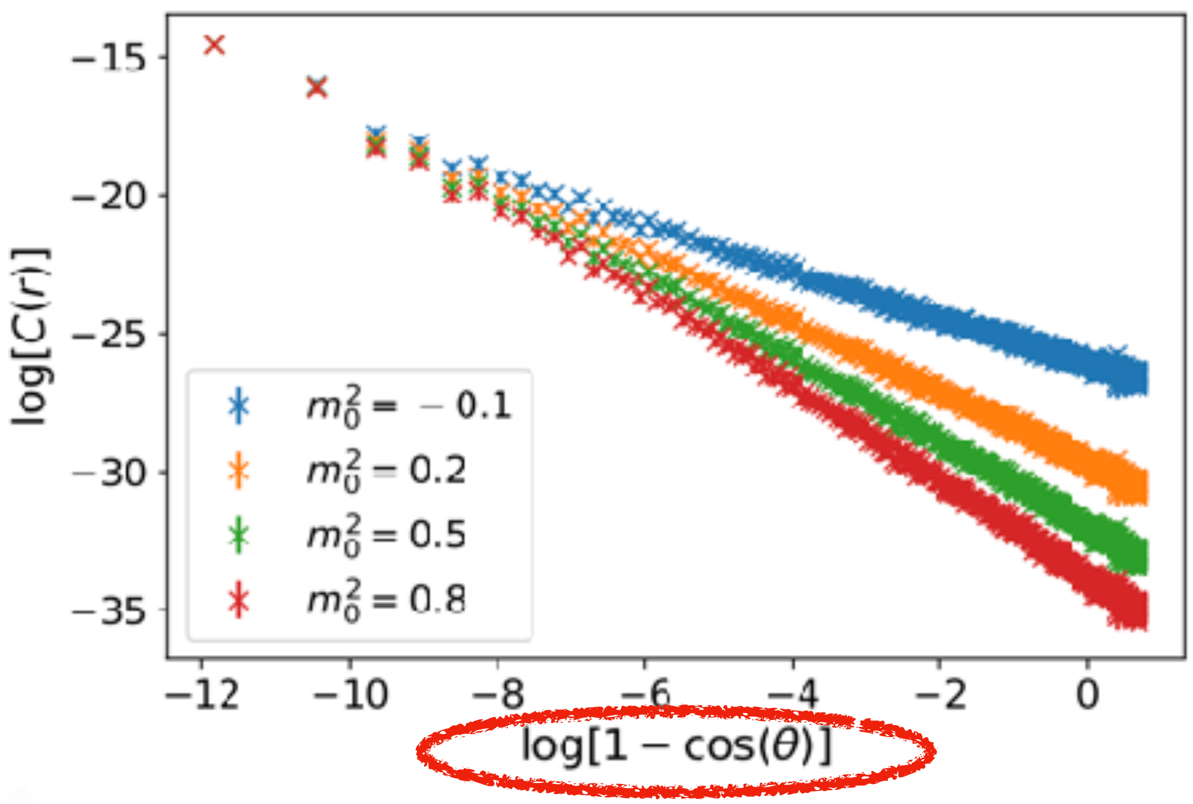
Scalar field in the bulk

Boundary - boundary 2-pt function:

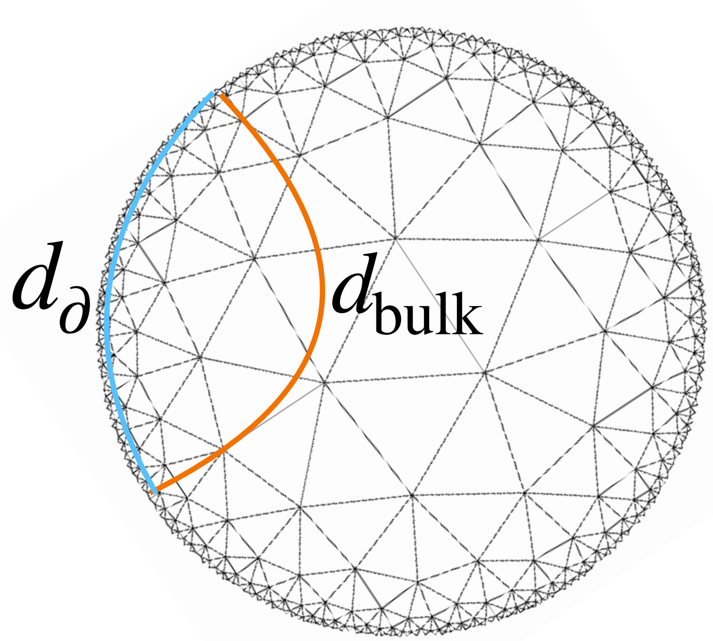
- Power law correlators are generic

- Simple consequence of geodesic through bulk gaining exponential shortcut vs. path along boundary:

$$e^{-m_{\text{eff}} d_{\text{bulk}}} \sim \frac{1}{d_\partial^{m_{\text{eff}}}}$$



Conformal symmetry broken as boundary has topology of circle
With radius R



Fitting the power

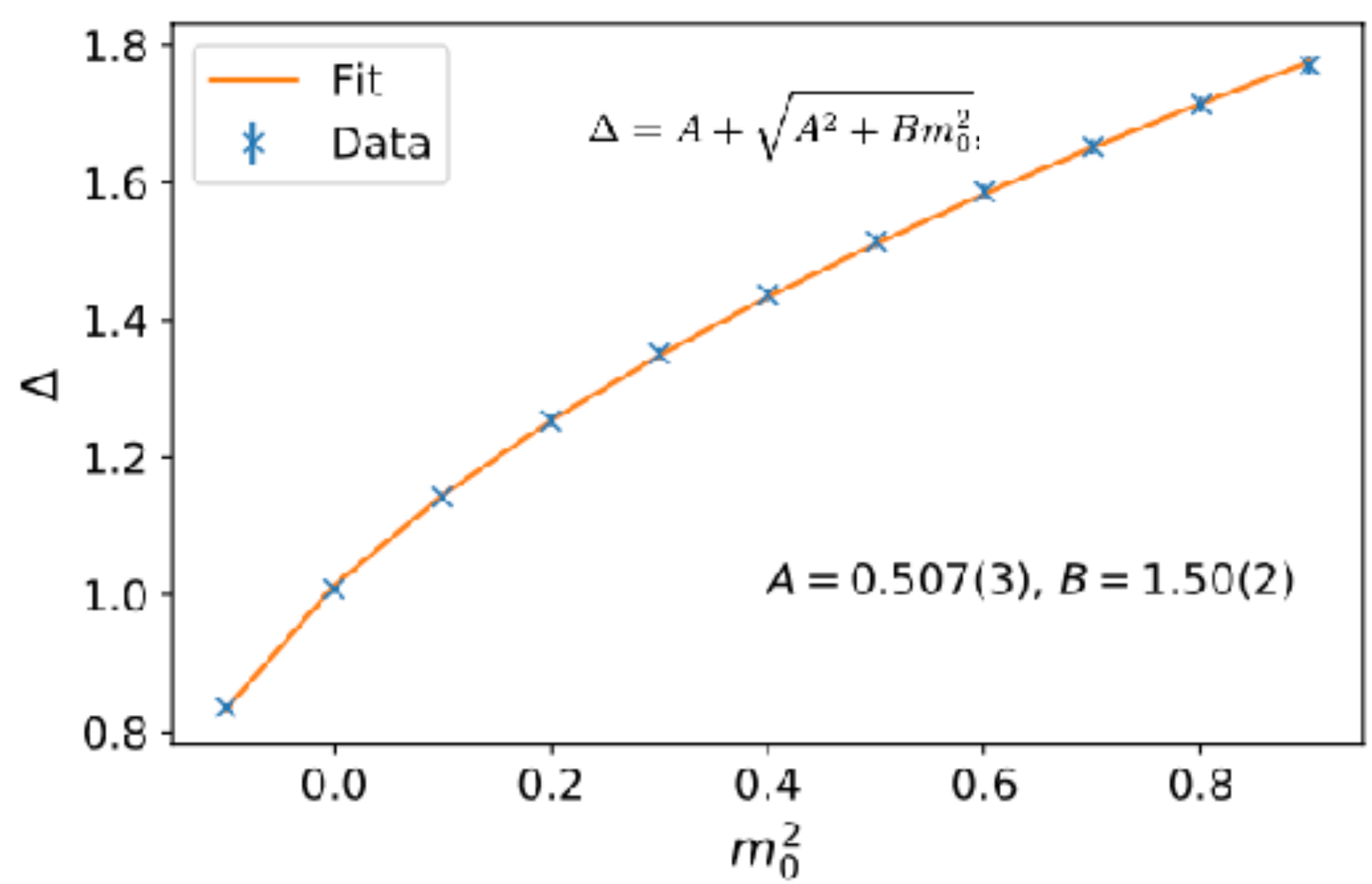
Continuum Expectation

Scaling dimension of corresponding operator:

$$\Delta = \frac{d}{2} \pm \sqrt{\frac{d^2}{4} + \frac{m^2}{k^2}}$$

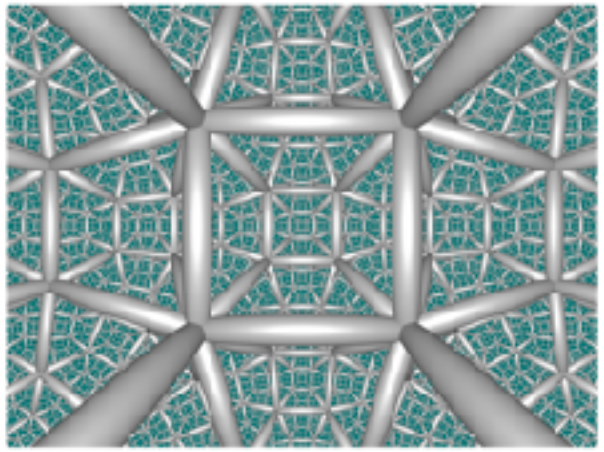
Dimension of the boundary is d=1

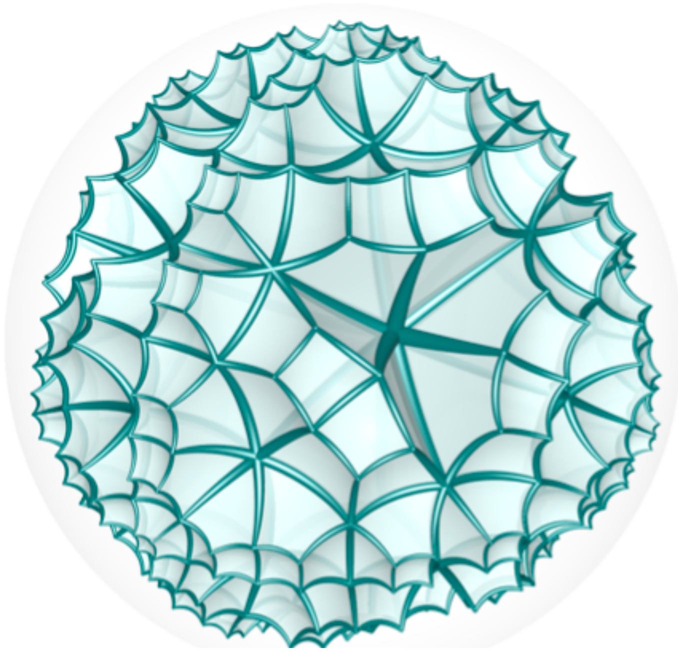
Both A and B match expectations, A = 1/2, and B being related to the radius of curvature

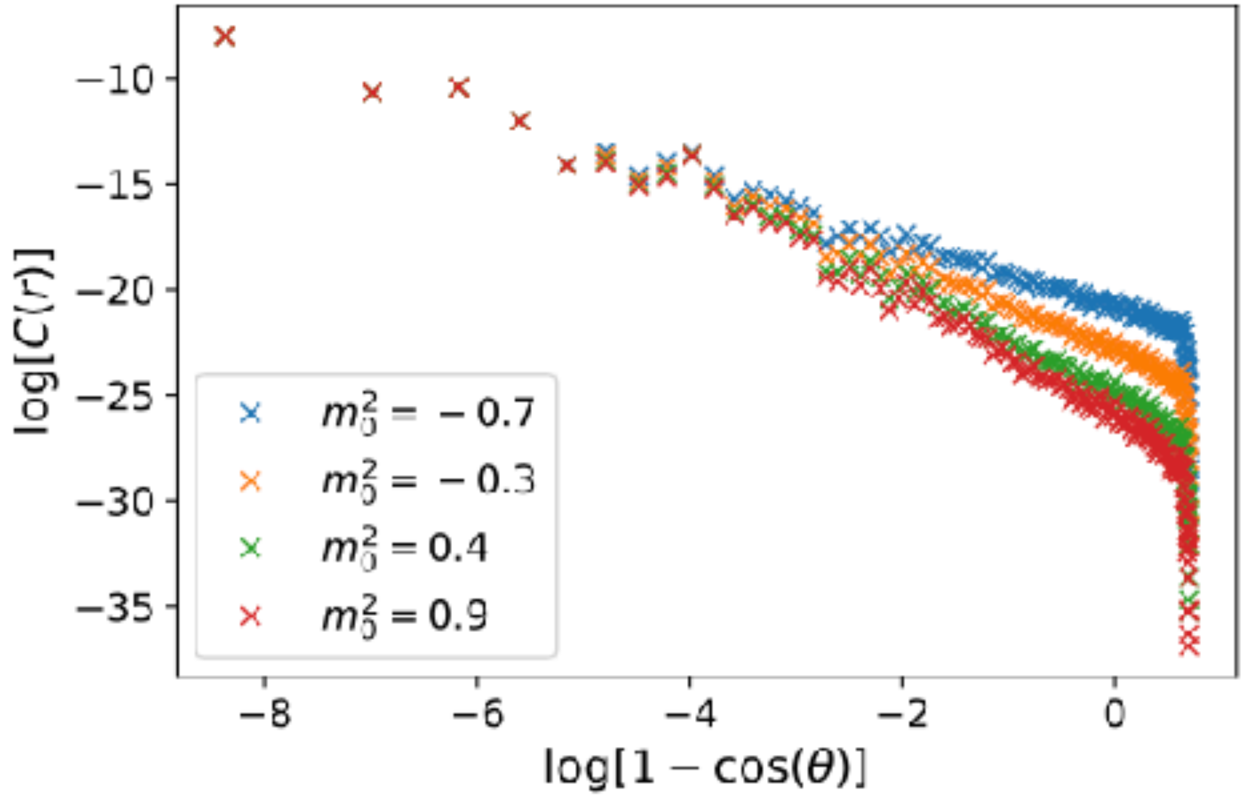


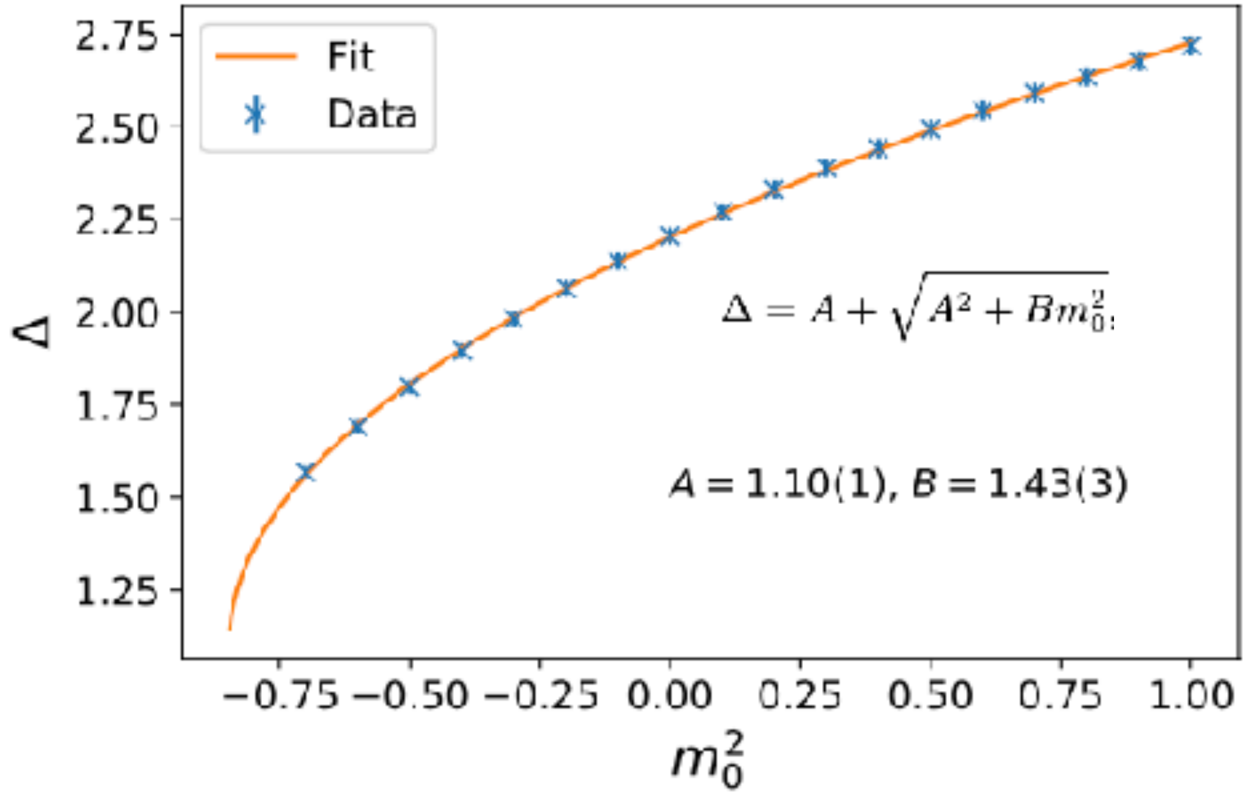
3D Hyperbolic Geometry

$$\Delta = \frac{d}{2} \pm \sqrt{\frac{d^2}{4} + \frac{m^2}{k^2}}$$







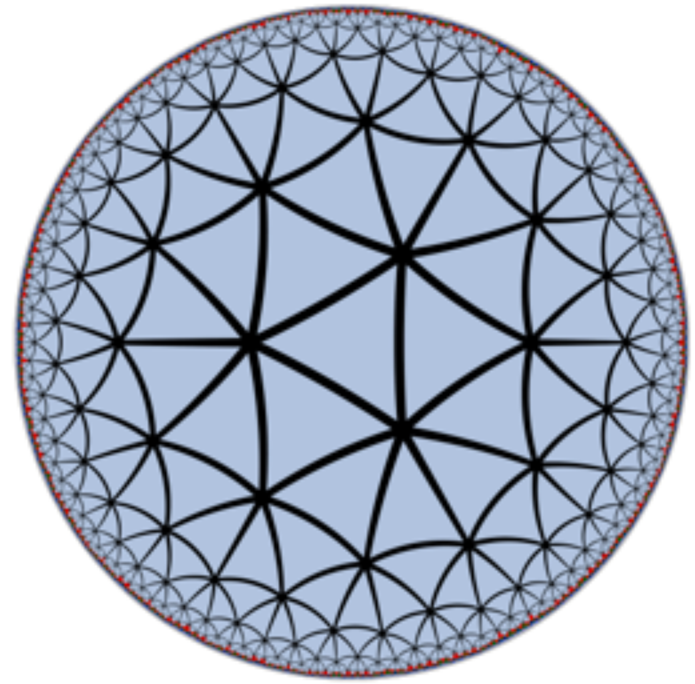


Conclusion to draw: despite using very crude lattice (spacing is nearly same as inverse curvature!) Aspects of continuum AdS/CFT expectations are preserved.

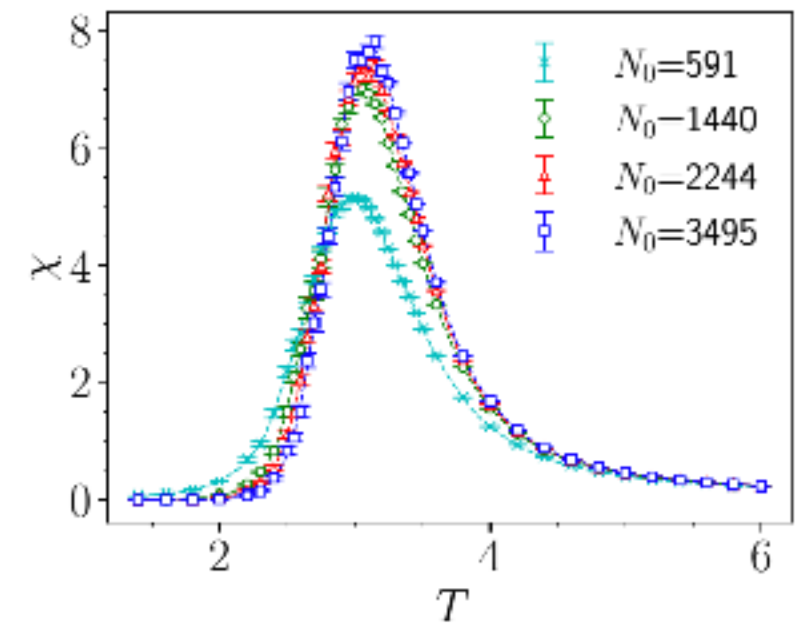
Strong dynamics in the bulk

Put Ising spins on the vertices -
links indicate nearest neighbor coupling

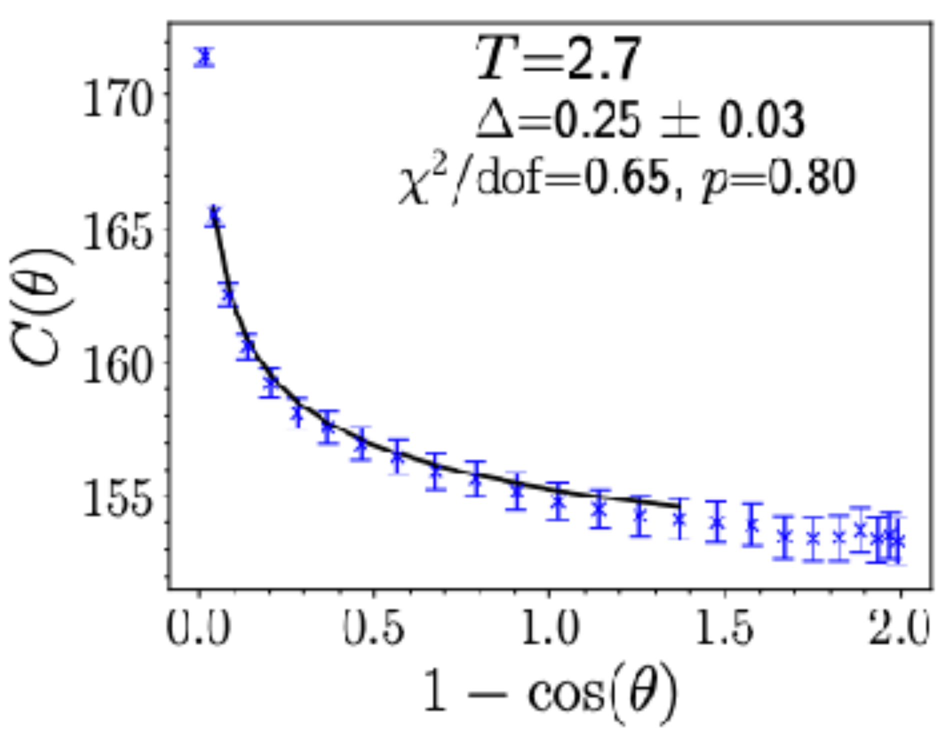
$$H = \sum_{<ij>} \sigma_i \sigma_j$$

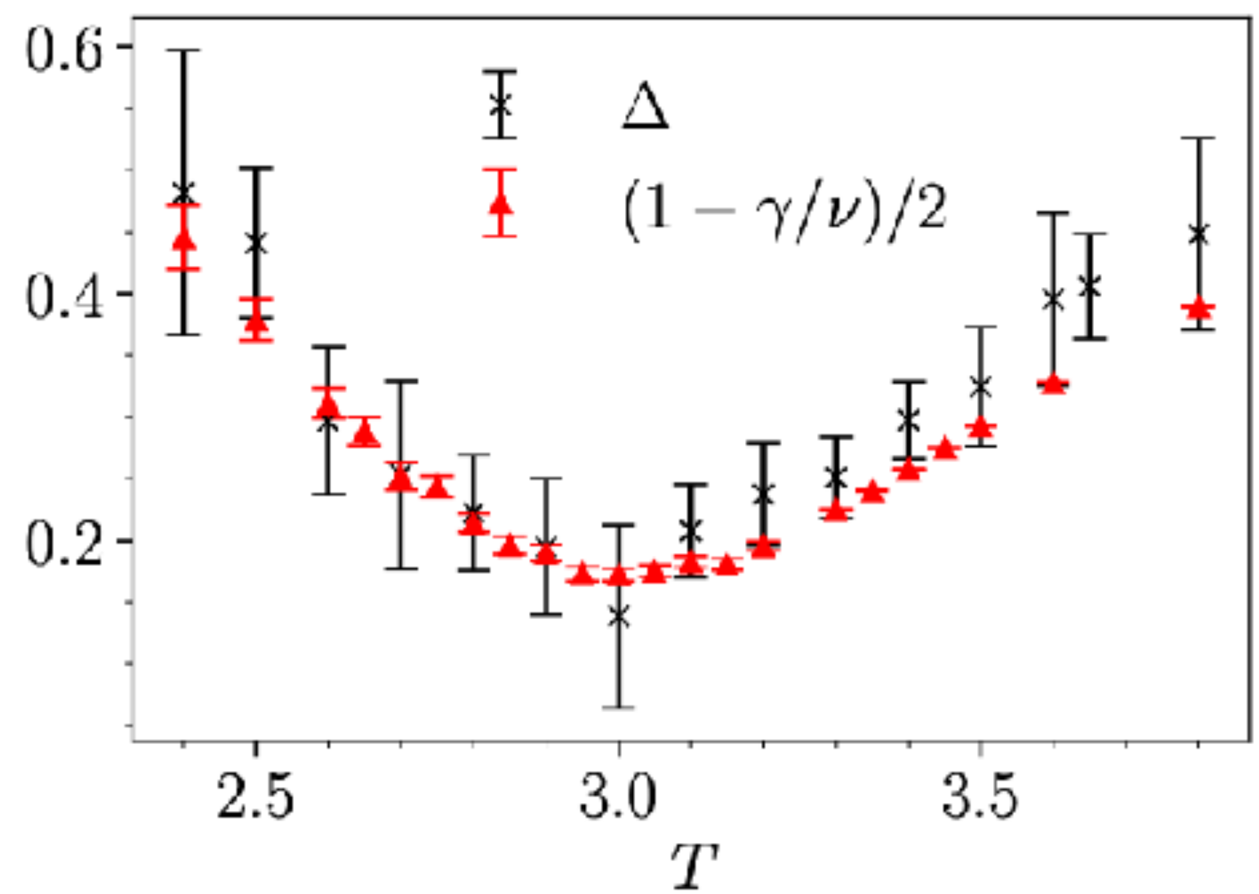


Ising Critical Temperature: T=~3



Boundary-Boundary correlators

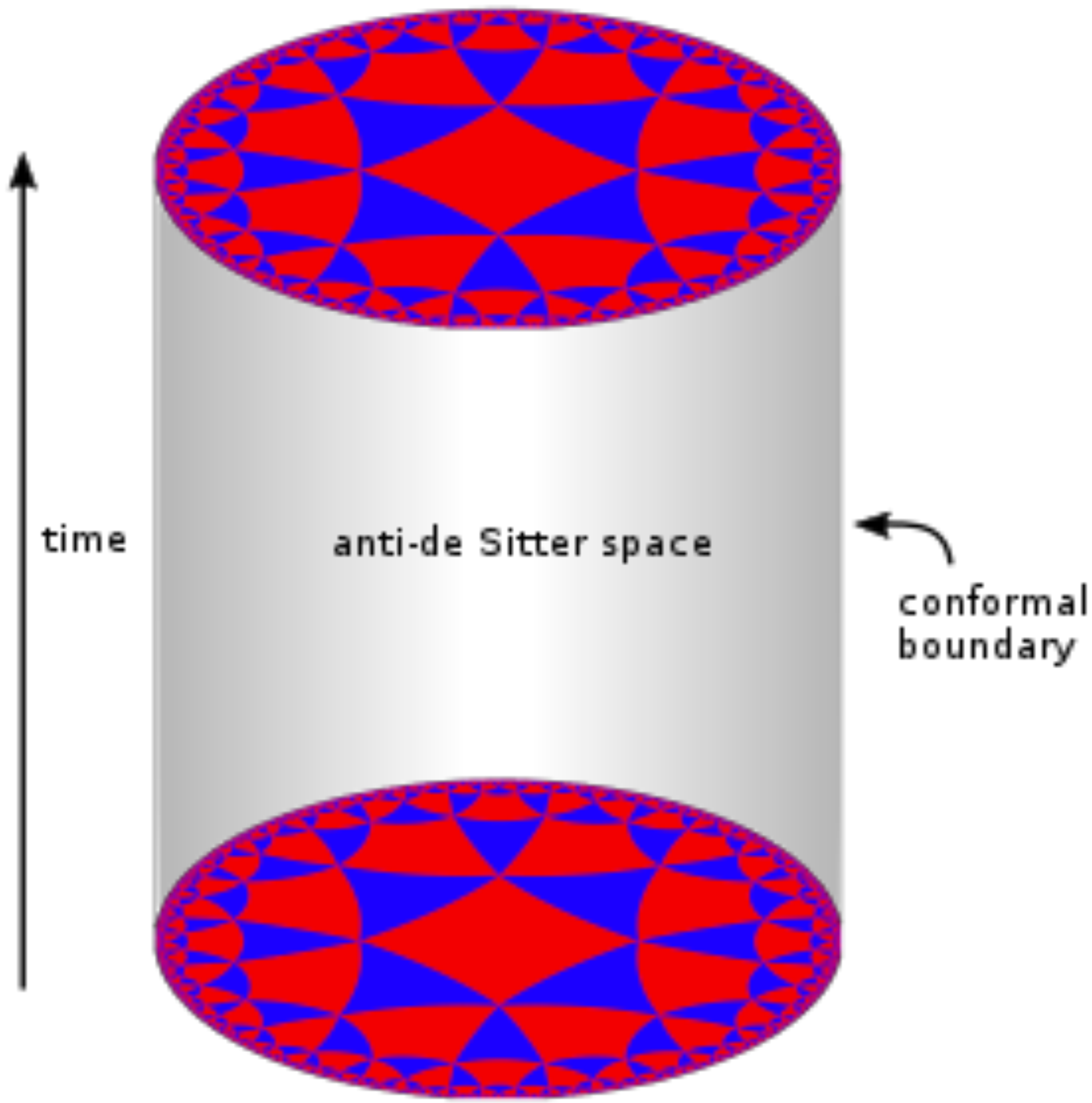




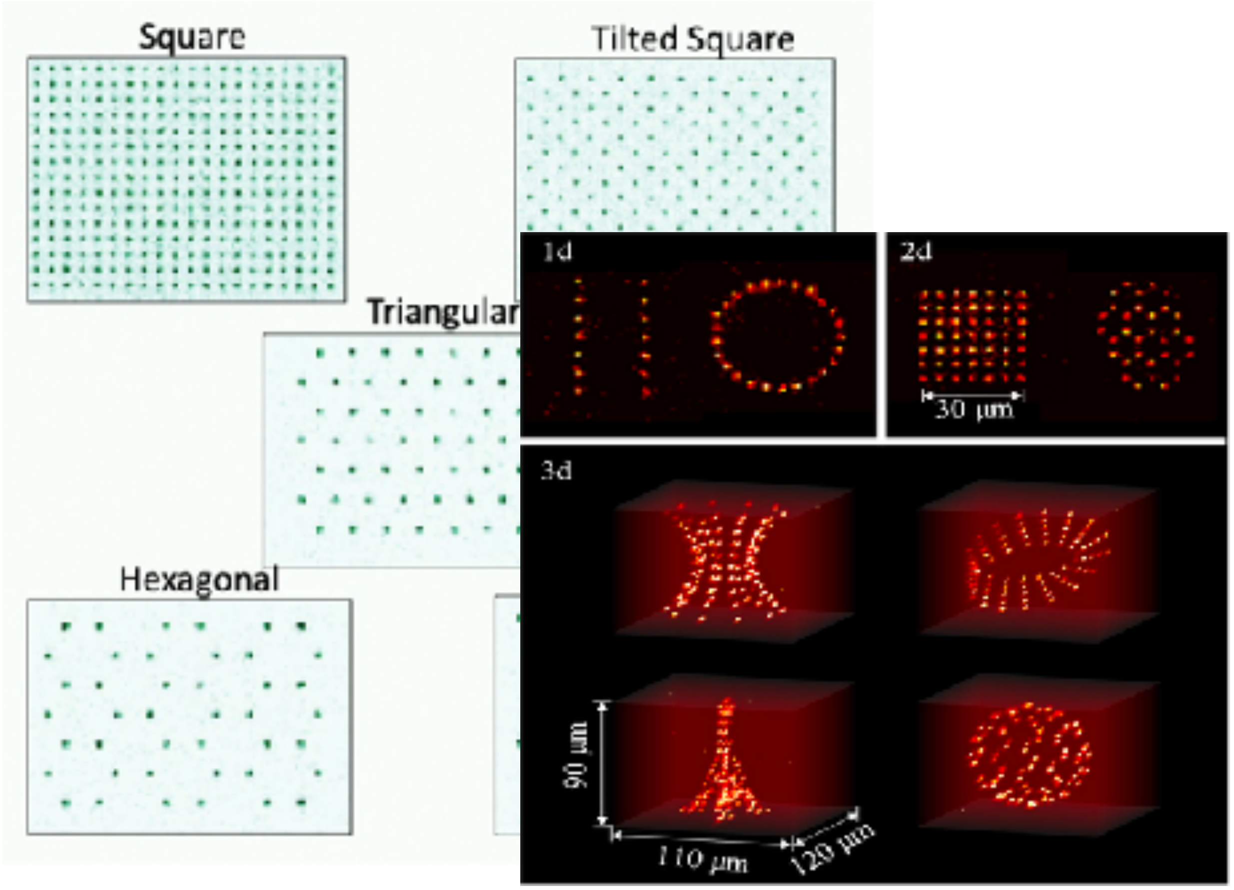
Physics in the bulk is non-mean field - expect non-trivial N-point correlators giving further info about boundary CFT

Quantum time evolution on H_2?

Rydberg arrays



Atoms trapped in geometry of your choice:

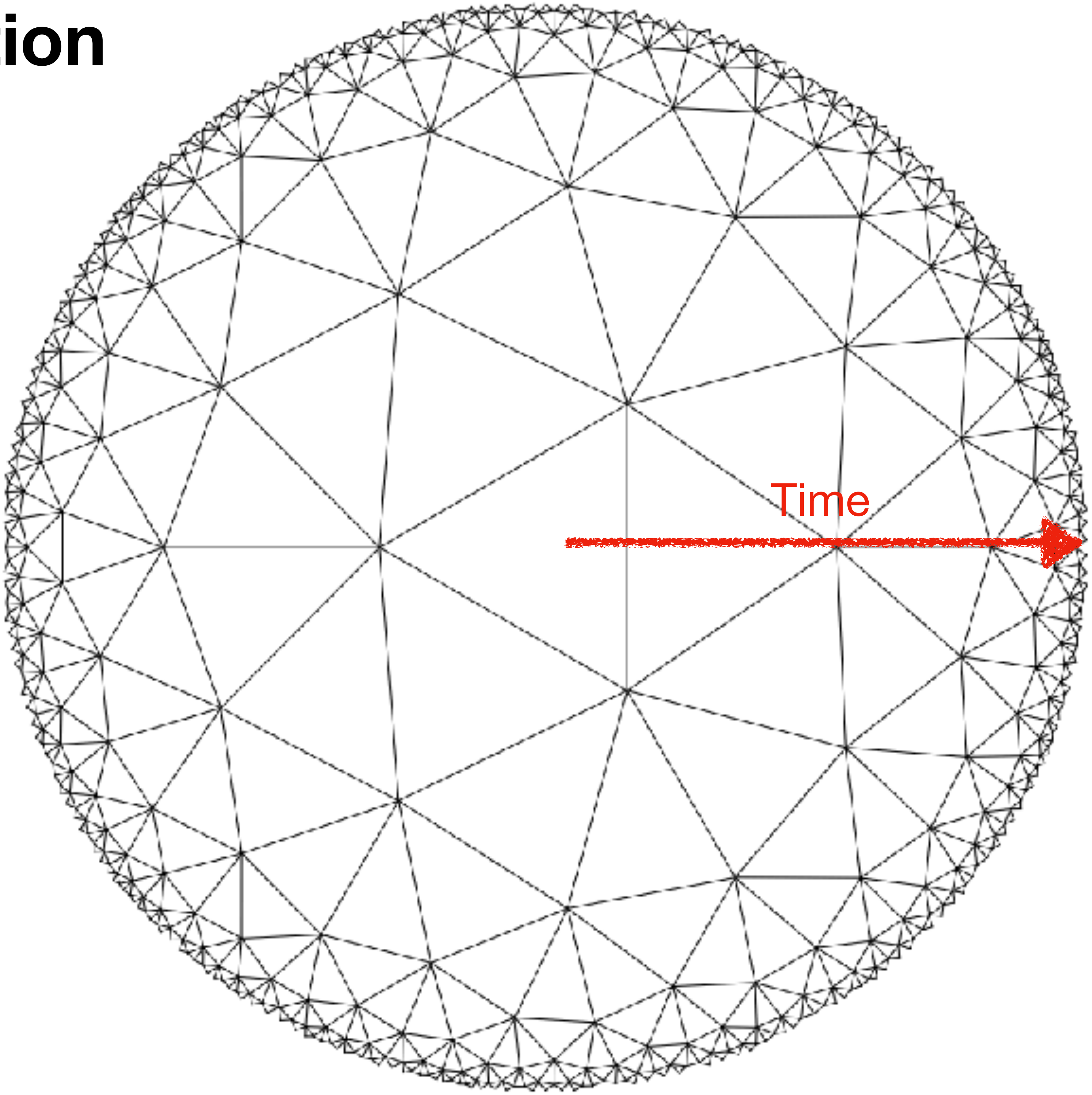


Bring together to interact with VdV interaction,
shine light for single qubit gates

Limited choice of Hamiltonian, slow refresh rate,
but long coherence time ~100μs

Qubit Cosmology

E.g. inflation



Can envision adding additional qubits at each time step - spacetime inflating! Initial state of new qubits ~ initial state selection

Conclusions

- Quantum computing holds promise for solving long-standing problems in particle theory
  - Overcoming the sign problem is the holy grail (both for particle theory and condensed matter physics)
- Some simple things can already be done
- **Lots** of work to do…e.g. hamiltonian gauge theories are finicky
  - How to truncate?
- It’s a very cool and fun playground, and hopefully one that will eventually yield real progress for our fields of study