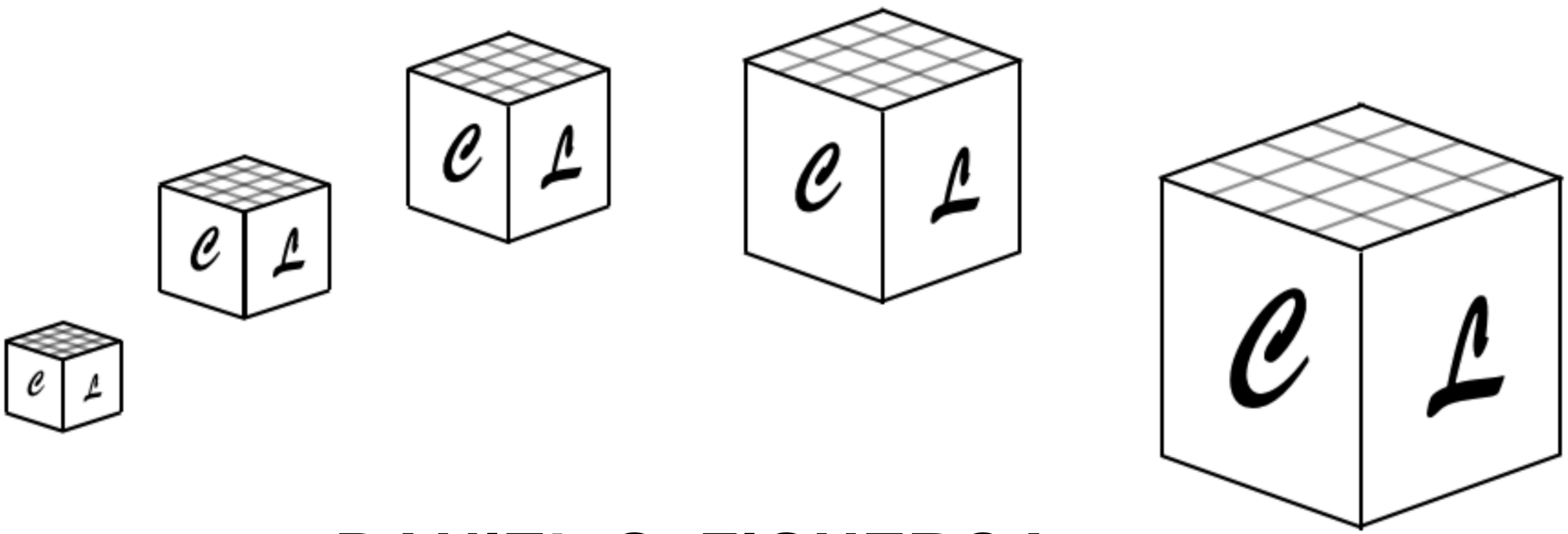
The early Universe

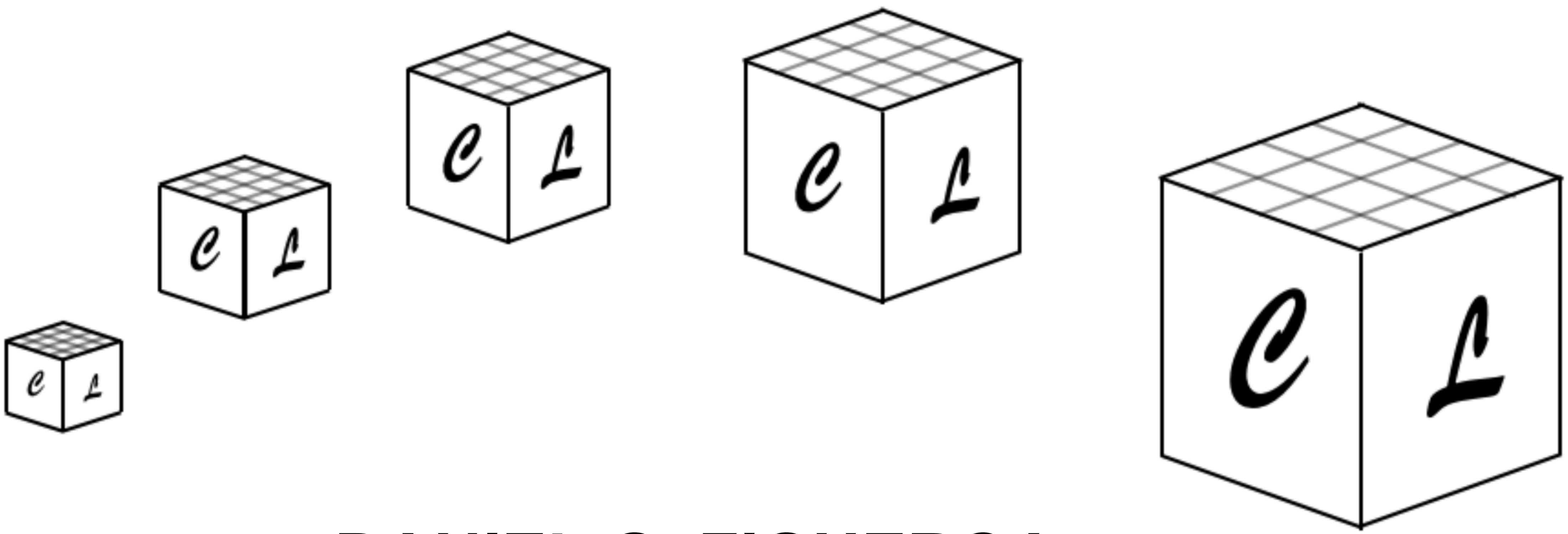


DANIEL G. FIGUEROA

IFIC, Valencia, Spain

BSM: PP meets Cosmo 2022 @ IFT, Madrid, Oct 3-28, 2022

The early Universe
… numerically speaking



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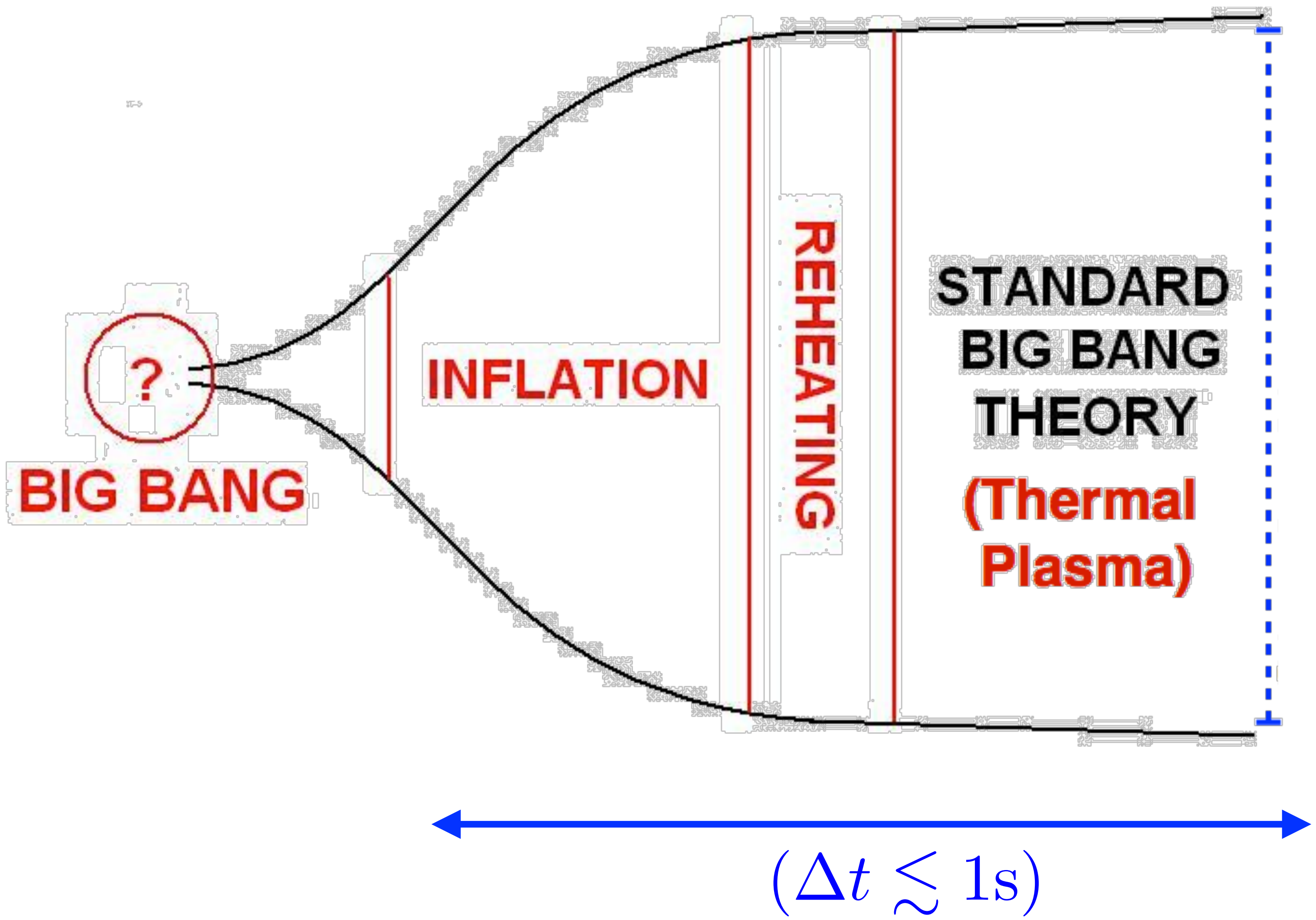

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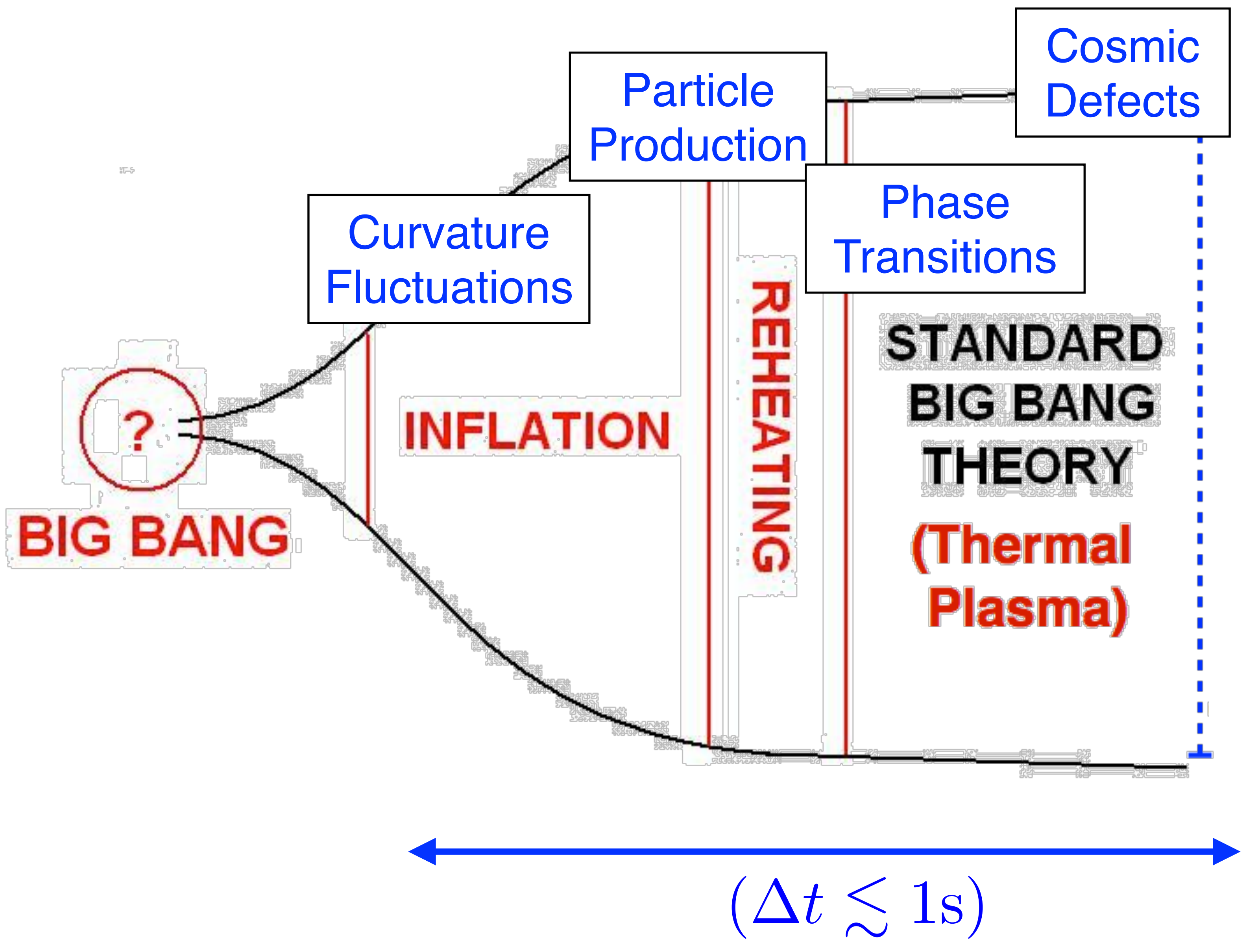

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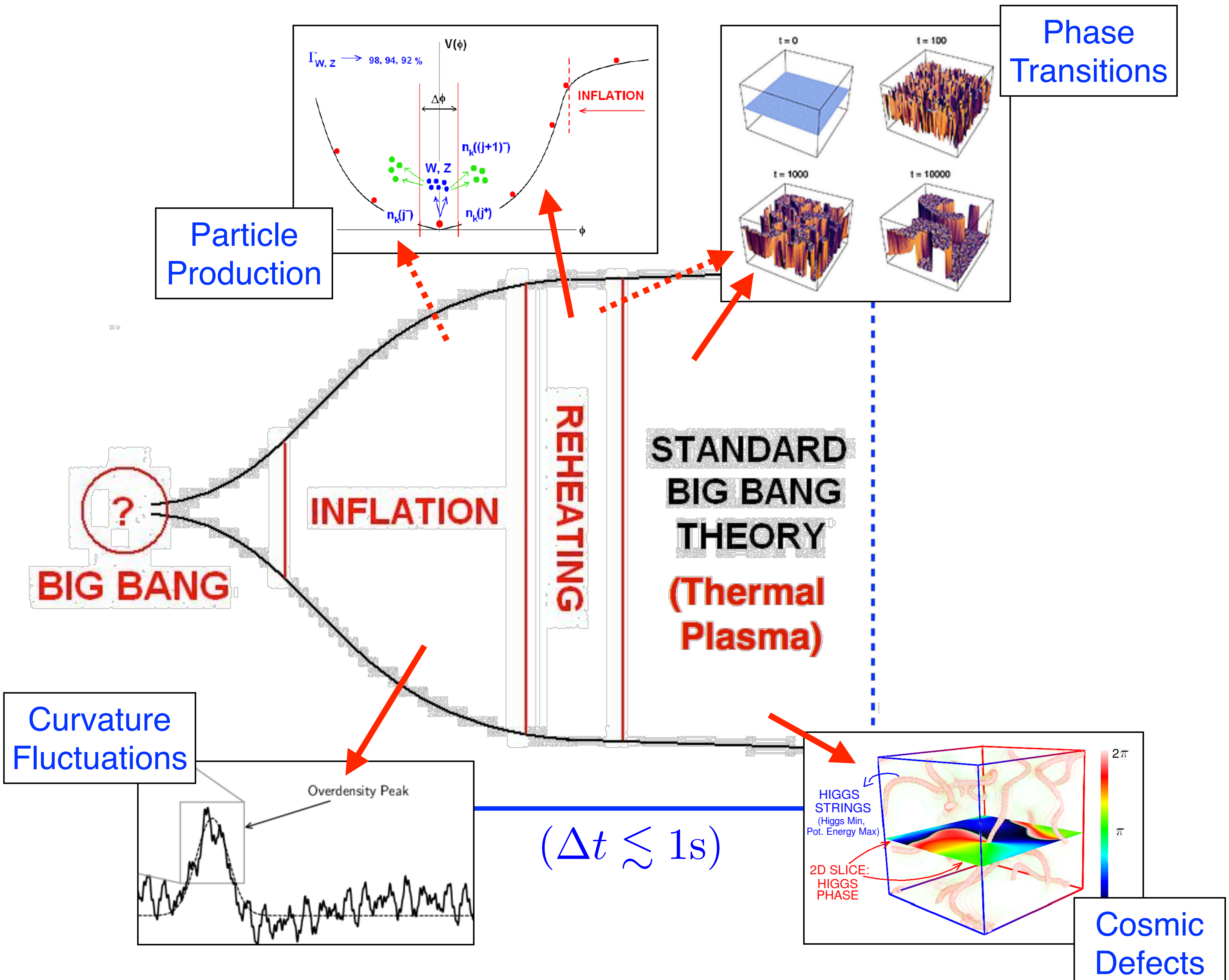

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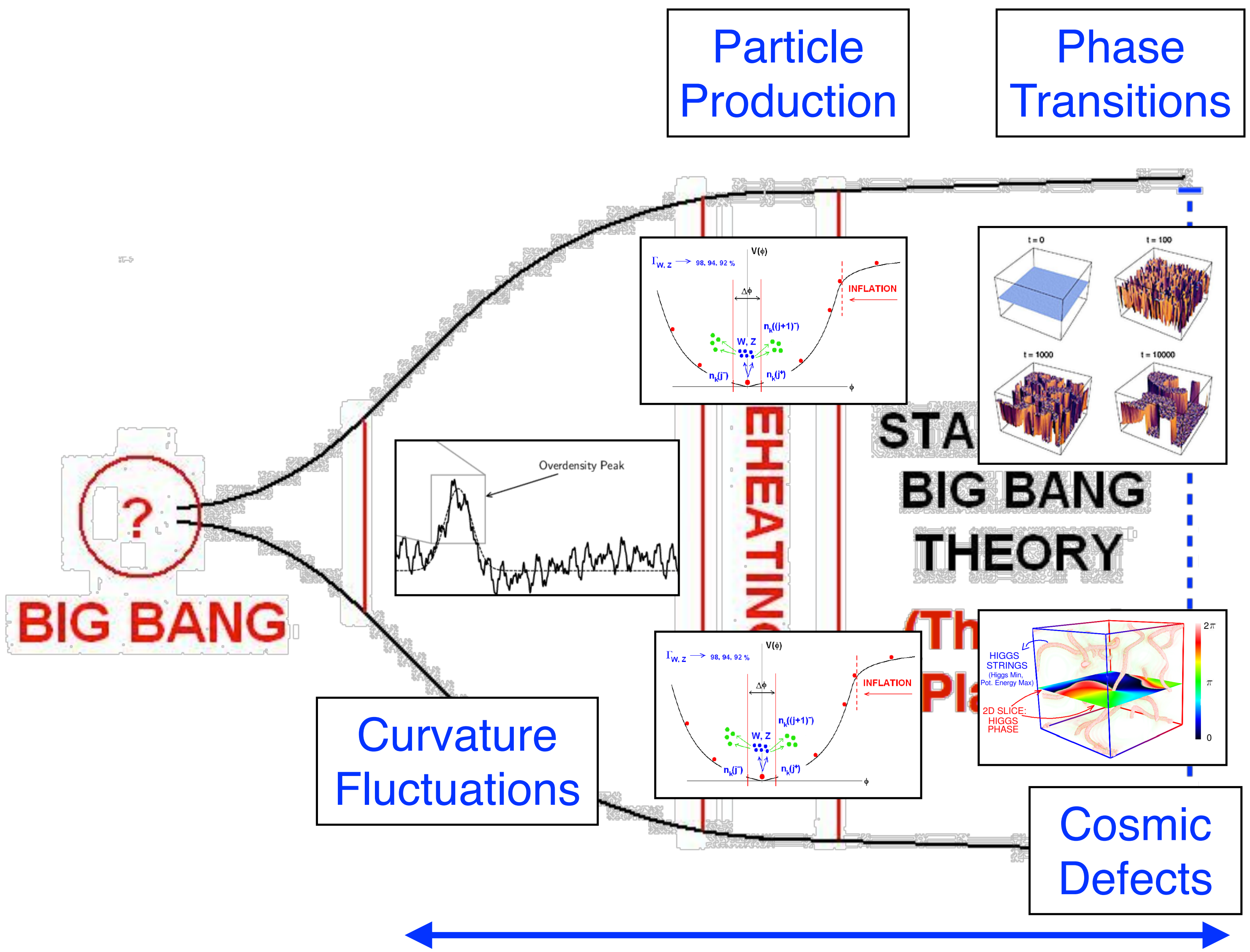
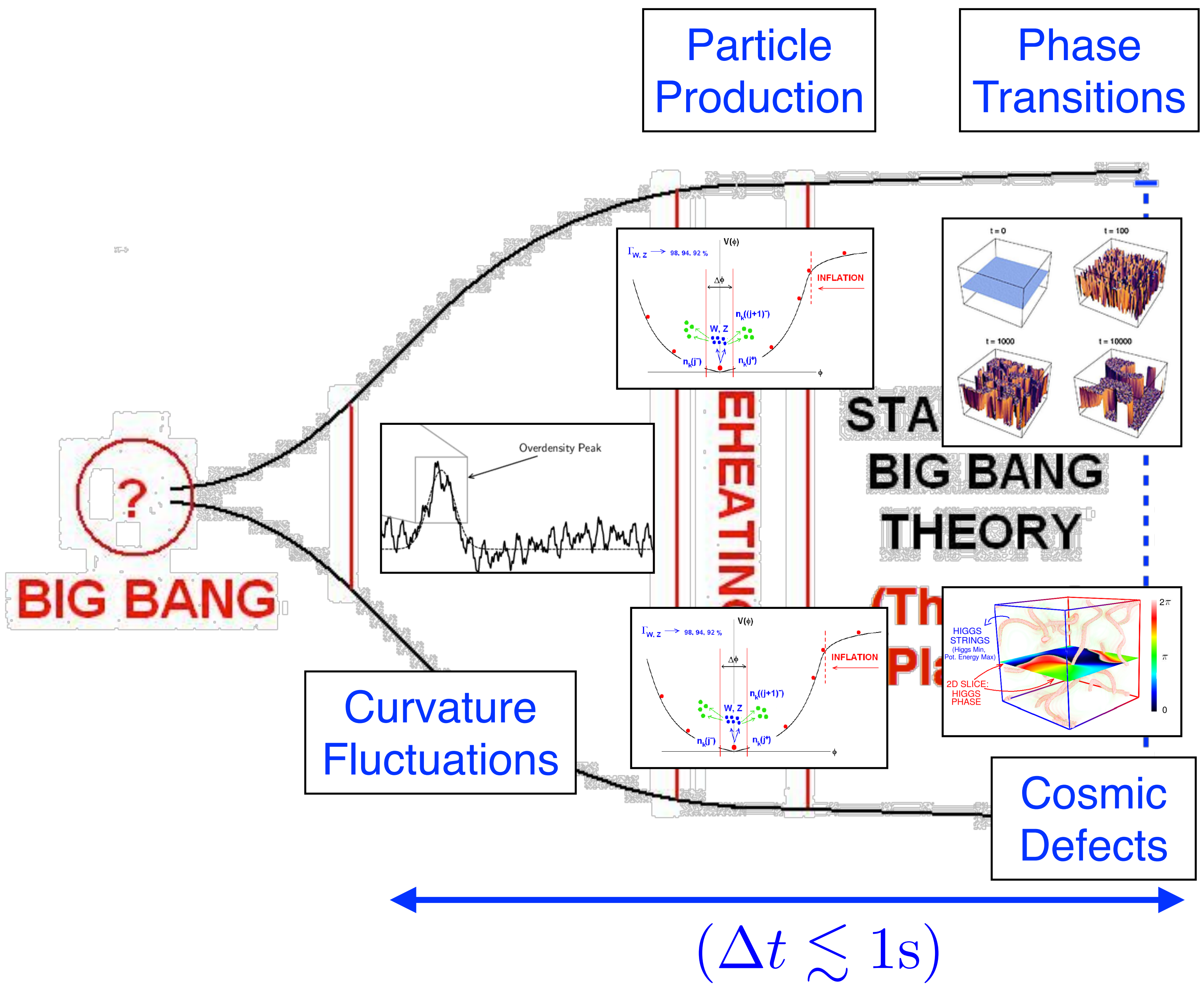


$(\Delta t \lesssim 1\mathrm{s})$

The Early Universe



Common Factor

Non-linear Field dynamics

The Early Universe

Particle Production

Phase Transitions

Curvature Fluctuations

Cosmic Defects

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Non-minimal Gravitational Coupling

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Non-linear Field dynamics

Non-minimal Kinetic Theories

Magneto-genesis

Turbulence Thermalisation ....

Curvature Fluctuations

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CosmoLattice

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CosmoLattice

Figueroa, Florio, Torrenti, Valkenburg

Lattice Theory: arXiv: 2006.15122

Code Manual: arXiv: 2102.01031

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- **Family** of evolution **algorithms**, accuracy ranging from $\delta\mathcal{O}(\delta t^2)$ – $\delta\mathcal{O}(\delta t^{10})$

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http://www.cosmolattice.net/

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CosmoLattice

A modern code for lattice simulations of scalar and gauge field dynamics in an expanding universe

CosmoLattice

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Physical Problem

* Init Conditions
* Eqs. of Motion

CosmoLattice

http://www.cosmolattice.net/

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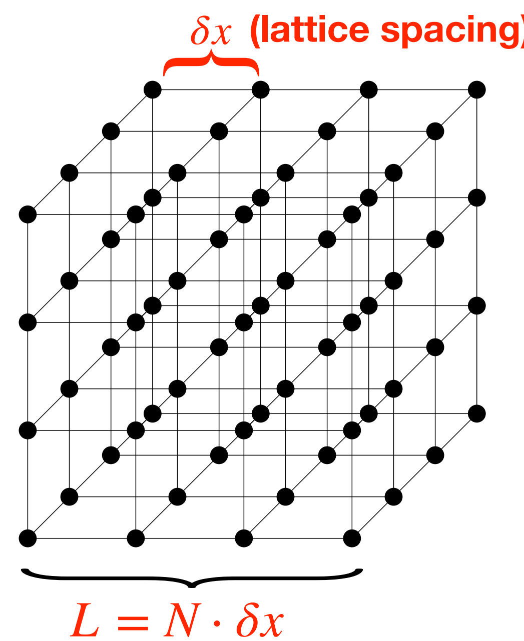

CosmoLattice

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CosmoLattice

Physical Problem

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* Choose Lattice: dt, N, dx
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```
#Output
outputfile = ./

#Evolution
expansion = true
evolver = VV2

#Lattice
N = 32
dt = 0.01
kIR = 0.75
nBinsSpectra = 55

#Times
tOutputFreq = 0.1
tOutputInfreq = 1
tMax = 300

#IC
kCutOff = 1.75
initial_amplitudes = 7.42675e18 0 # homogeneous amplitudes in GeV
initial_momenta = -6.2969e30 0    # homogeneous amplitudes in GeV2

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lambda = 9e-14
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CosmoLattice

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CosmoLattice

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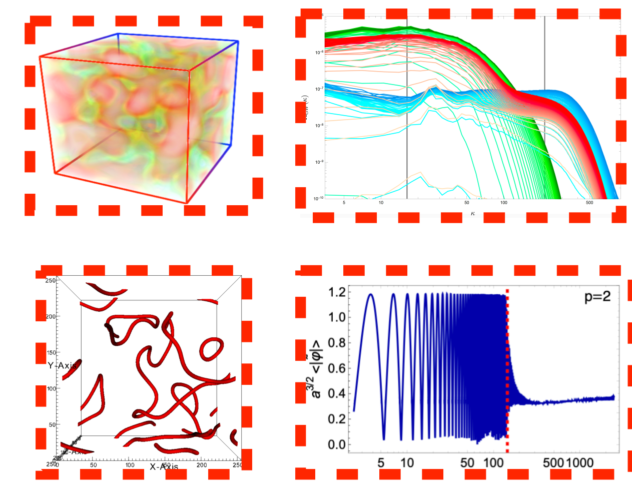


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CosmoLattice

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CosmoLattice

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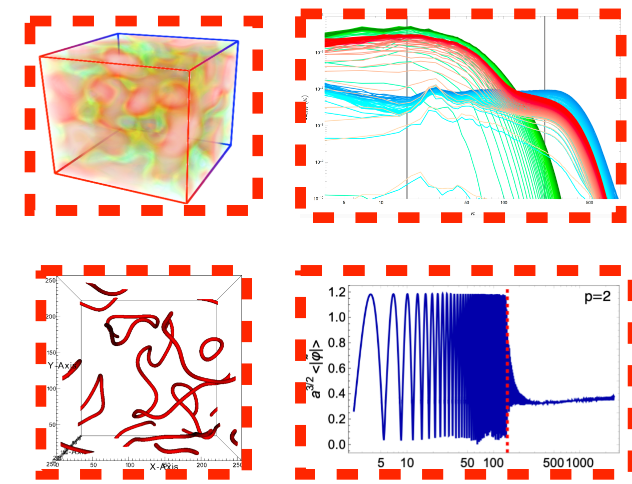

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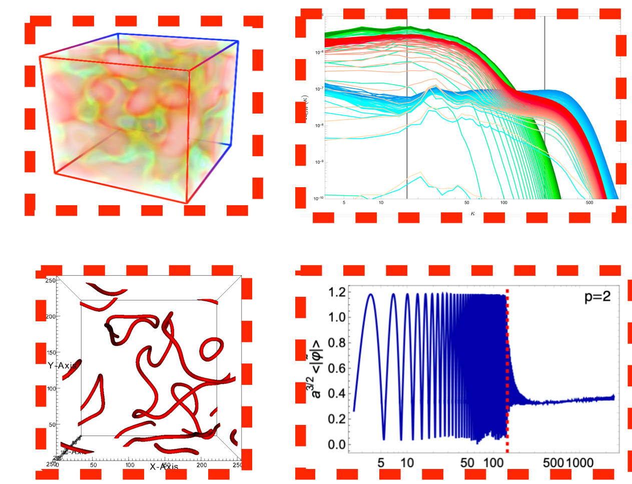

CL is a platform for field theories
You choose the problem to solve !

CosmoLattice

http://www.cosmolattice.net/

- **CL so far (v1.0, Public):**
  - Global scalar field dynamics
  - U(1) scalar-gauge dynamics
  - SU(2) scalar-gauge dynamics

$\Leftrightarrow$

- Canonical Kinetic Terms
- Self-consistent Evolution
- Hom. IC & Quantum Fluct.
- Symplectic Integrators $\delta\mathcal{O}(\delta t^2) - \delta\mathcal{O}(\delta t^{10})$

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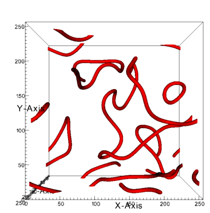
- **CL update (v2.0, to be released by ~2023):**
  - ✓ Gravitational waves $\Box h_{ij} = 2\Pi_{ij}^{TT}$
  - ✓ Axion-like couplings $\phi F_{\mu\nu}\tilde{F}^{\mu\nu}$
  - ✓ Non-minimal coupling $\xi\phi^2 R$
  - ✓ Cosmic String Networks



  - ...

  $\Leftrightarrow$
  - Non-Canonical Kinetic Terms
  - Non-symplectic integrators ✓
  - Self-consistent Evolution ✓
  - Arbitrary IC: external PS or fld config. ✓
  - ...

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CL v1.1: Just released in May 2022 !

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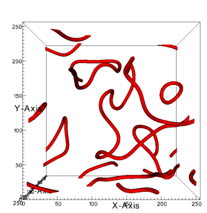
  - ✓ Cosmic String Networks 
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Applications

1) Preheating & Equation of State after inflation

2) GW production from non-linear dynamics

3) Non-linear inflation dynamics (e.g Axion-inflation)

4) Cosmic string networks (axions, AH, …)

5) Single string loop dynamics

6) Non-minimal gravitational Interactions

7) Phase transitions

….

X) Your project !

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Applications

I) GW production from non-linear dynamics

II) Non-linear inflation dynamics (e.g Axion-inflation)

III) Single string loop dynamics (if time permits)

Example I

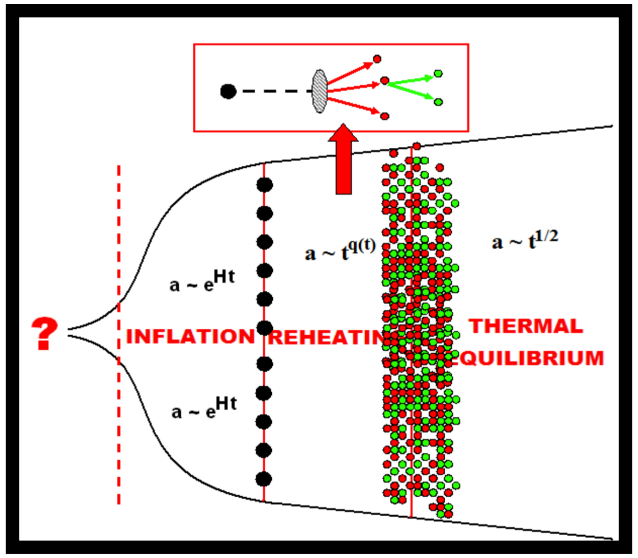
Particle coupling reconstruction with gravitational waves

with

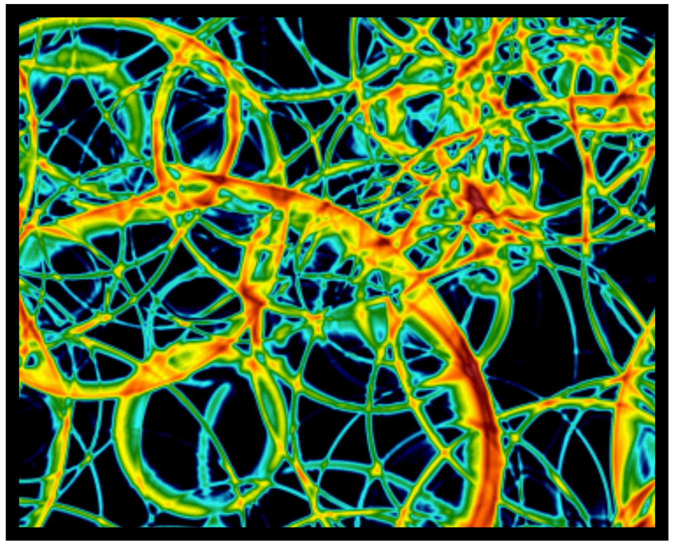
A. Florio, N. Loayza and M. Pieroni

Phys. Rev. D 106 (2022) 6, 063522 ; [2202.05805](2202.05805)

MOTIVATION



(p)Reheating non-perturbative dynamics



Strong 1st Order Phase Transitions

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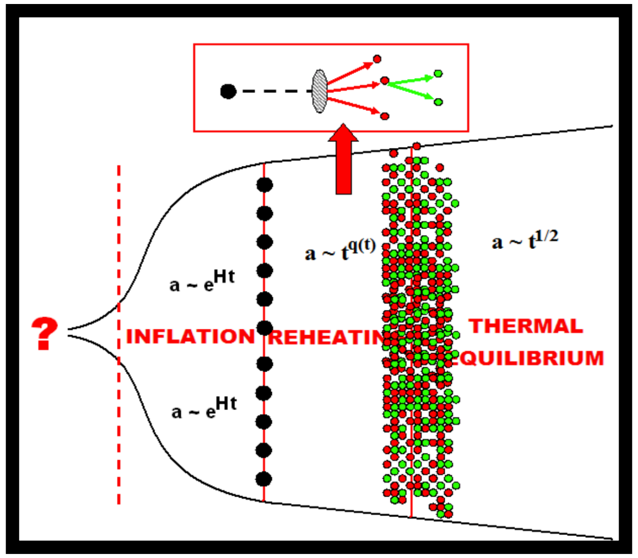
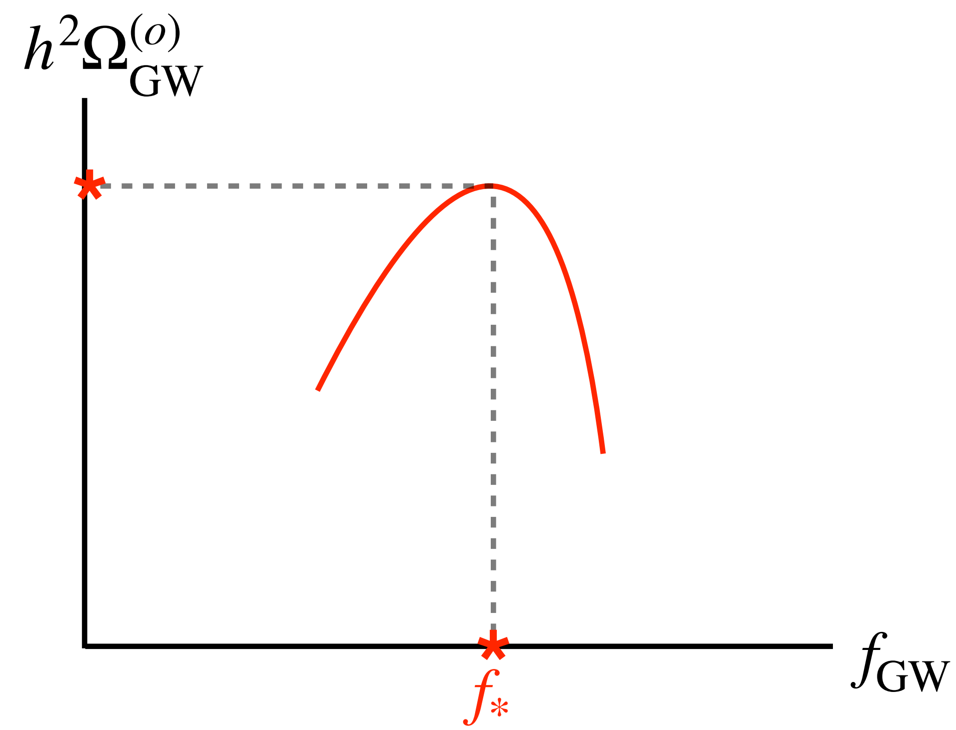
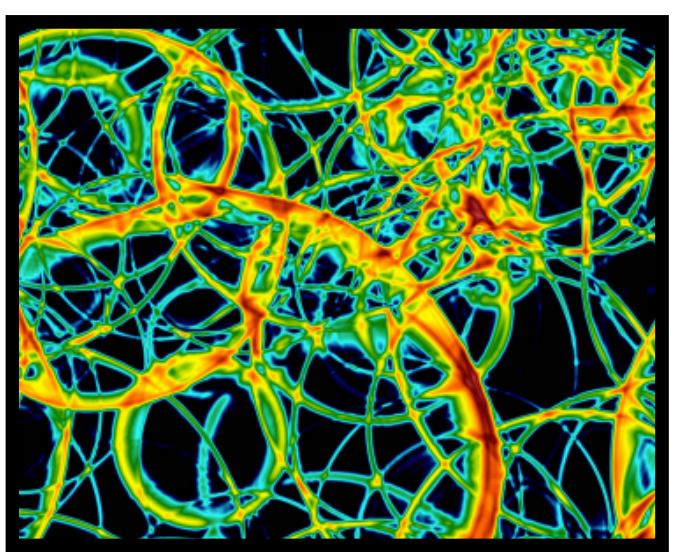


(p)Reheating non-perturbative dynamics

Causally produced Grav. Wave (GW) Backgrounds





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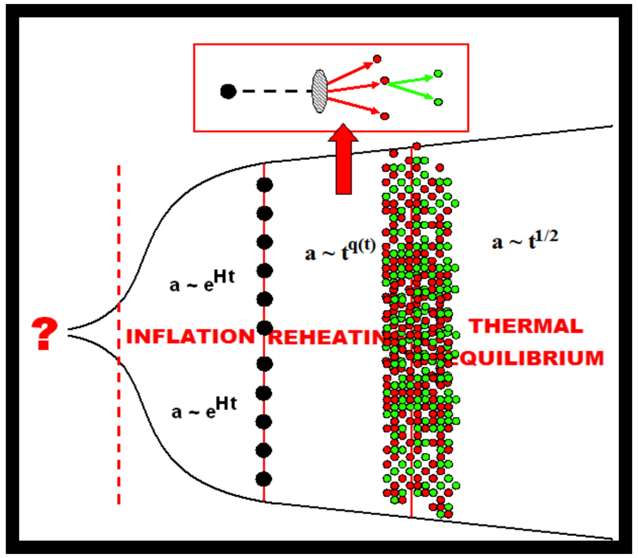

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What happens if
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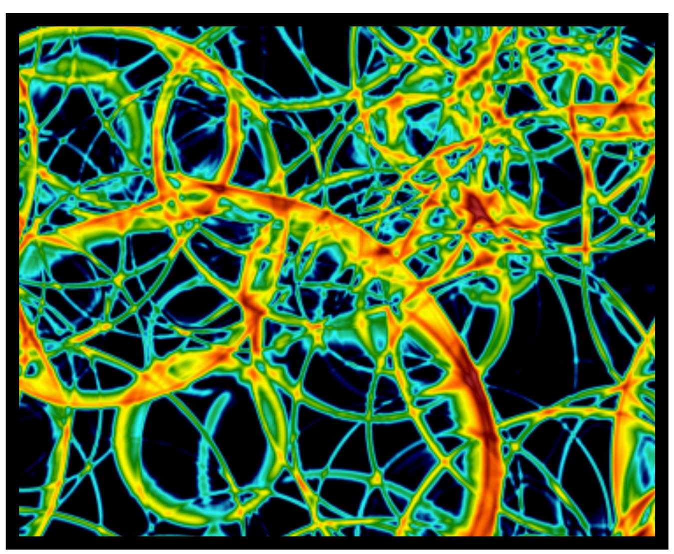

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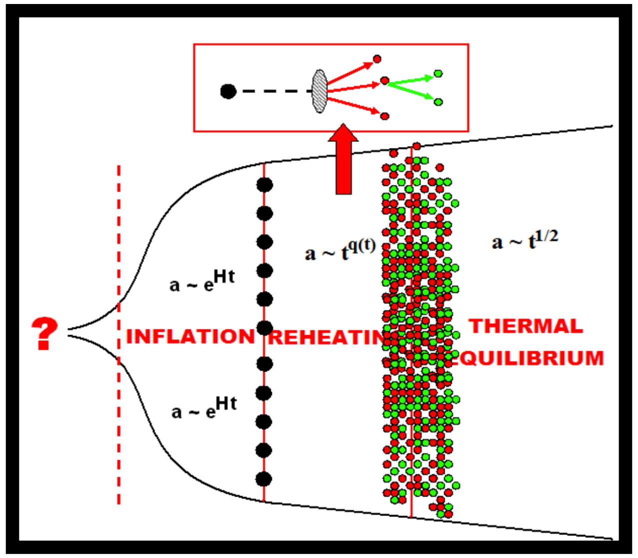

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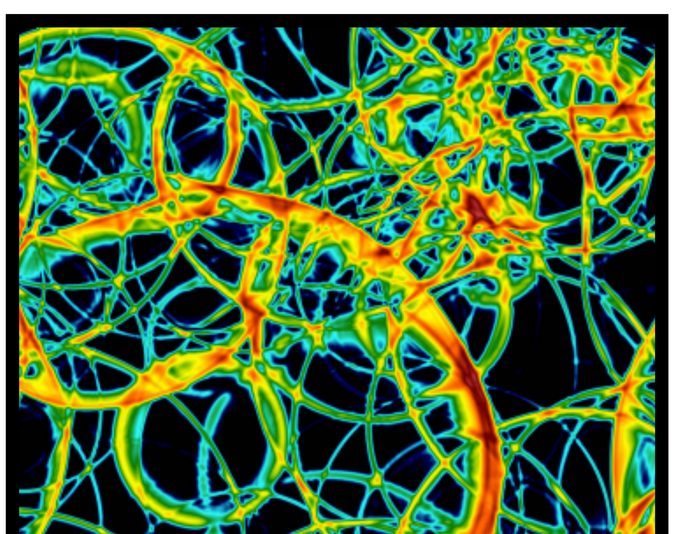

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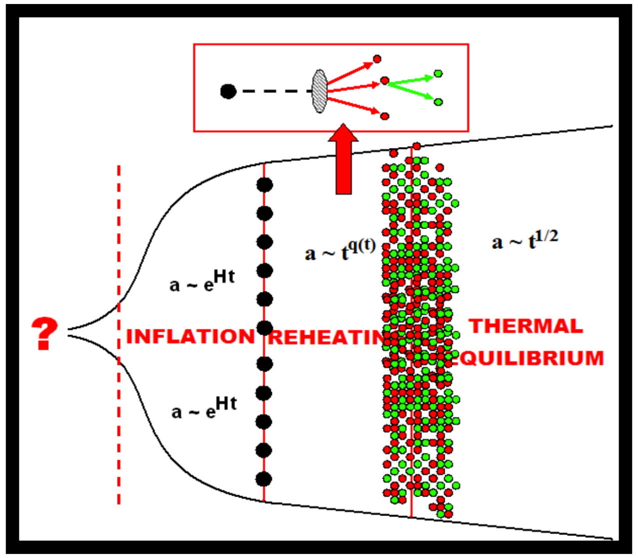

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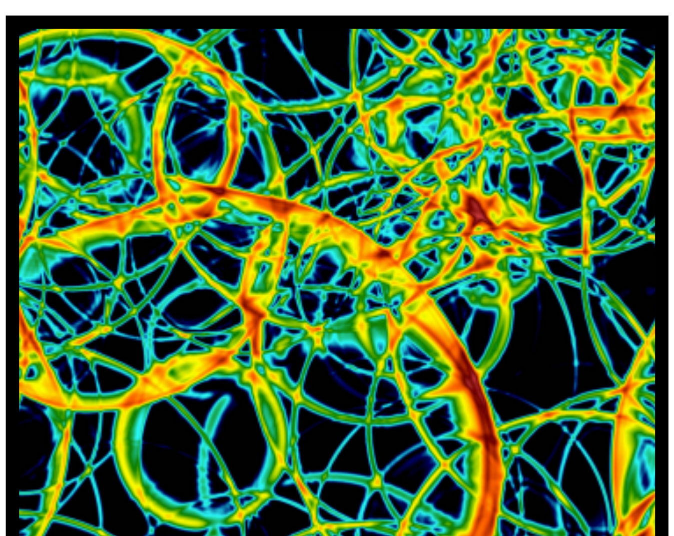

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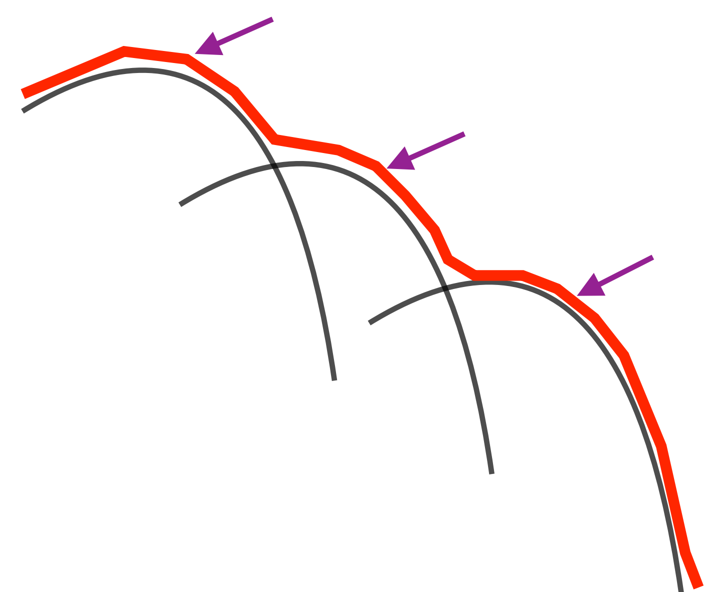
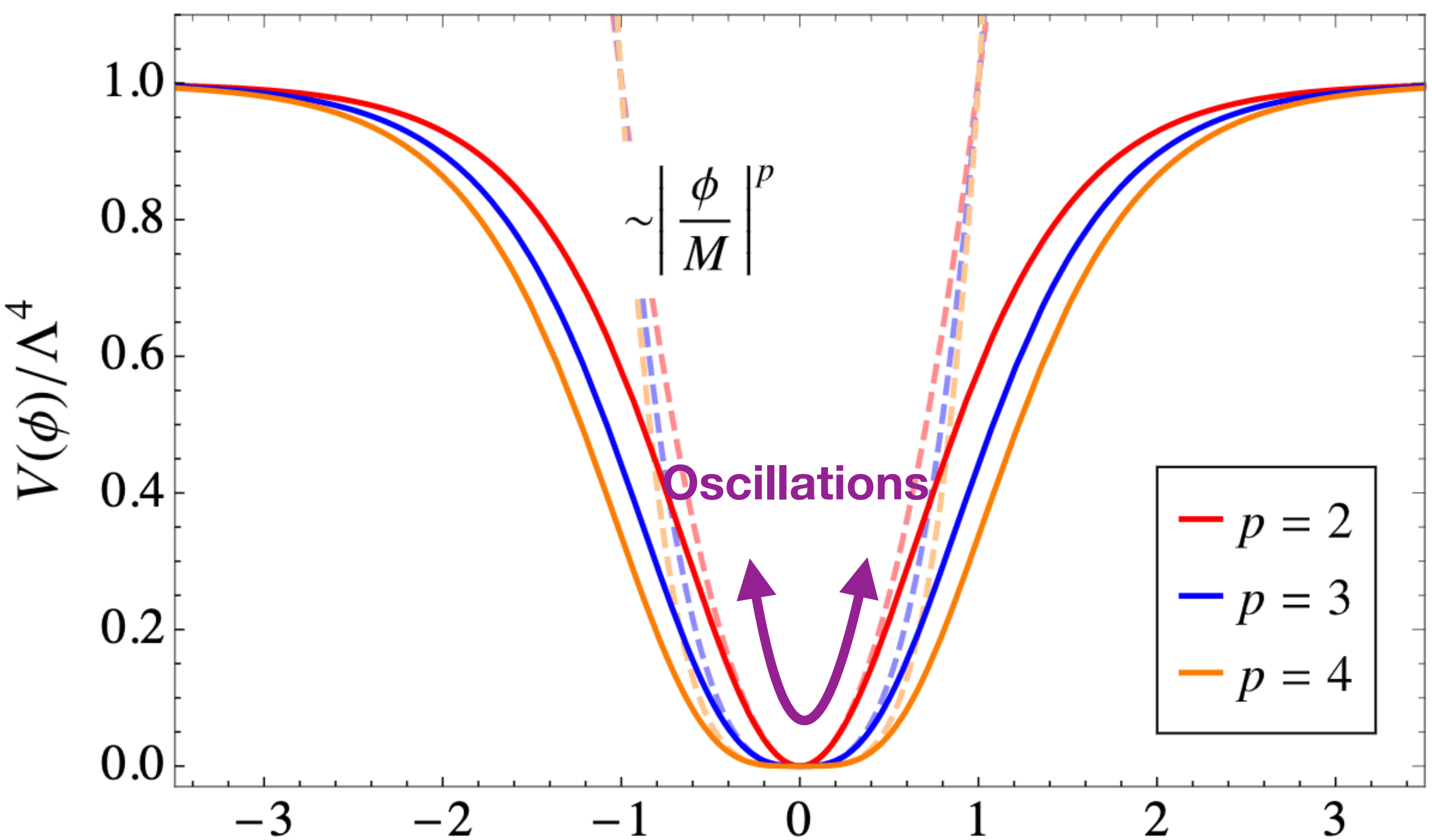


SCALAR (P)REHEATING

$$V(\phi,\chi) = \frac{1}{p}\Lambda^4 \tanh\left(\frac{\phi}{M}\right)^p + \frac{1}{2}g^2\chi^2\phi^2$$

(*e.g.* $\alpha -$ attractors, Kallosh, Linde 2013)



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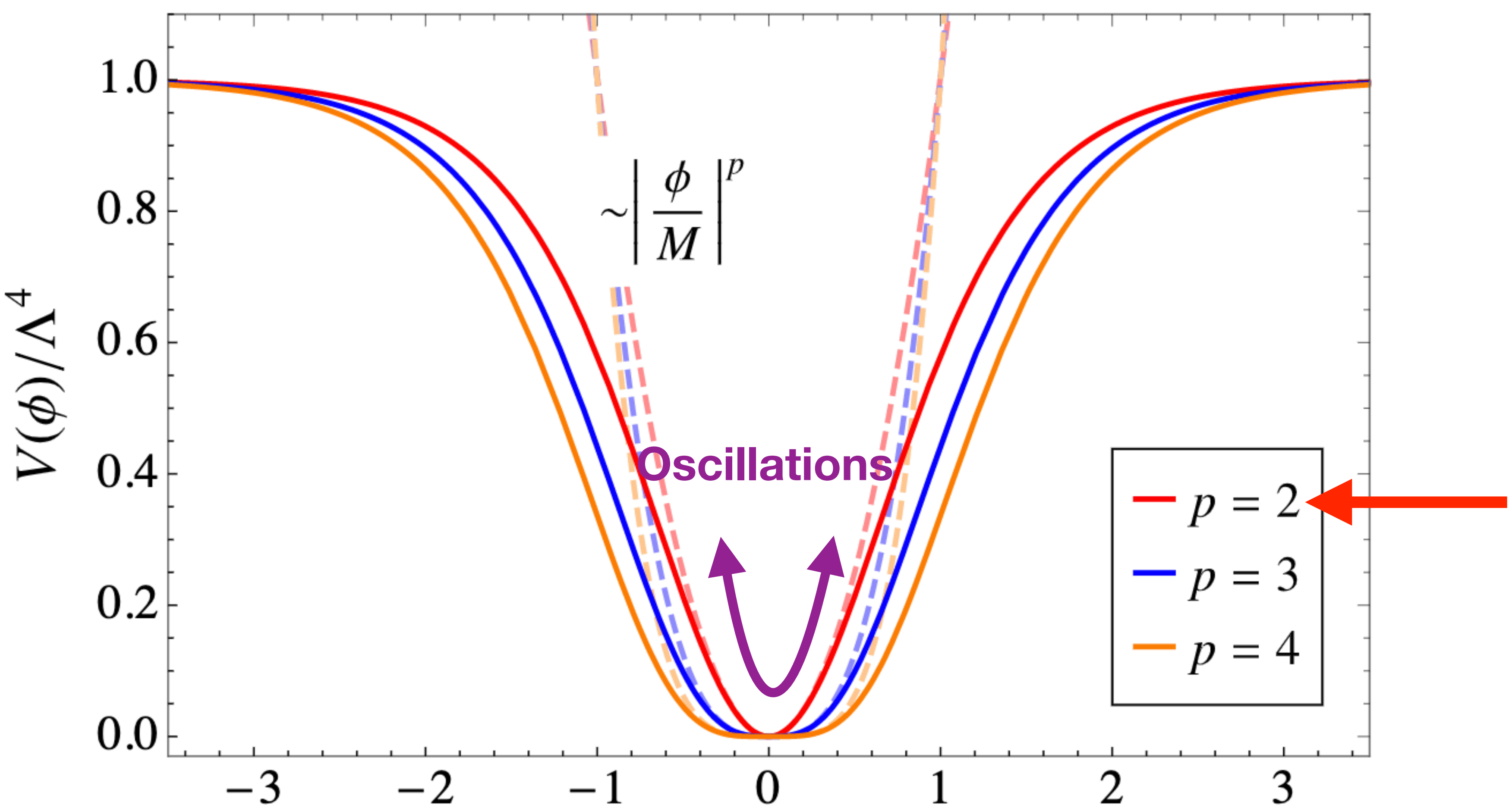

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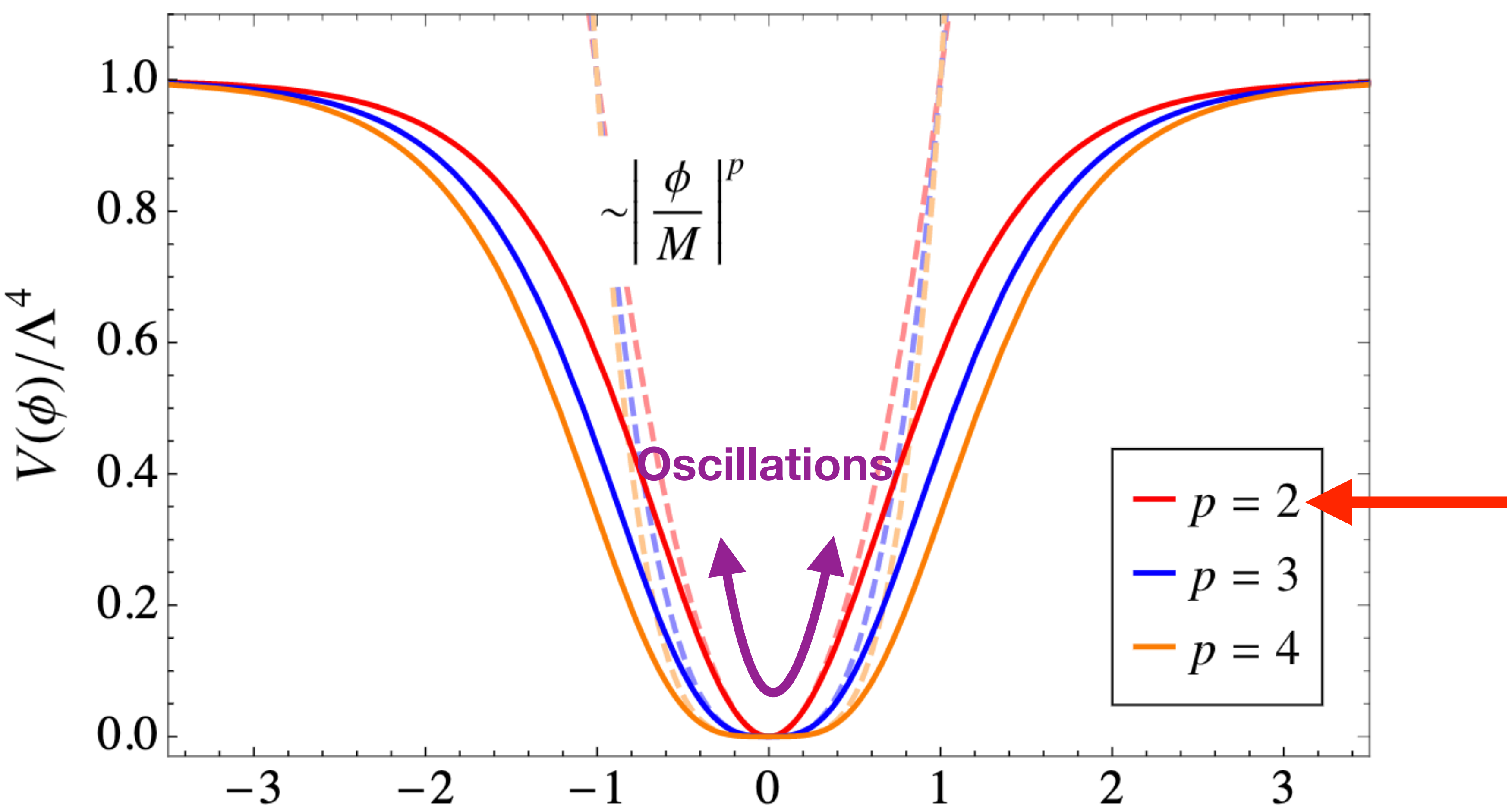
$\chi_k'' + [\kappa^2 + m^2(\phi)]\chi_k = 0$ **(Preheat fld excitation)**



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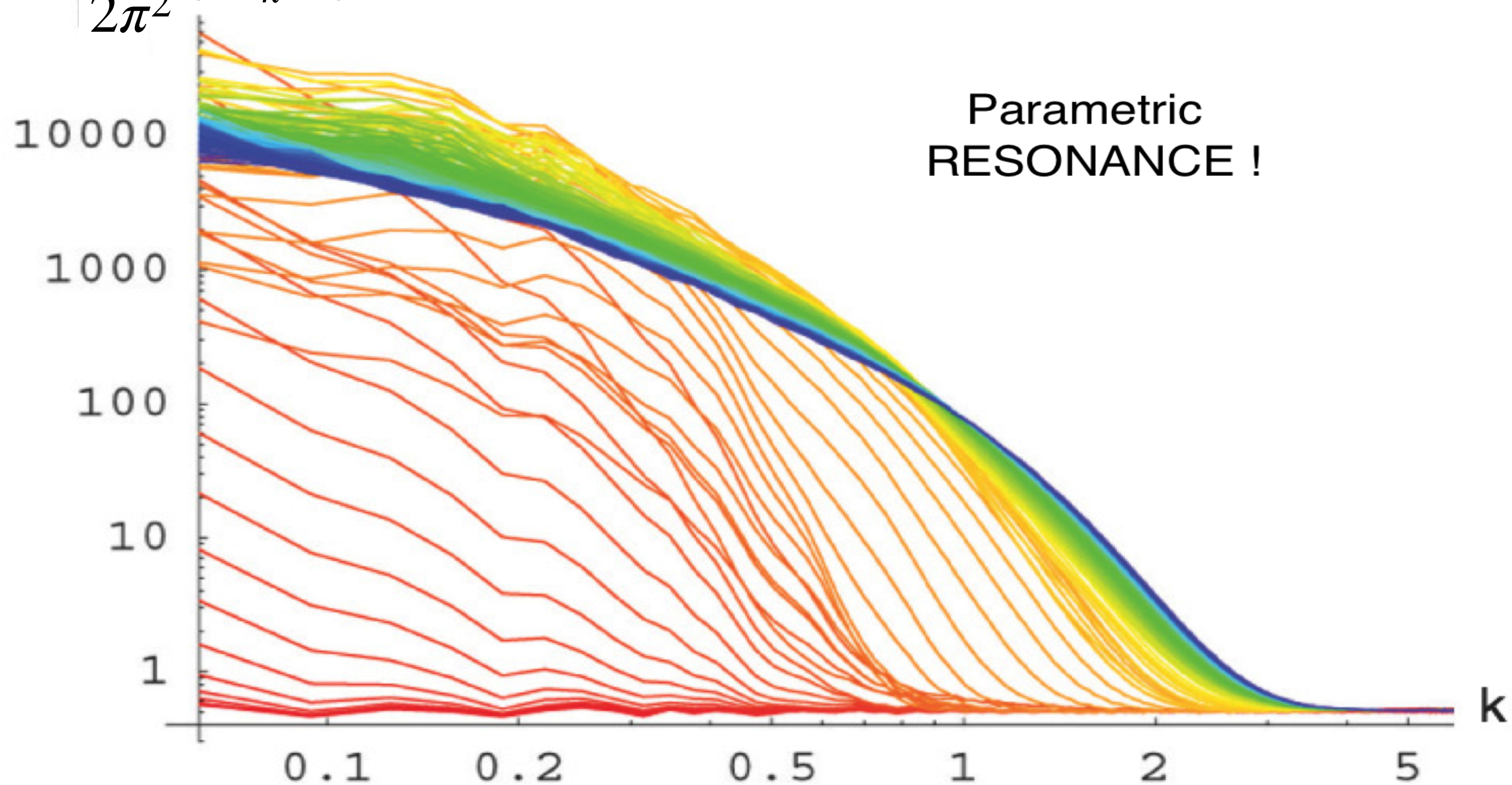
$\mathscr{P}_\chi(k) = \frac{k^3}{2\pi^2}\langle|\chi_k|^2\rangle$

(Kofman, Linde, Starobinsky 1997)



SCALAR (P)REHEATING

Physics of (p)REHEATING: $\ddot{\varphi}_k + \omega^2(k,t)\varphi_k = 0$ **(linear regime)**

Non-perturbative & Out-of-Equilibrium

SCALAR (P)REHEATING

Physics of (p)REHEATING: $\ddot{\varphi}_k + \omega^2(k,t)\varphi_k = 0$ **+ mode-coupling**

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At $\mathbf{k}_i$: $\varphi_{k_i}, n_{k_i} \sim e^{\mu(k,t)t} \Rightarrow$ Inhomogeneities: $\begin{cases} L_i \sim 1/k_i \\ \delta\rho/\rho \gtrsim 1 \\ v \approx c \end{cases}$

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Preheating: Very Effective GW generator !

Khlebnikov, Tkachev '97
Easther, Giblin, Lim '06-'08
DGF, G[a]-Bellido, et al '07-'10
Kofman, Dufaux et al '07-'09

INFLATIONARY PREHEATING

Non - linear dynamics

Lattice Simulations
which include GWs

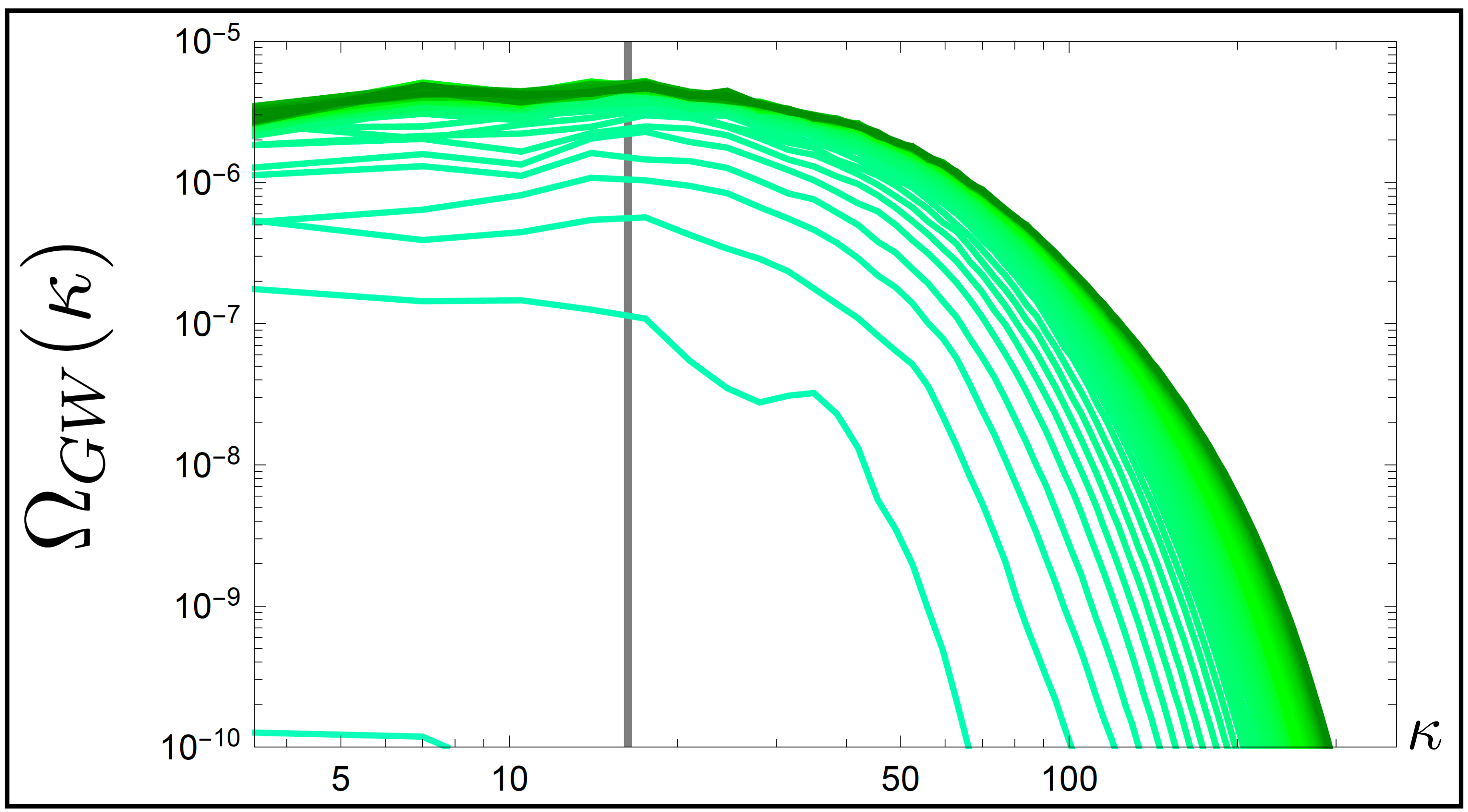
GW Spectrum

Parameter Dependence (Peak amplitude**)**

Monomial Models: **Single peak spectrum !**

(single daughter fld)

$$\Omega_{\rm GW} \equiv \frac{1}{\rho_c}\frac{d\rho_{\rm GW}}{d\log k}$$



GW Spectrum

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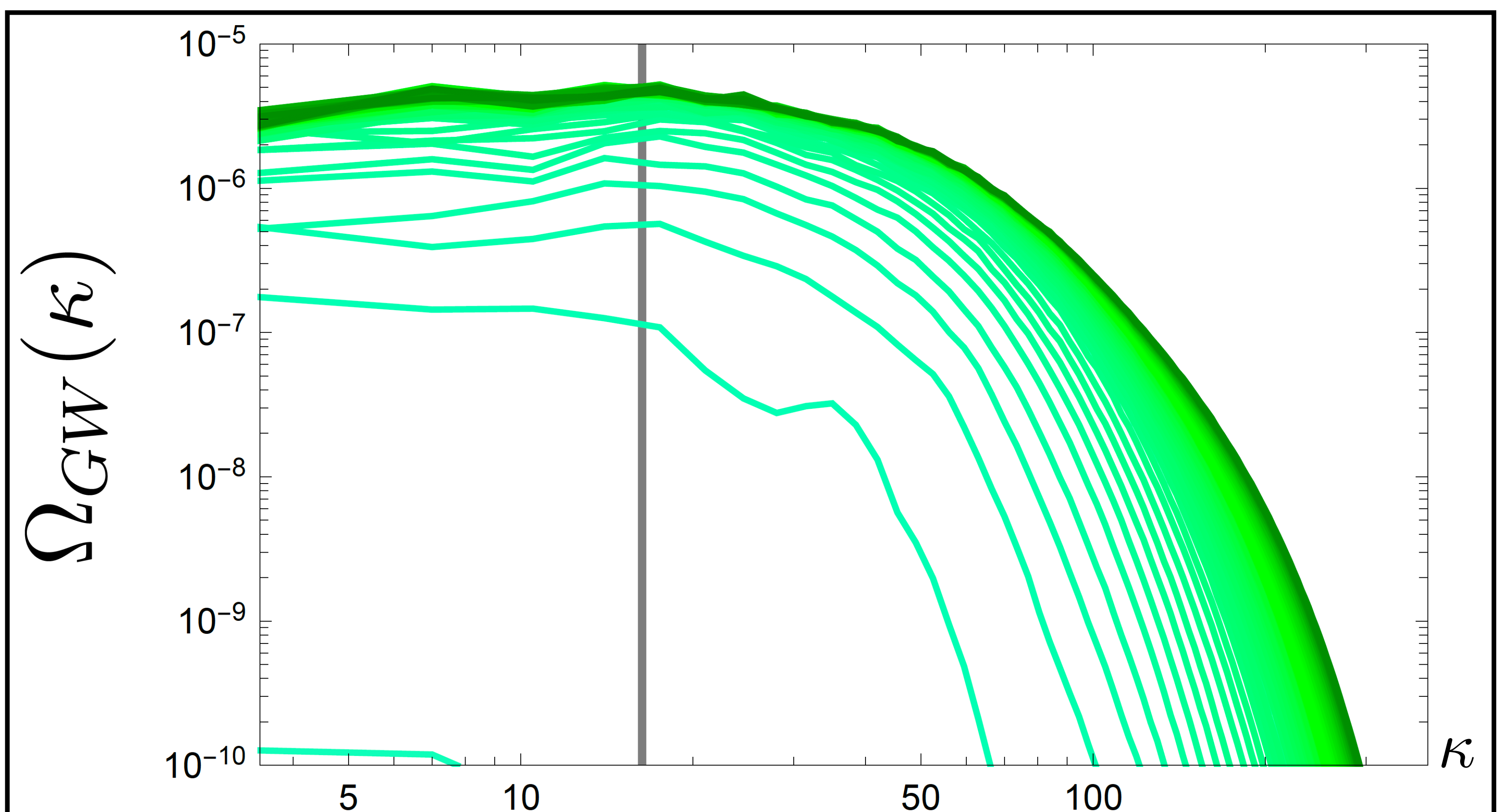
$$q \equiv \frac{g^2\Phi_{\rm i}^2}{\omega^2}$$

Resonance Param.



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Parameter Dependence (Peak amplitude**)**

$$\boxed{\omega^2 \equiv V''(\Phi_I)}$$

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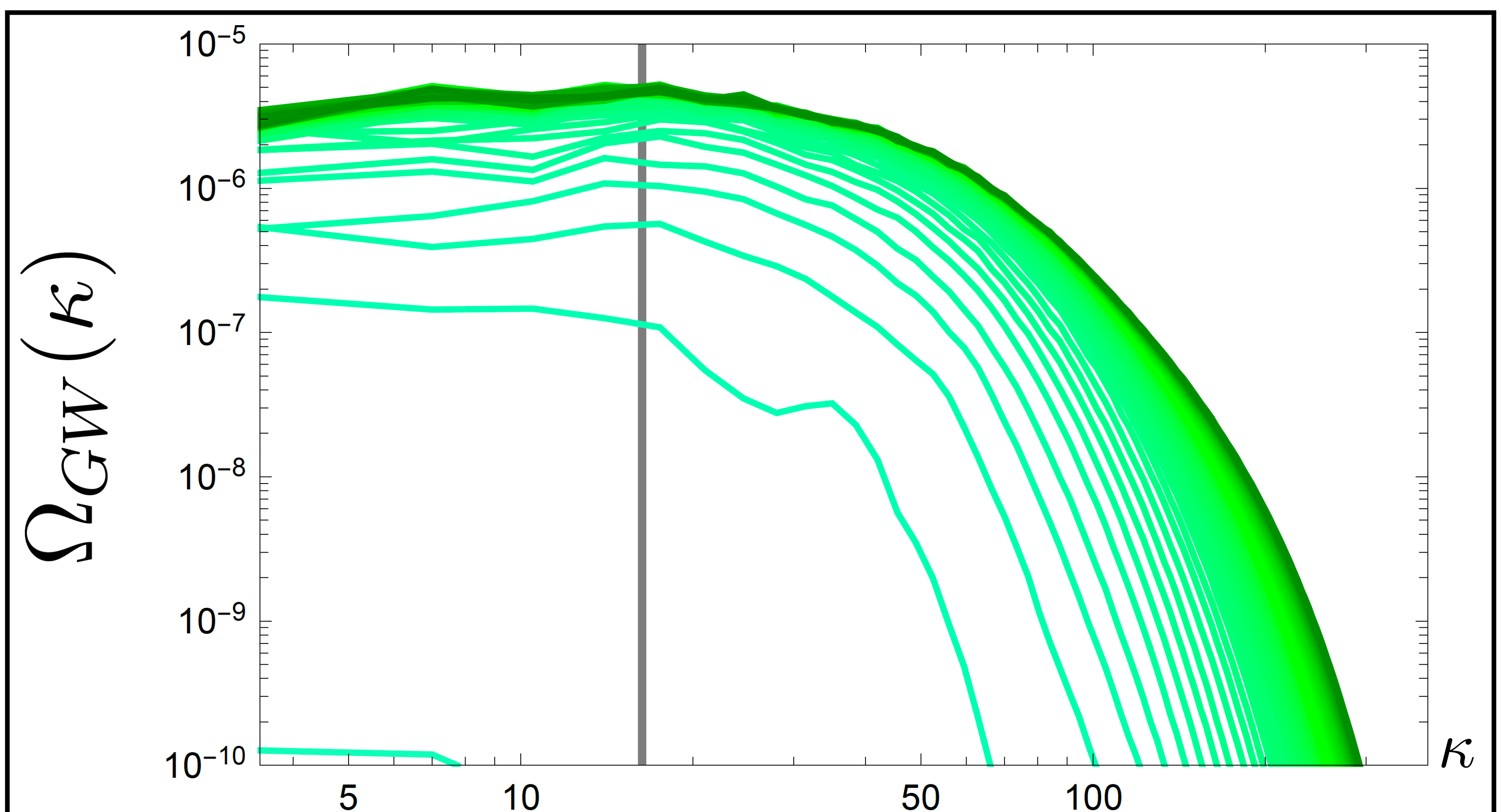

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Resonance Param.

GW Spectrum

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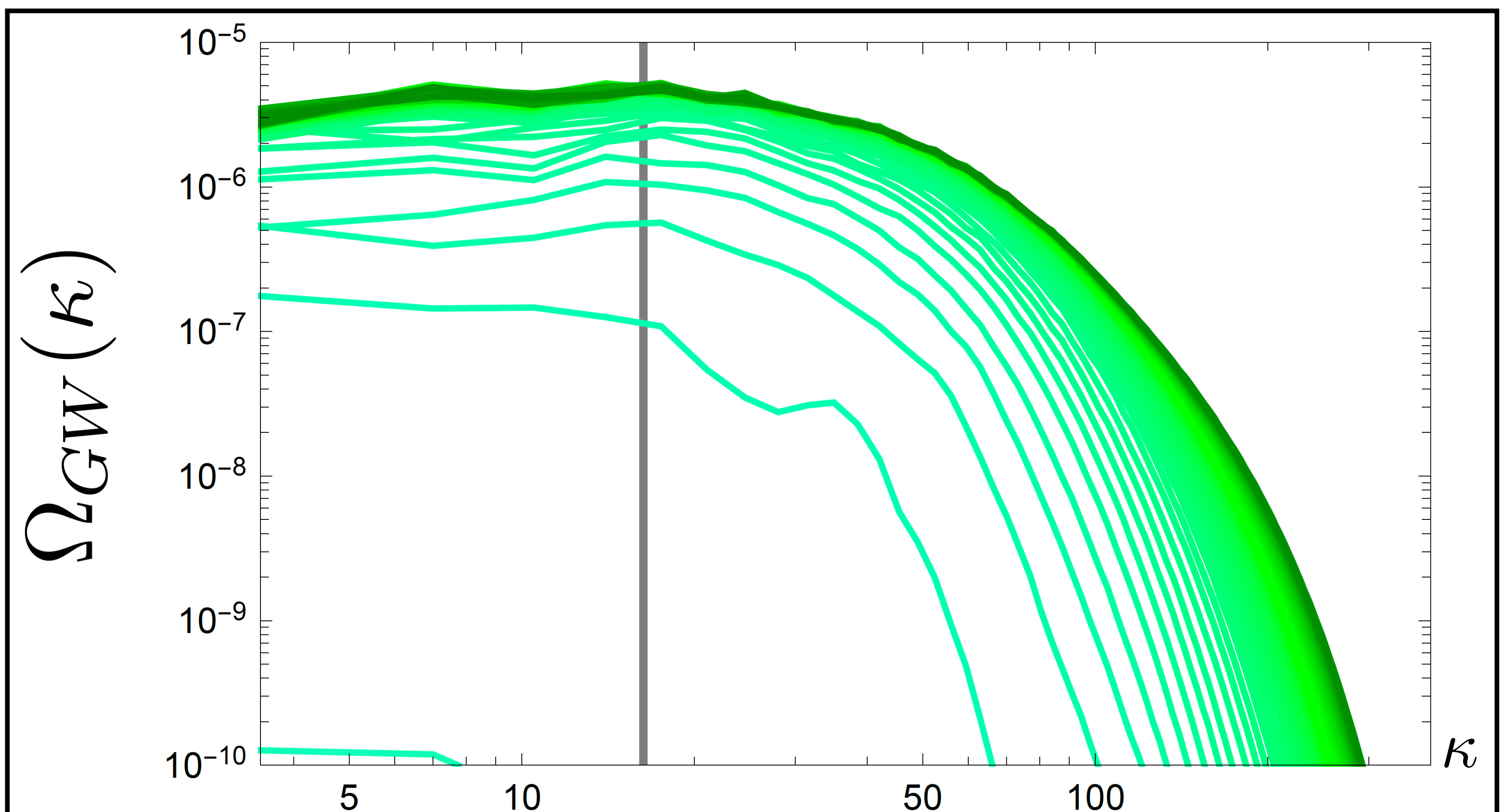
$\boxed{\omega^2 \equiv V''(\Phi_I)}$

$\boxed{q \equiv \frac{g^2\Phi_{\rm i}^2}{\omega^2}}$

Resonance Param.



(DGF, Torrentí JCAP 2017)

GW Spectrum

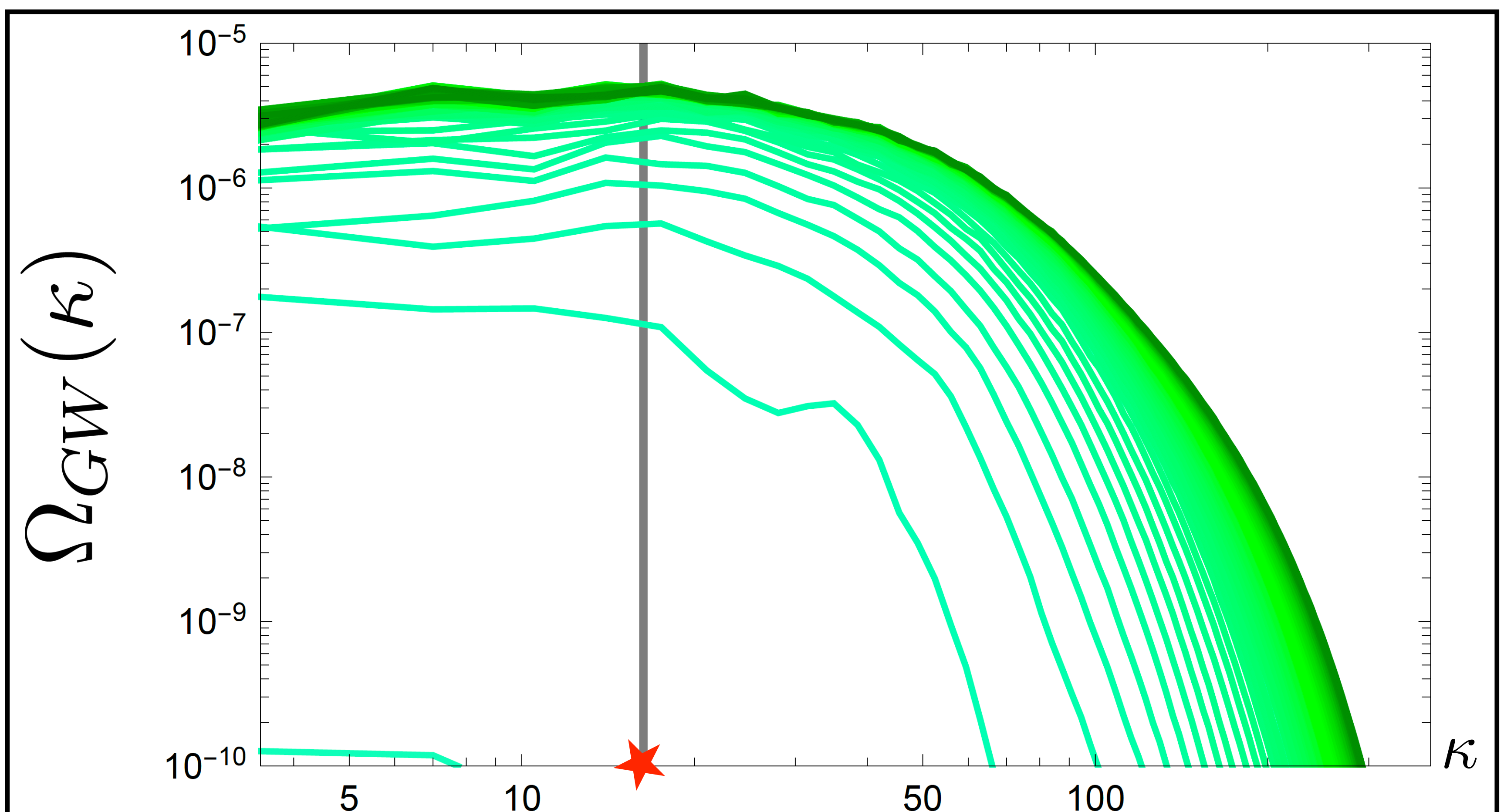
Parameter Dependence (Peak amplitude**)**

Monomial Models: $$\Omega_{\rm GW}^{(o)} \sim A^2 \frac{\omega^6}{\rho m_p^2}\, q^{-1}$$

$$\boxed{\omega^2 \equiv V''(\Phi_I)}$$

$$\boxed{q \equiv \frac{g^2\Phi_{\rm i}^2}{\omega^2}}$$

Resonance Param.



$k_p \propto q^{1/2}$

Peak Position

(DGF, Torrentí JCAP 2017)

GW Spectrum

Parameter Dependence (Peak amplitude**)**

Monomial Models: $\Omega_{\rm GW}^{(o)} \sim 10^{-9}\,,$

Large amplitude !

Khlebnikov, Tkachev '97
Easther, Giblin, Lim '06-'08
DGF, G[a]-Bellido, et al '07-'10
Kofman, Dufaux et al '07-'09
Many others afterwards ...

GW Spectrum

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Many others afterwards ...

$$\Omega_{\rm GW} \propto q^{-1} \propto 1/g^2$$

$$(f_{\rm peak} \propto q^{1/2} \propto g)$$

(DGF, Torrentí JCAP 2017)

GW Spectrum

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$\Omega_{\rm GW} \propto q^{-1} \propto 1/g^2$ ⟶ **What if multiple species with $g_i \neq g_j$?**

$(f_{\rm peak} \propto q^{1/2} \propto g)$

(DGF, Torrentí JCAP 2017)

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Spectroscopy of particle couplings ?

(DGF, Torrentí JCAP 2017)

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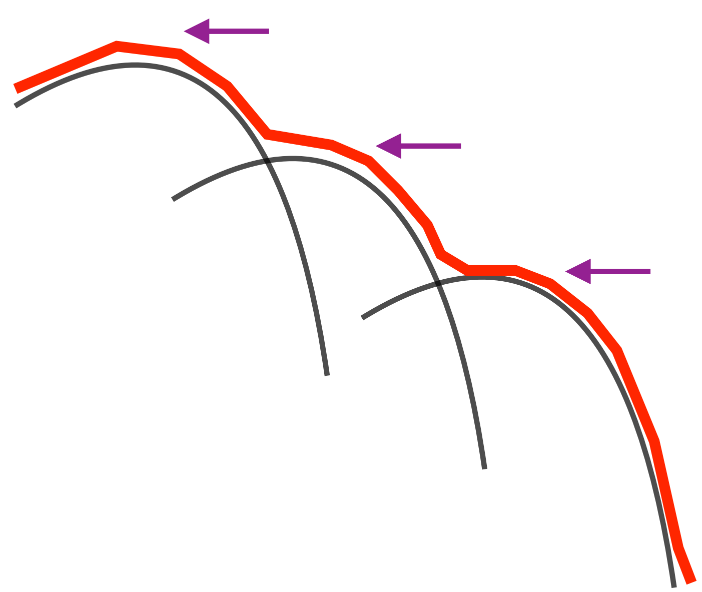
(DGF, Torrentí JCAP 2017)

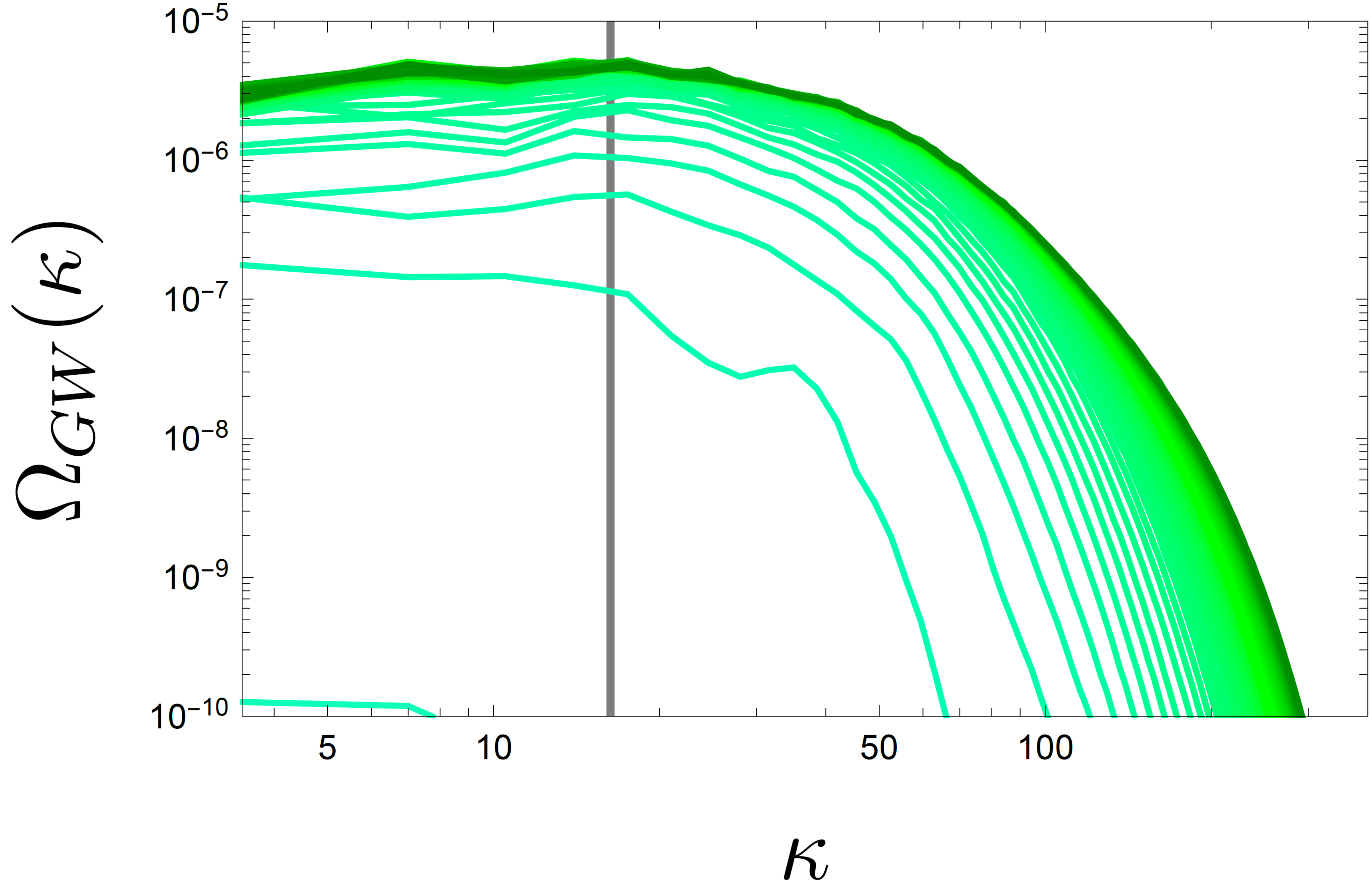


different couplings ... different peaks ?

GW Spectroscopy

Parameter Dependence (Peak amplitude**)**

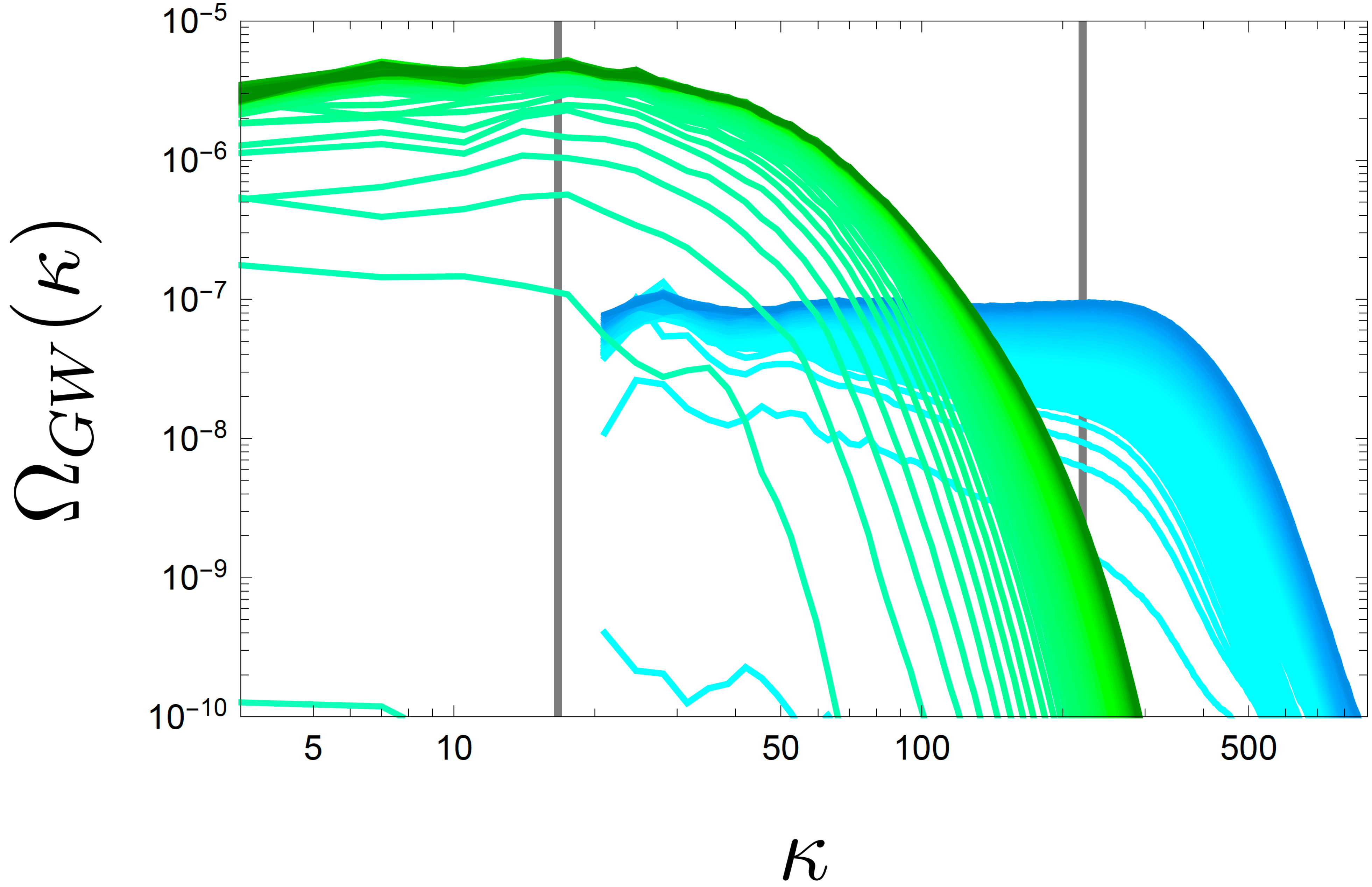
$$V(\phi) \; + \; \frac{1}{2} g_1^2 \phi^2 \chi_1^2$$



GW Spectroscopy

Parameter Dependence (Peak amplitude)

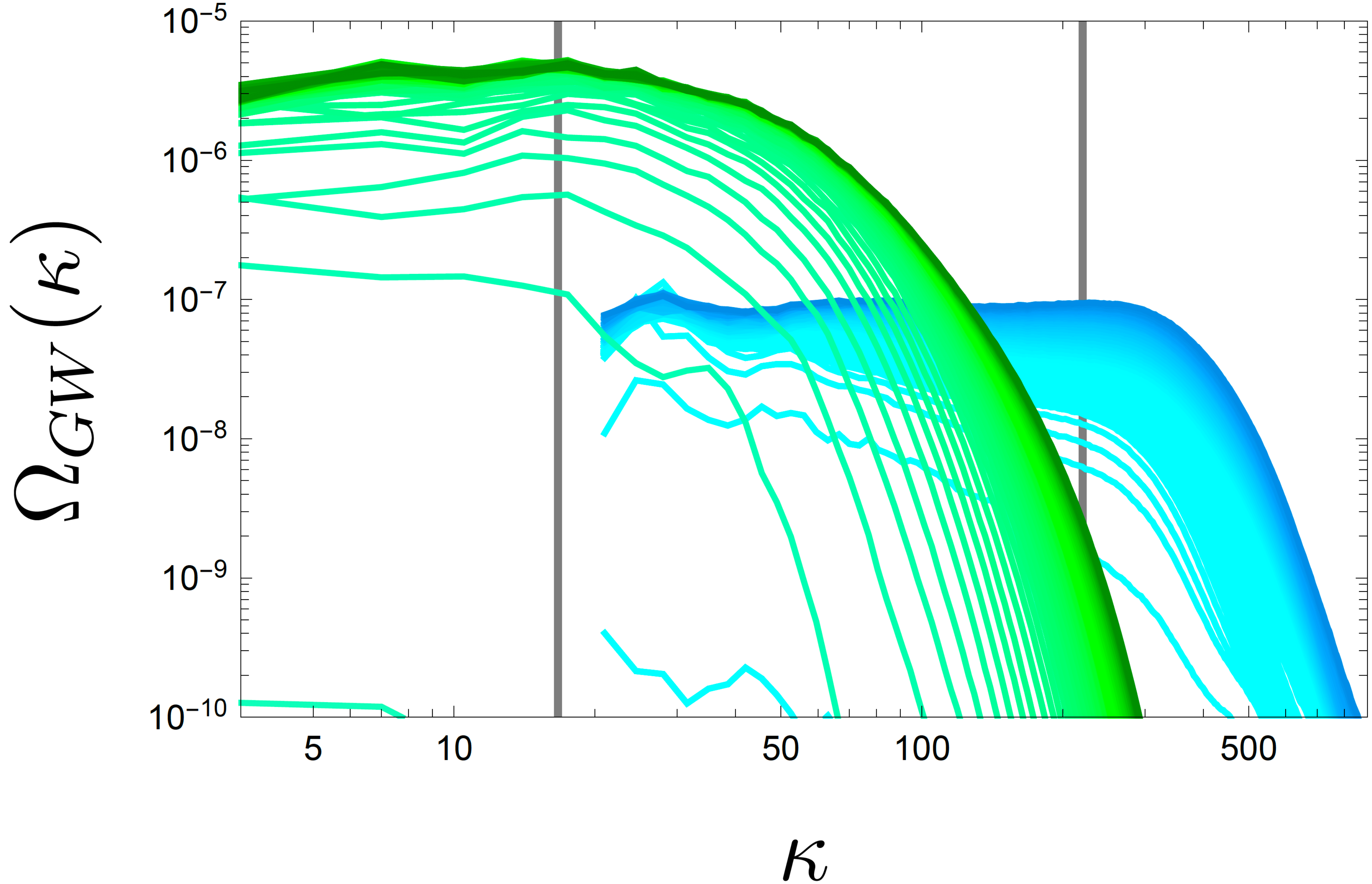
$$V(\phi) + \frac{1}{2}g_1^2\phi^2\chi_1^2 \quad ; \quad V(\phi) + \frac{1}{2}g_2^2\phi^2\chi_2^2$$



GW Spectroscopy

Parameter Dependence (Peak amplitude**)**

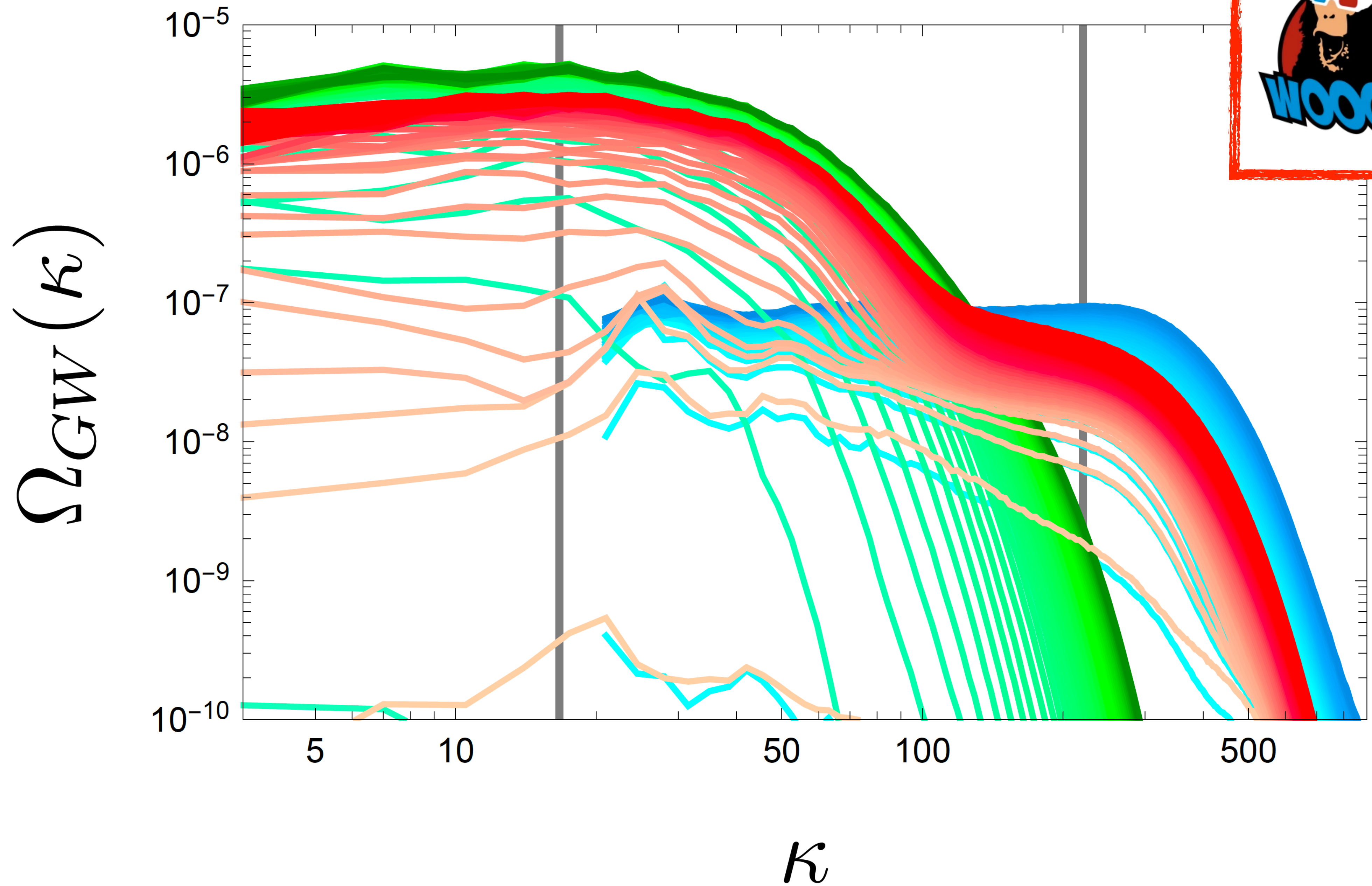
$$V(\phi) \;+\; \frac{1}{2}g_1^2\phi^2\chi_1^2 \;+\; \frac{1}{2}g_2^2\phi^2\chi_2^2 \quad \textbf{?}$$



GW Spectroscopy

Parameter Dependence (Peak amplitude)

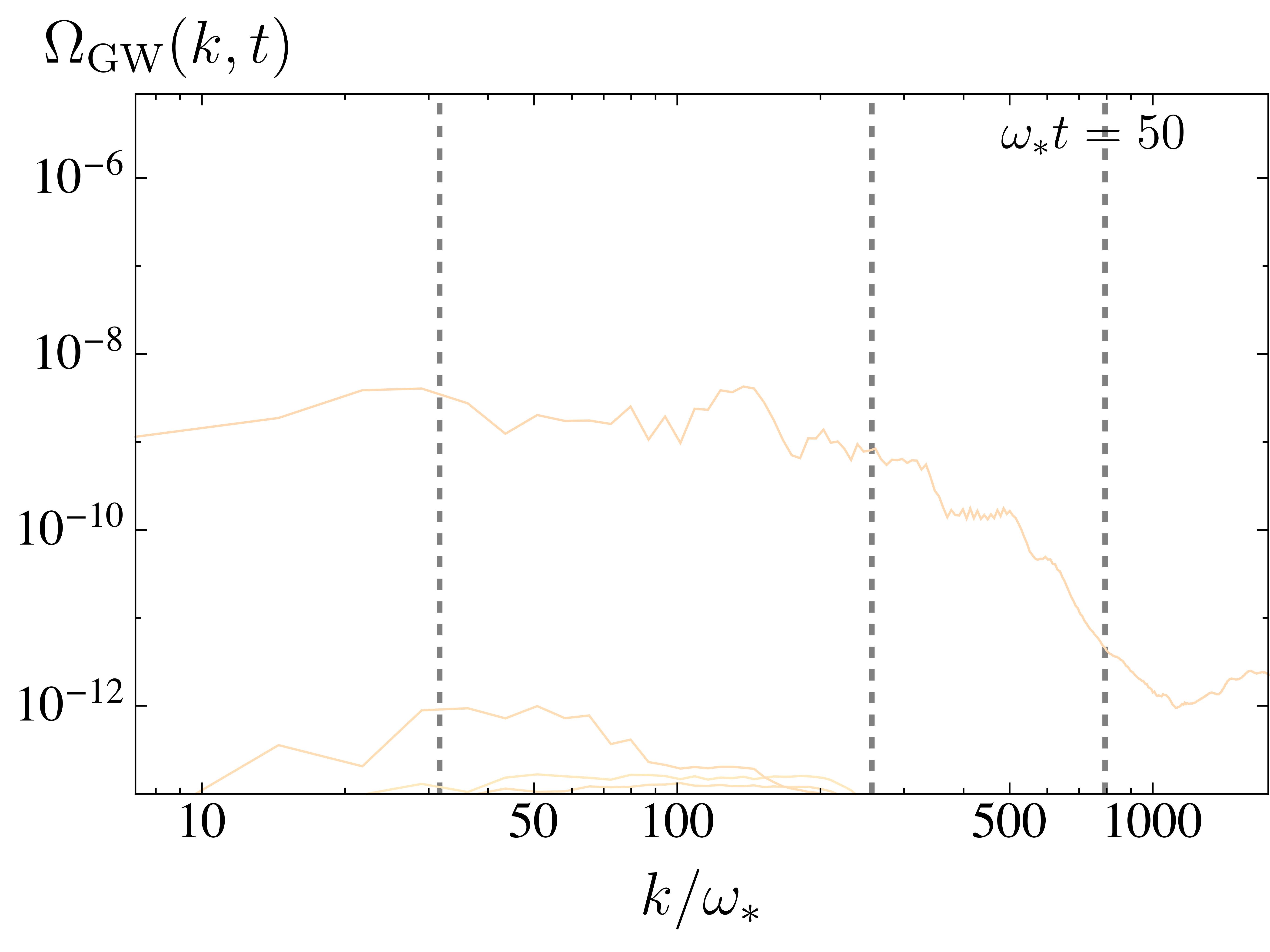
$$V(\phi) + \frac{1}{2}g_1^2\phi^2\chi_1^2 + \frac{1}{2}g_2^2\phi^2\chi_2^2$$



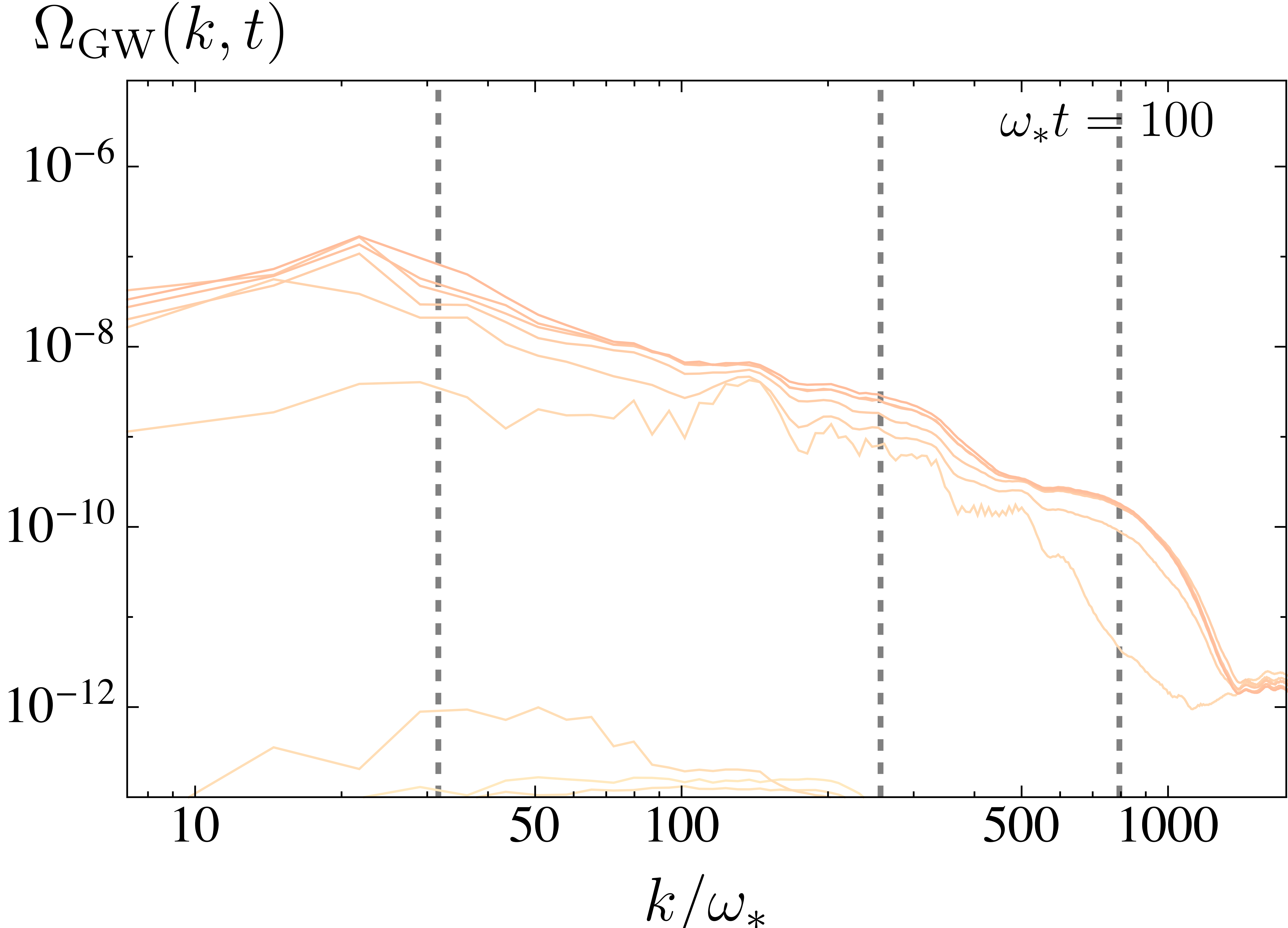
Three-peak signature
(three preheat flds)

$$V(\phi) \; + \; \frac{1}{2}g_1^2\phi^2\chi_1^2 \; + \; \frac{1}{2}g_2^2\phi^2\chi_2^2 \; + \; \frac{1}{2}g_3^2\phi^2\chi_3^2$$

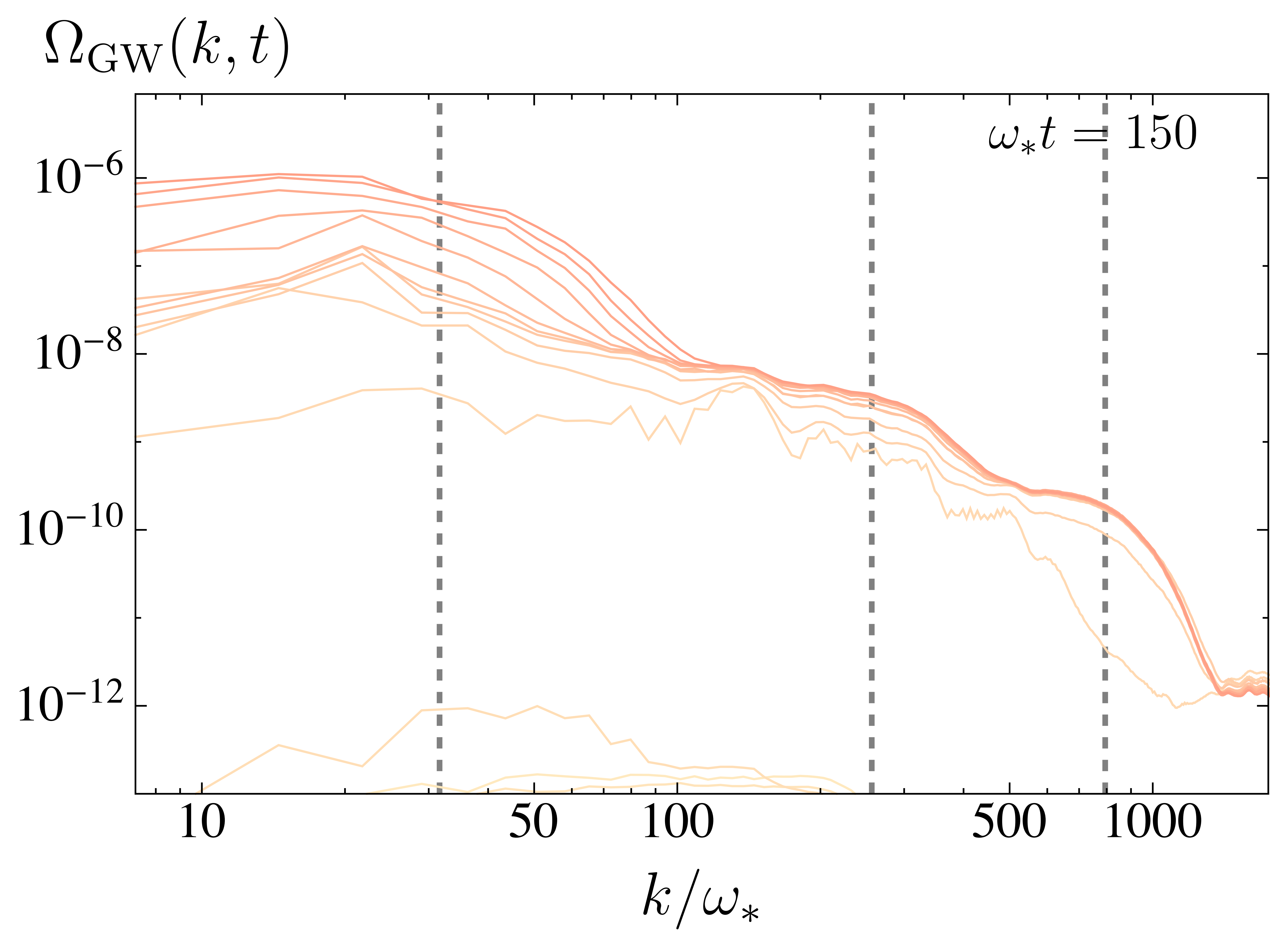
ANIMATION
(by Nico Loayza)

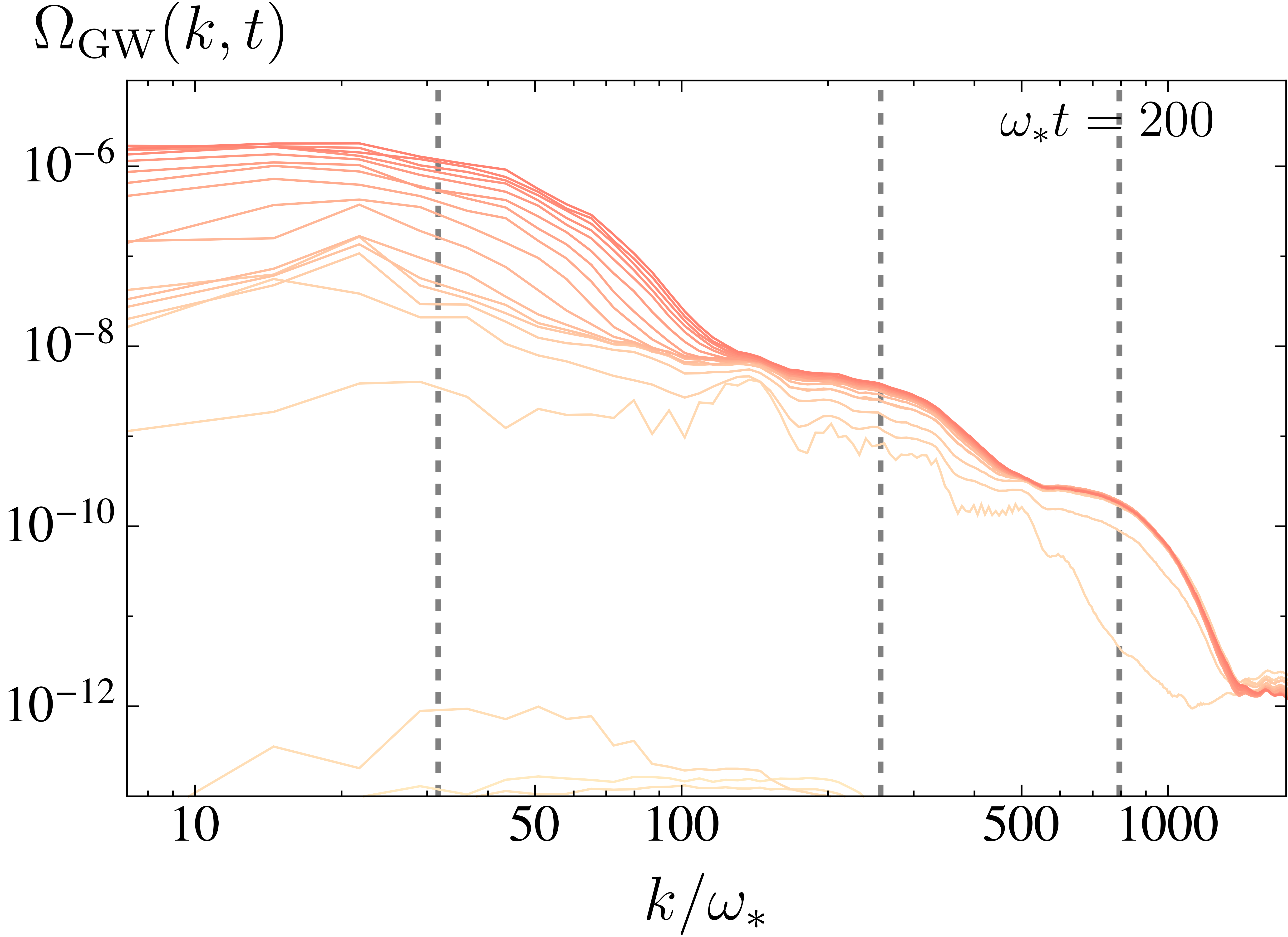


$\Omega_{\mathrm{GW}}(k,t)$
$\omega_* t = 50$
10^{-6}
10^{-8}
10^{-10}
10^{-12}
10
50
100
500
1000
k/ω_*

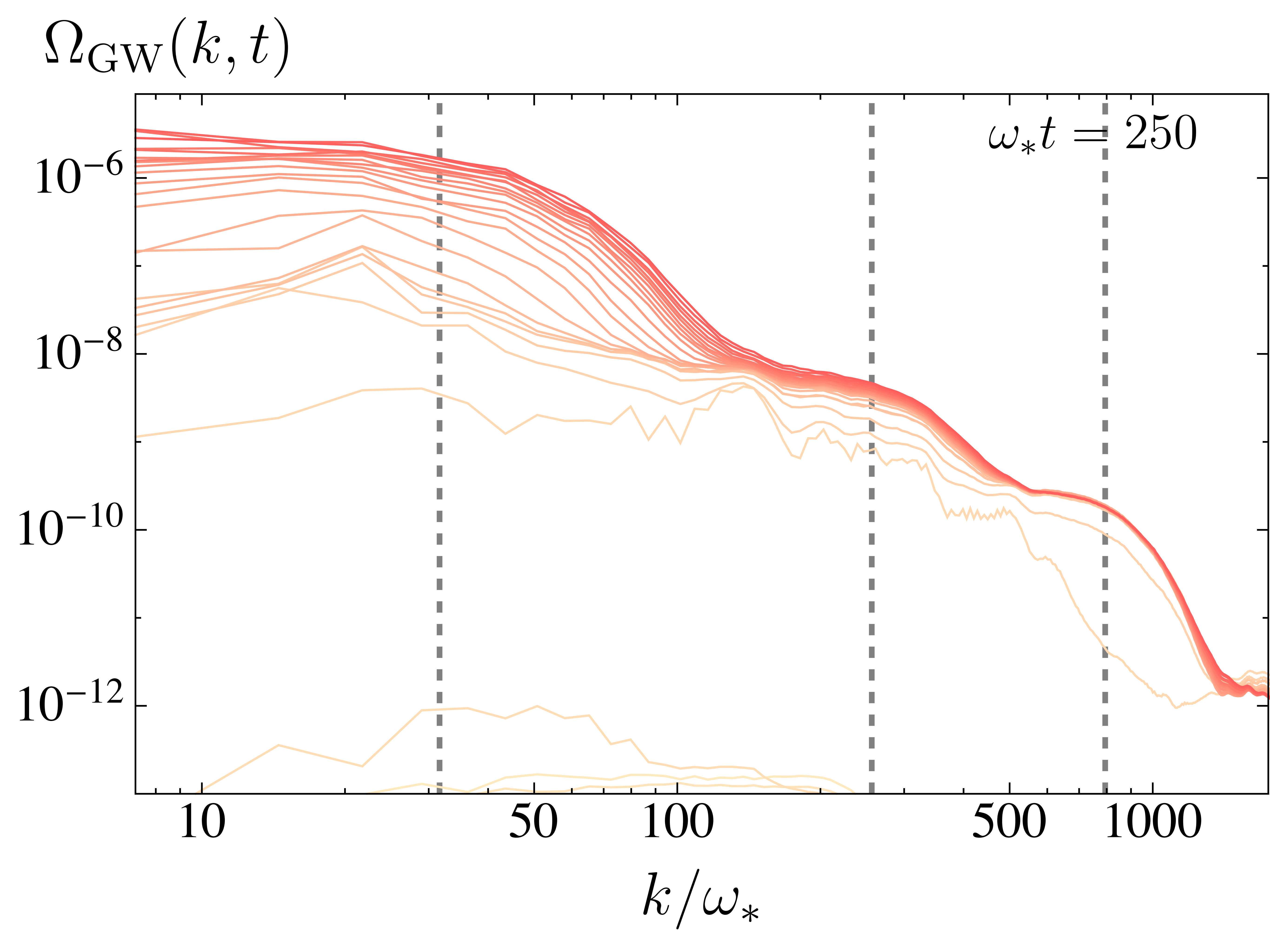


$\Omega_{\rm GW}(k,t)$
$\omega_* t = 100$
10^{-6}
10^{-8}
10^{-10}
10^{-12}
10
50
100
500
1000
k/ω_*

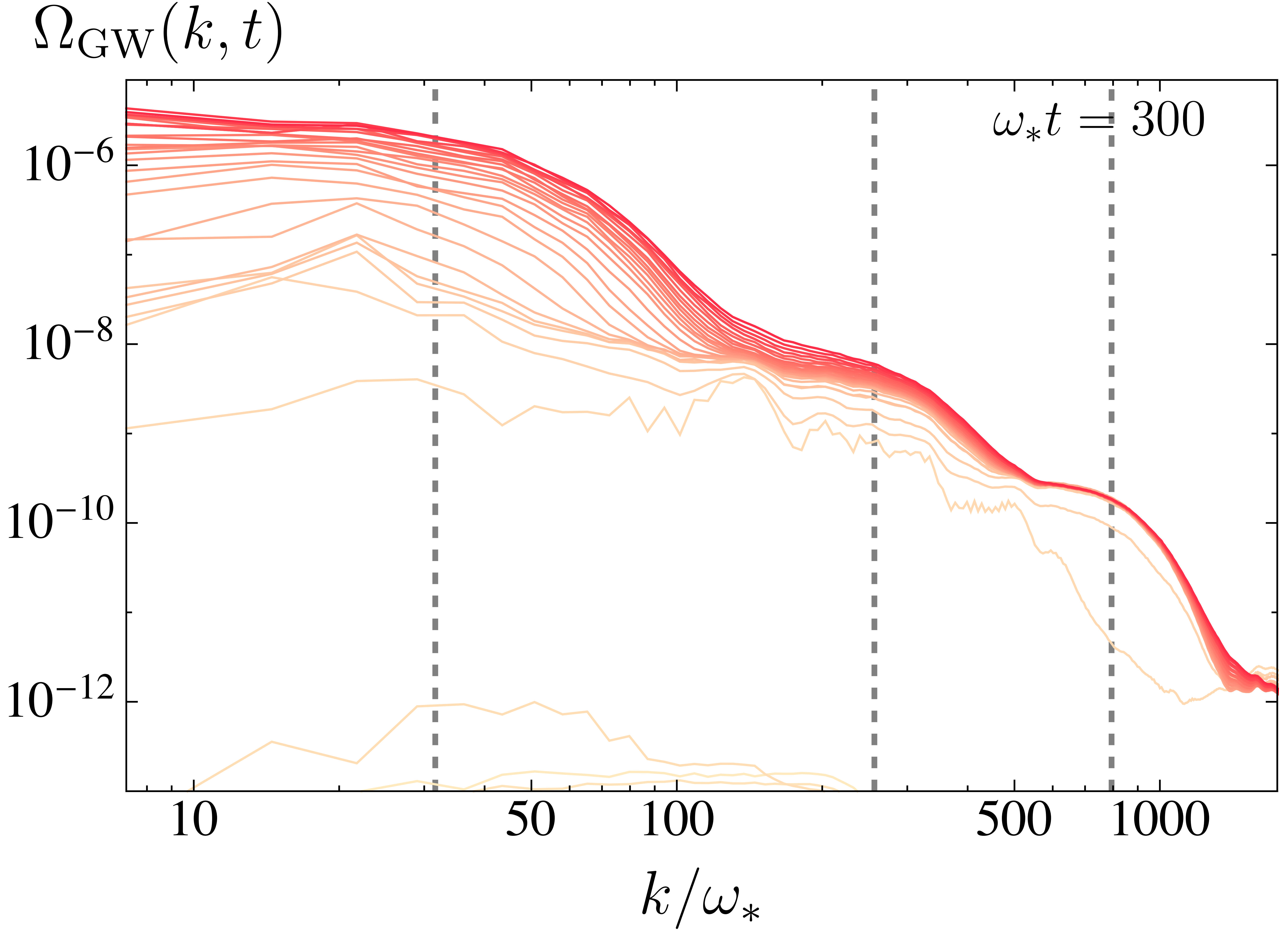

$\Omega_{\rm GW}(k,t)$
$\omega_* t = 150$
10^{-6}
10^{-8}
10^{-10}
10^{-12}
10
50
100
500
1000
k/ω_*



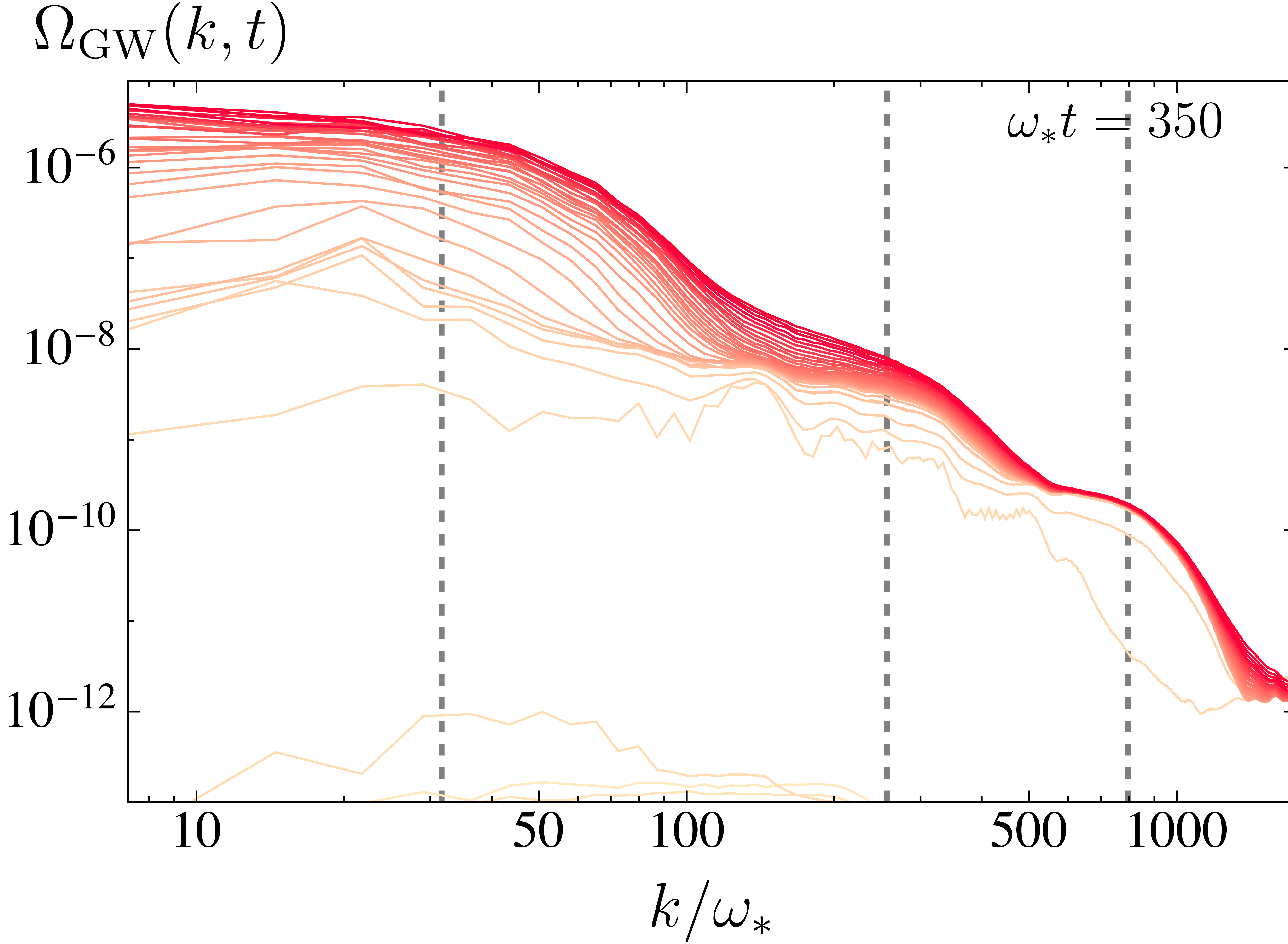
$\Omega_{\rm GW}(k,t)$
$\omega_* t = 200$
10^{-6}
10^{-8}
10^{-10}
10^{-12}
10
50
100
500
1000
k/ω_*



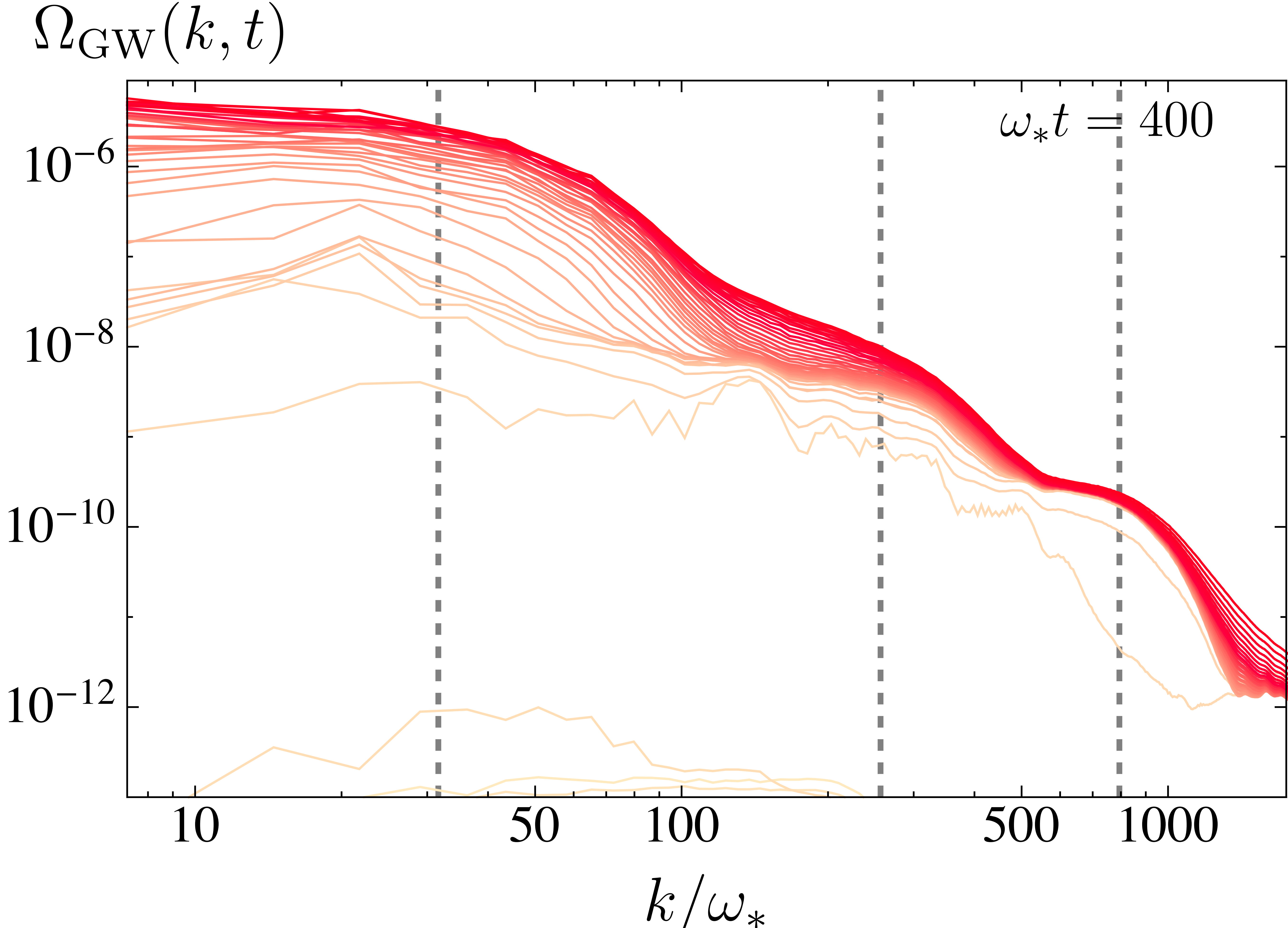
$\Omega_{\rm GW}(k,t)$
$\omega_* t = 250$
10^{-6}
10^{-8}
10^{-10}
10^{-12}
10
50
100
500
1000
k/ω_*



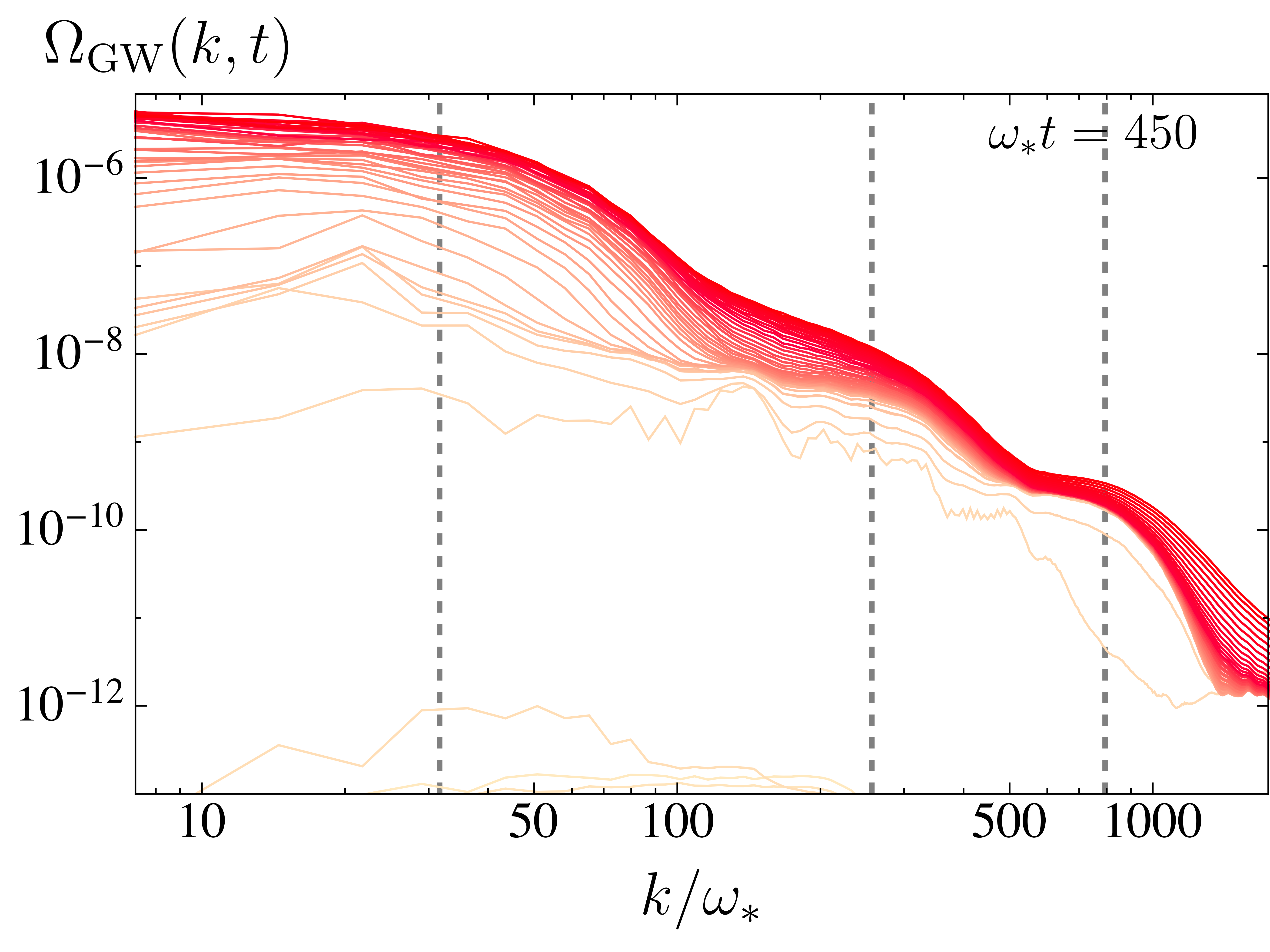
$\Omega_{\rm GW}(k,t)$
$\omega_* t = 300$
10^{-6}
10^{-8}
10^{-10}
10^{-12}
10
50
100
500
1000
k/ω_*



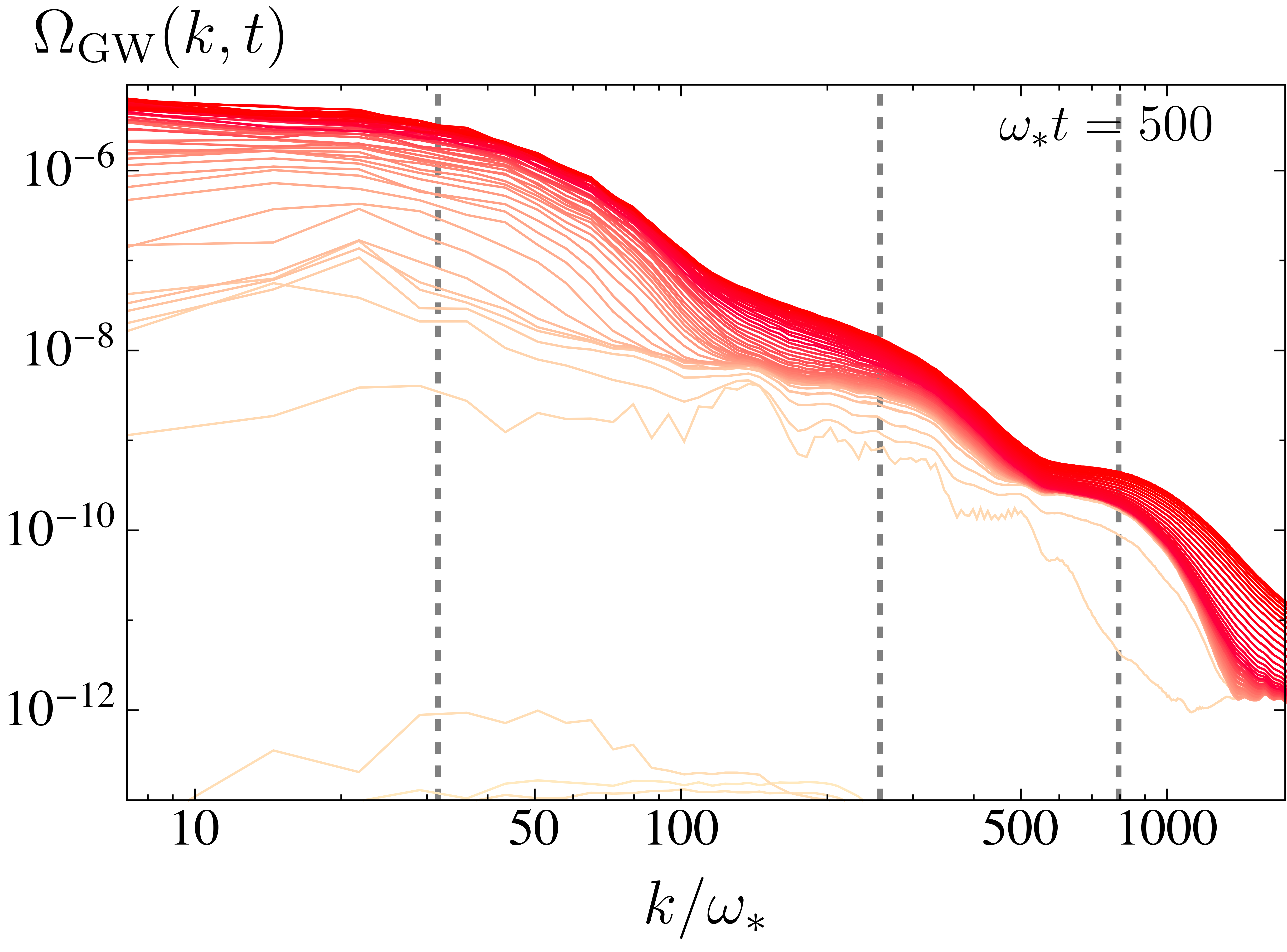
$\Omega_{\rm GW}(k,t)$
$\omega_* t = 350$
10^{-6}
10^{-8}
10^{-10}
10^{-12}
10
50
100
500
1000
k/ω_*



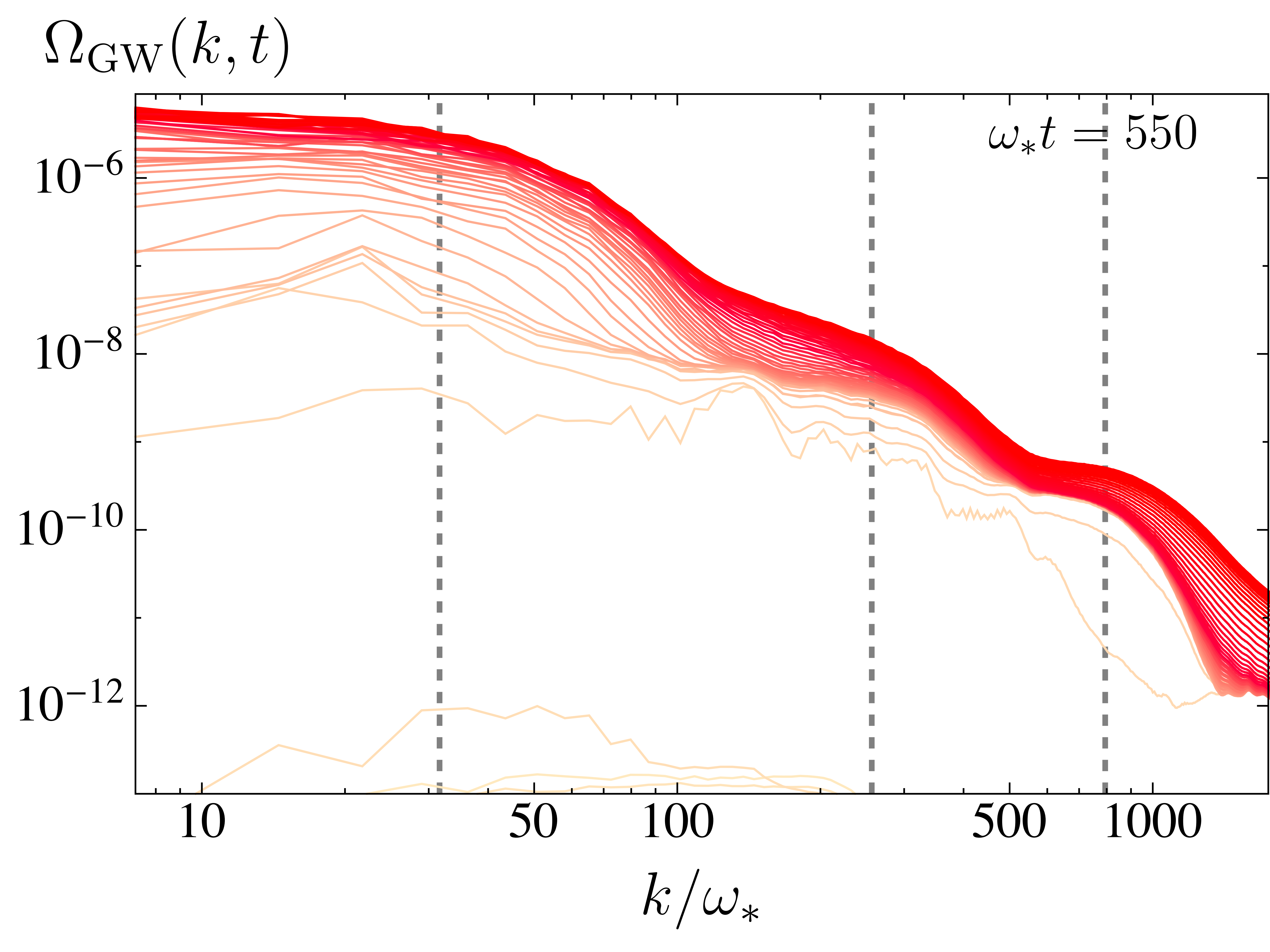
$\Omega_{\rm GW}(k,t)$
$\omega_* t = 400$
10^{-6}
10^{-8}
10^{-10}
10^{-12}
10
50
100
500
1000
k/ω_*

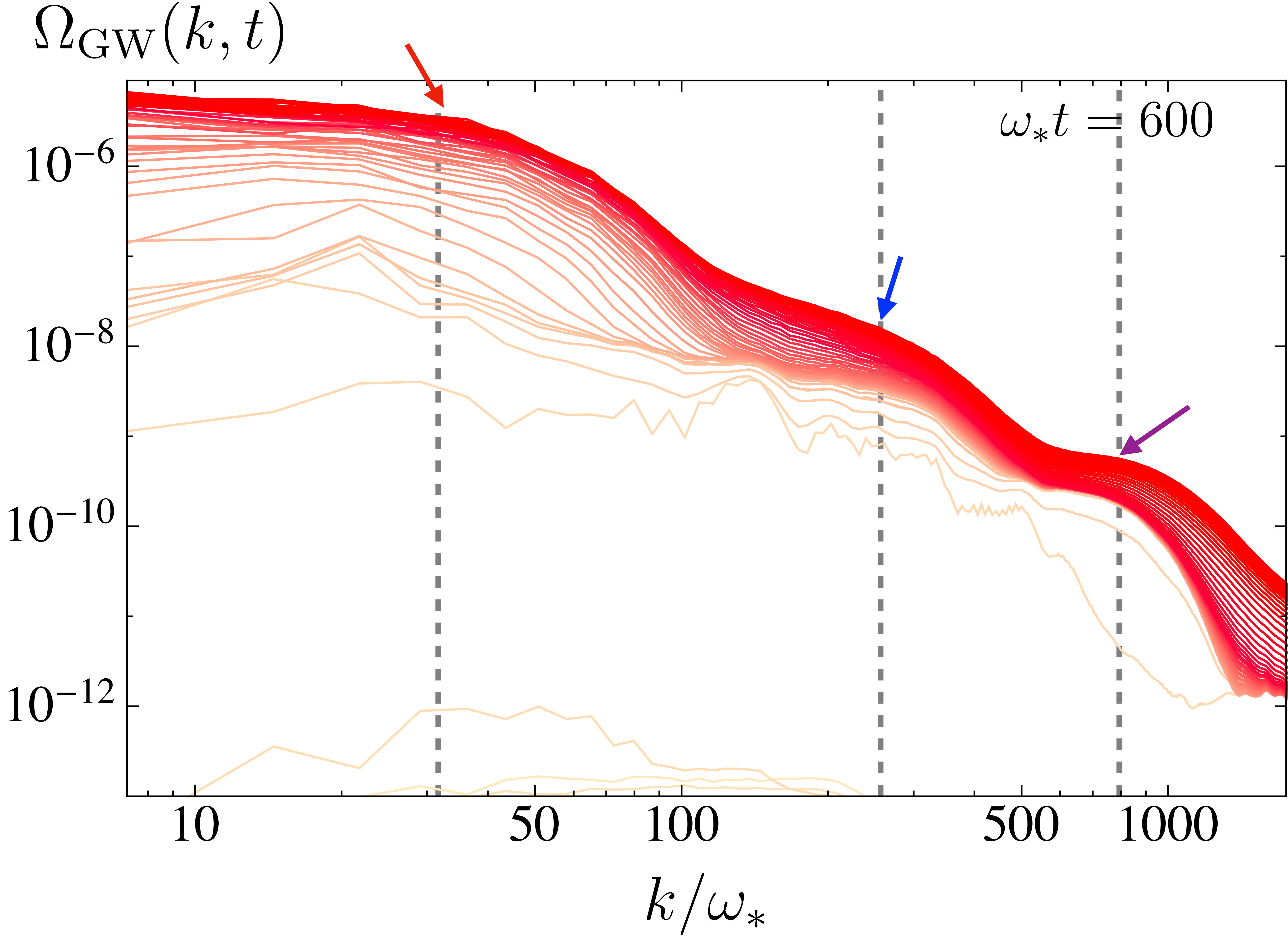


$\Omega_{\mathrm{GW}}(k,t)$
$\omega_* t = 450$
10^{-6}
10^{-8}
10^{-10}
10^{-12}
10
50
100
500
1000
k/ω_*



$\Omega_{\rm GW}(k,t)$
$\omega_* t = 500$
10^{-6}
10^{-8}
10^{-10}
10^{-12}
10
50
100
500
1000
k/ω_*


$\Omega_{\rm GW}(k,t)$
$\omega_* t = 550$
10^{-6}
10^{-8}
10^{-10}
10^{-12}
10
50
100
500
1000
k/ω_*

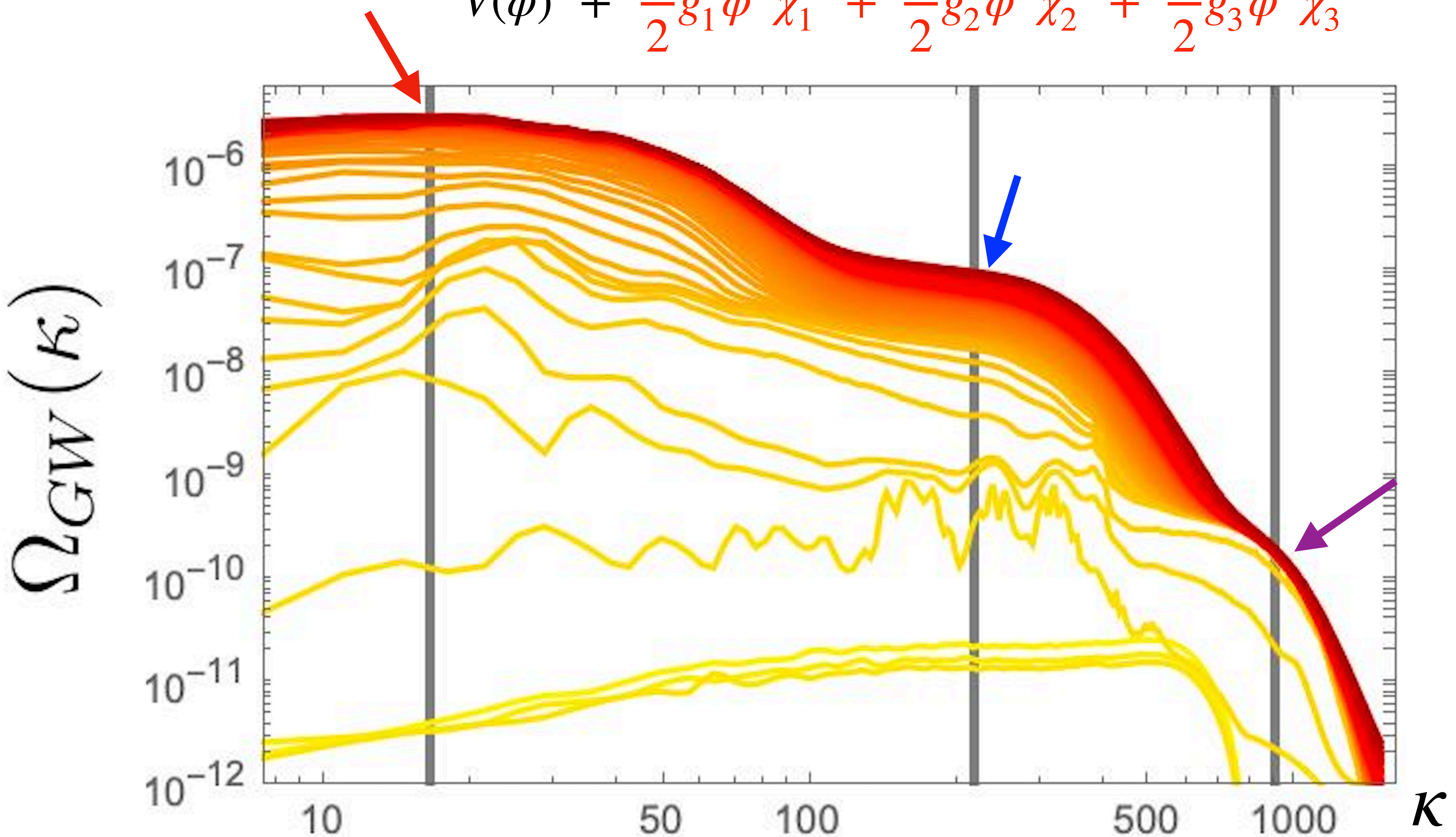


$\Omega_{\rm GW}(k,t)$
$\omega_* t = 600$
10^{-6}
10^{-8}
10^{-10}
10^{-12}
10
50
100
500
1000
k/ω_*

GW Spectroscopy

Parameter Dependence (Peak amplitude**)**

$$V(\phi) + \frac{1}{2}g_1^2\phi^2\chi_1^2 + \frac{1}{2}g_2^2\phi^2\chi_2^2 + \frac{1}{2}g_3^2\phi^2\chi_3^2$$



DGF, Florio, Loayza, Pieroni, [2202.05805]

GW Spectroscopy

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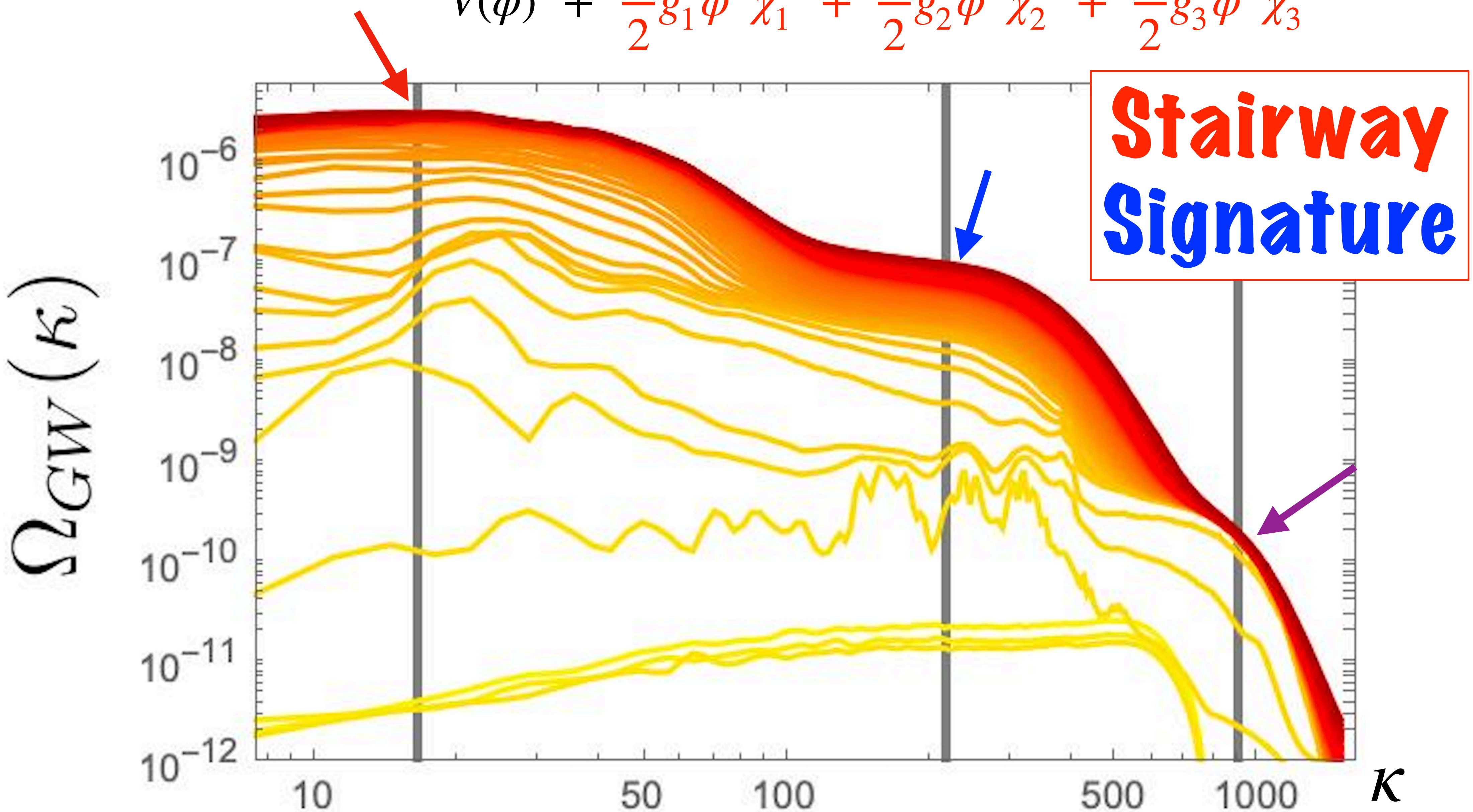

DGF, Florio, Loayza, Pieroni, [2202.05805](2202.05805)

GW Spectroscopy

Reconstruction (2-peak signal)

@ LISA

GW Spectroscopy

Reconstruction (2-peak signal)

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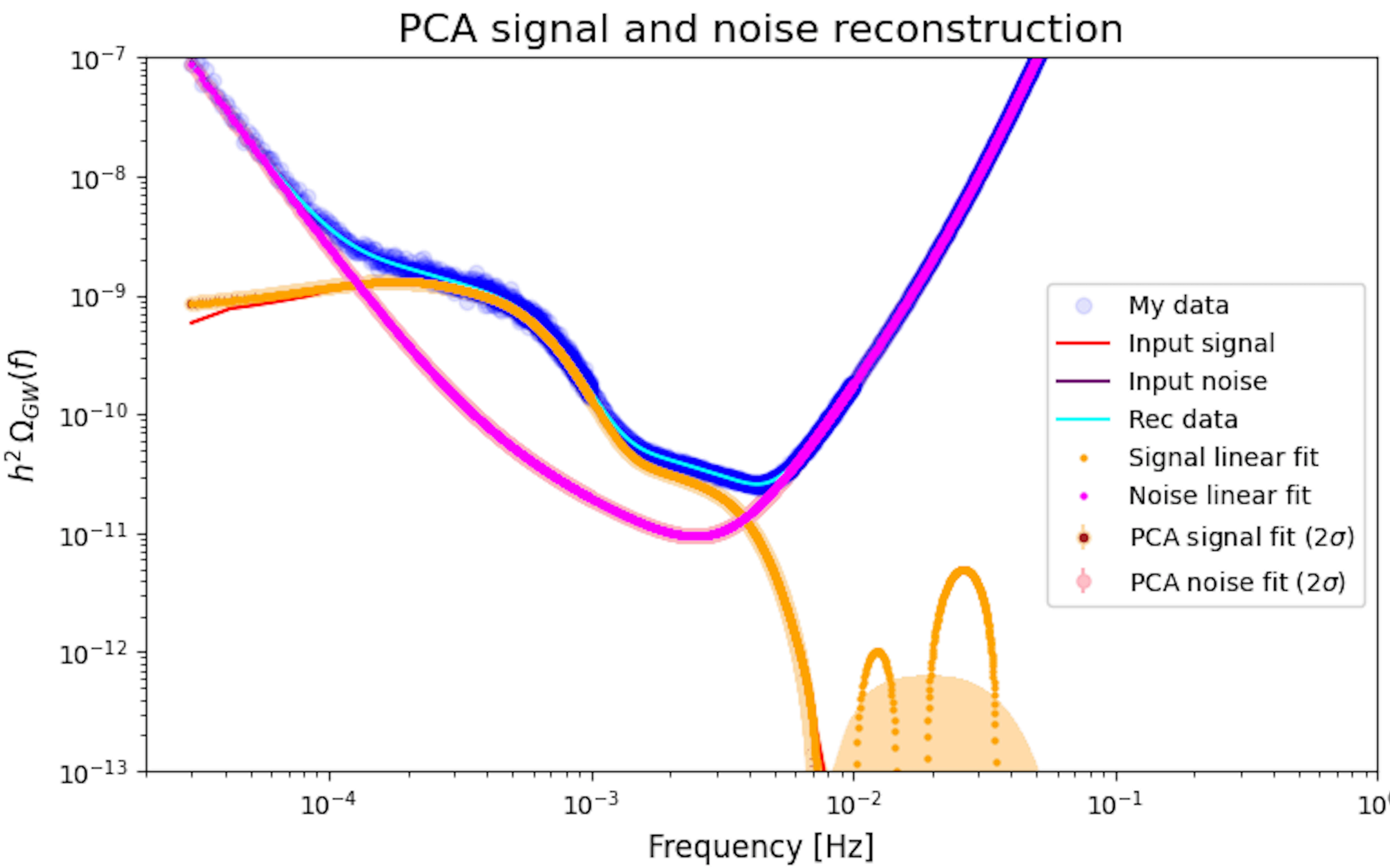
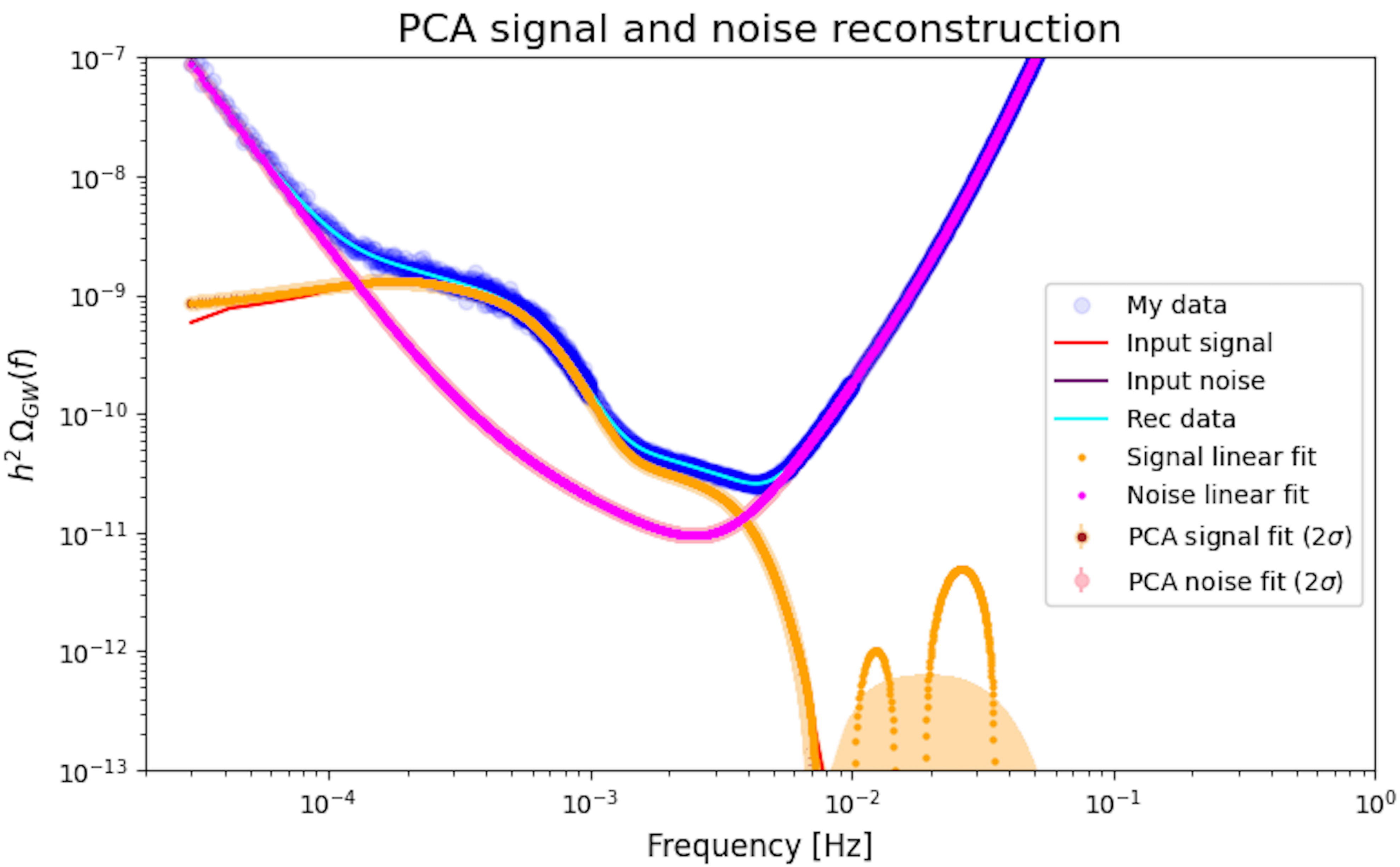


GW Spectroscopy

Reconstruction (2-peak signal)

@ LISA

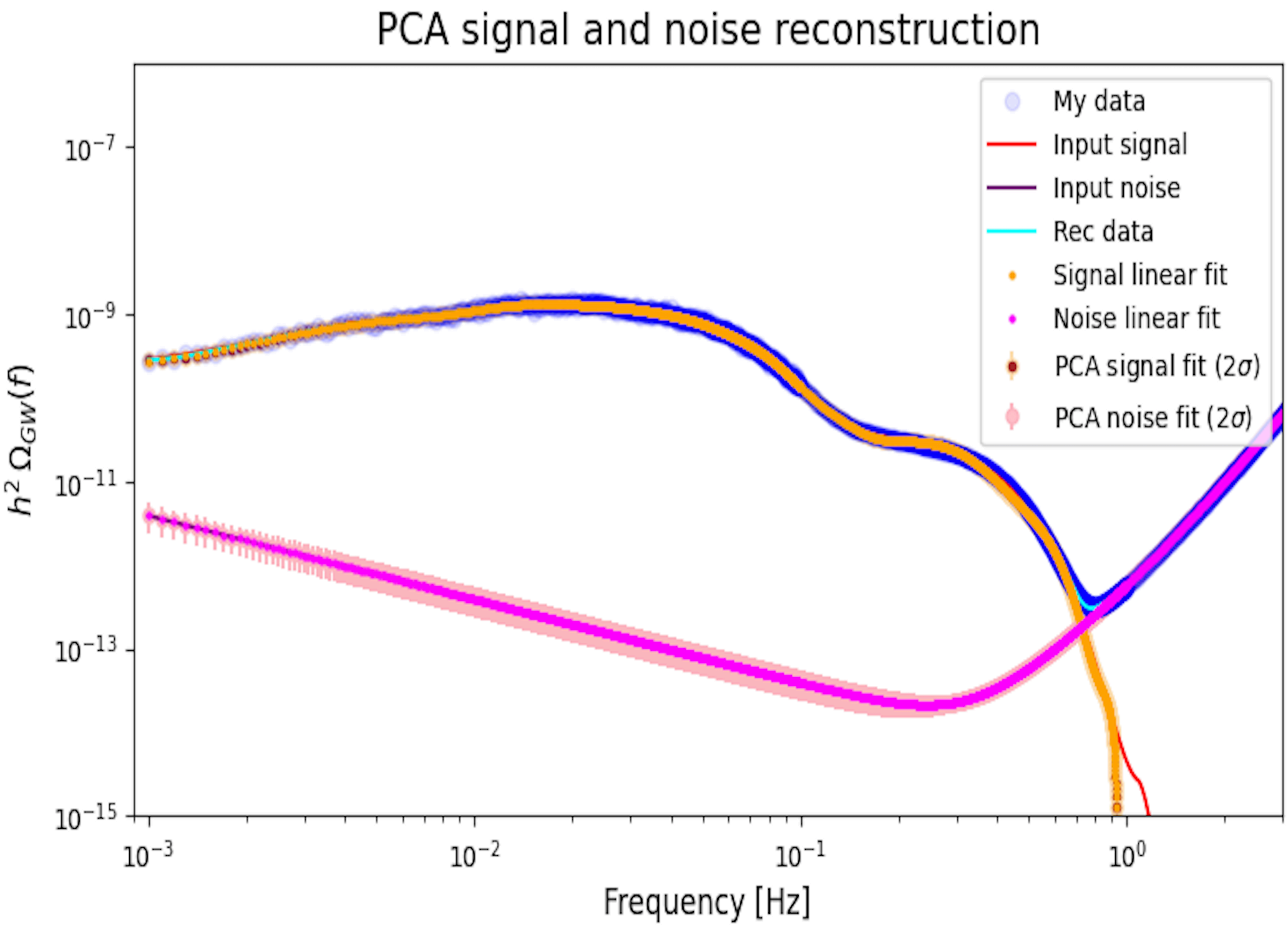


Note: Shift by hand to LISA frequencies

GW Spectroscopy

Reconstruction (2-peak signal)

@ BBO



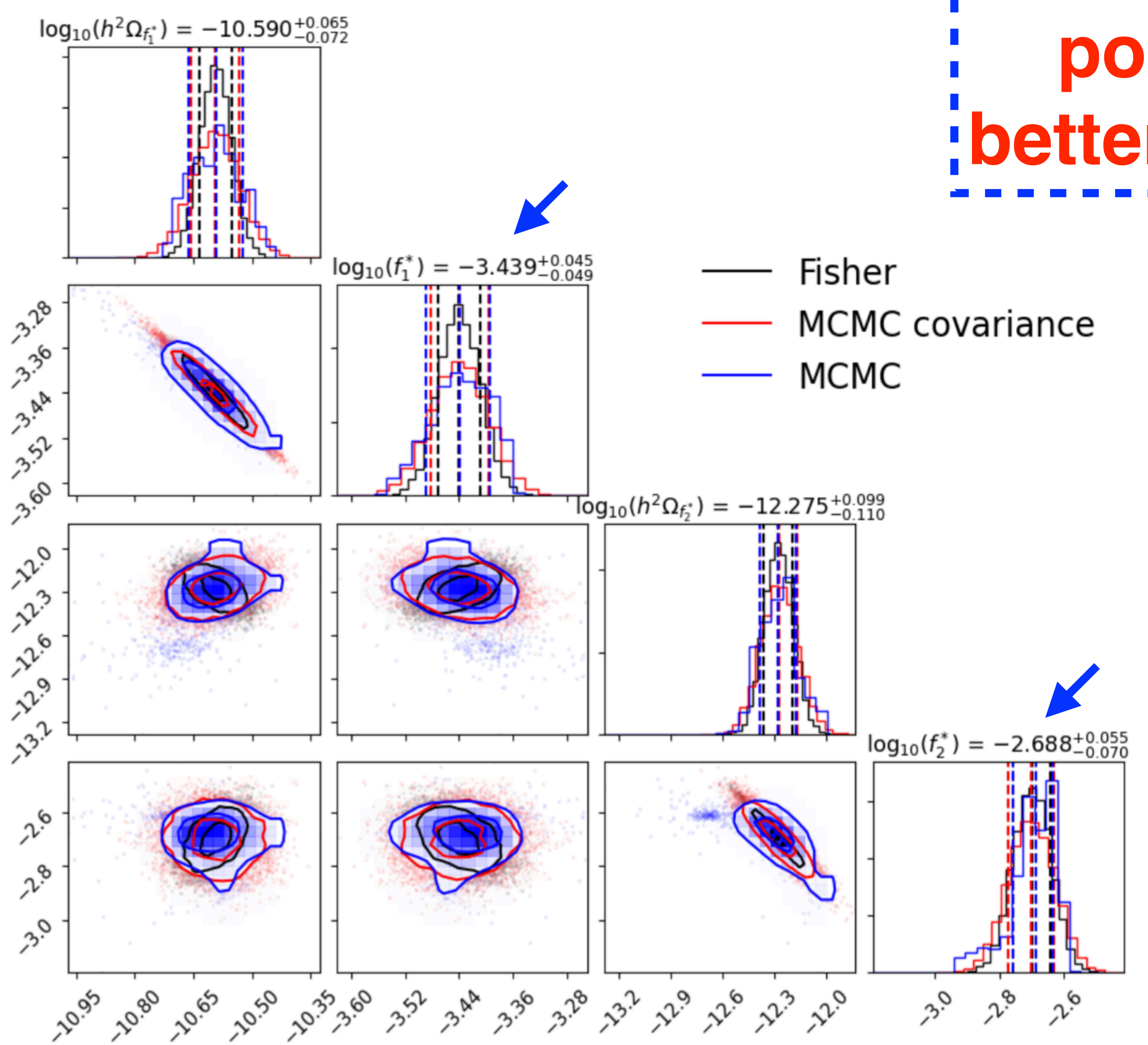
Note: Shift by hand to BBO frequencies

GW Spectroscopy

Reconstruction (2-peak signal)

@ LISA

Peak positions better than 1%



GW Spectroscopy

Reconstruction (2-peak signal)

@ LISA

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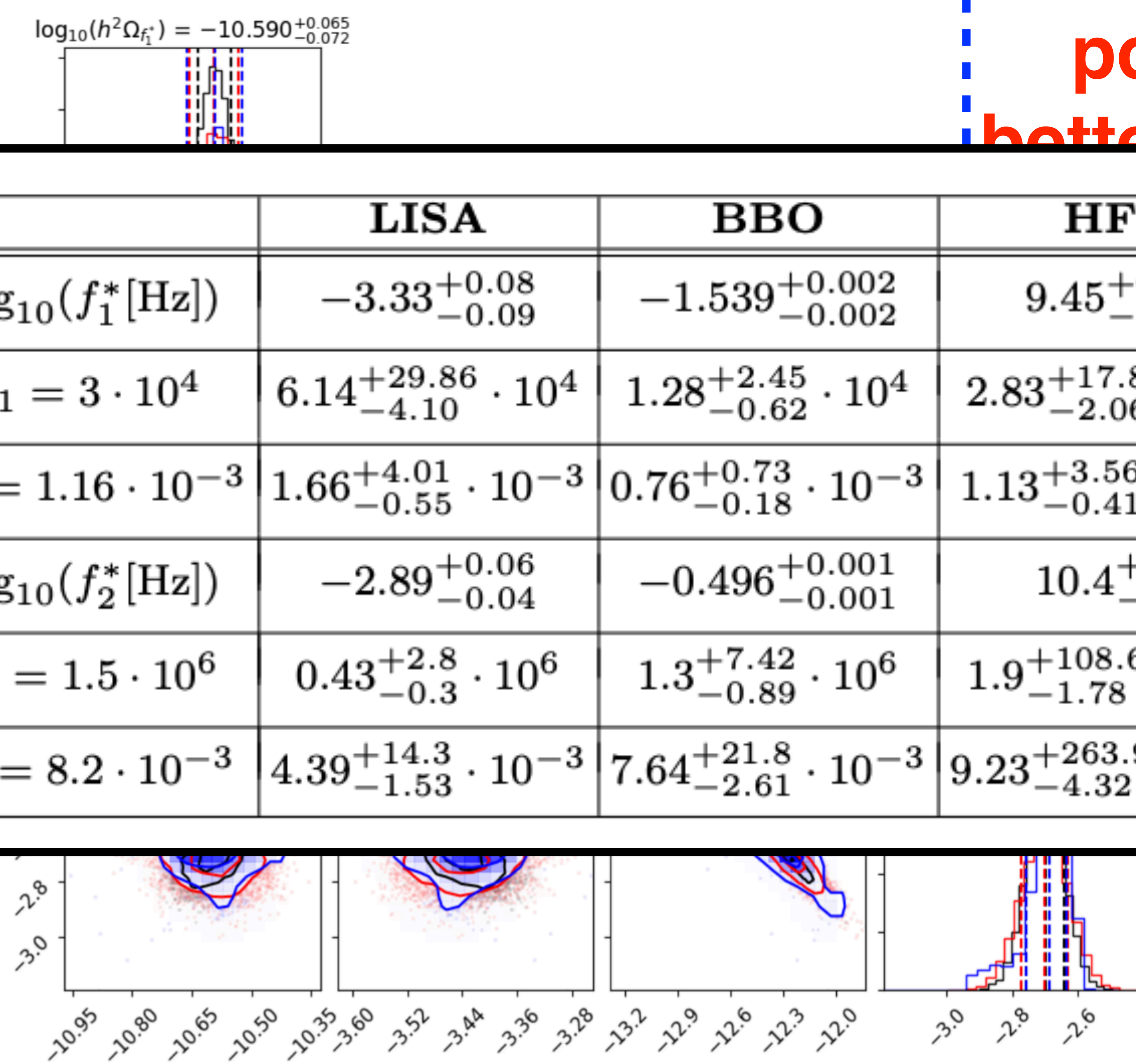

	LISA	BBO	HFE
$\log_{10}(f_1^*[\mathrm{Hz}])$	$-3.33^{+0.08}_{-0.09}$	$-1.539^{+0.002}_{-0.002}$	$9.45^{+0.15}_{-0.15}$
$q_1 = 3 \cdot 10^4$	$6.14^{+29.86}_{-4.10} \cdot 10^4$	$1.28^{+2.45}_{-0.62} \cdot 10^4$	$2.83^{+17.88}_{-2.06} \cdot 10^4$
$g_1 = 1.16 \cdot 10^{-3}$	$1.66^{+4.01}_{-0.55} \cdot 10^{-3}$	$0.76^{+0.73}_{-0.18} \cdot 10^{-3}$	$1.13^{+3.56}_{-0.41} \cdot 10^{-3}$
$\log_{10}(f_2^*[\mathrm{Hz}])$	$-2.89^{+0.06}_{-0.04}$	$-0.496^{+0.001}_{-0.001}$	$10.4^{+0.4}_{-0.4}$
$q_2 = 1.5 \cdot 10^6$	$0.43^{+2.8}_{-0.3} \cdot 10^6$	$1.3^{+7.42}_{-0.89} \cdot 10^6$	$1.9^{+108.65}_{-1.78} \cdot 10^6$
$g_2 = 8.2 \cdot 10^{-3}$	$4.39^{+14.3}_{-1.53} \cdot 10^{-3}$	$7.64^{+21.8}_{-2.61} \cdot 10^{-3}$	$9.23^{+263.9}_{-4.32} \cdot 10^{-3}$

GW Spectroscopy

Reconstruction (2-peak signal)

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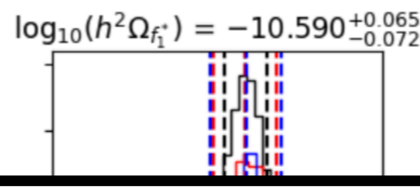

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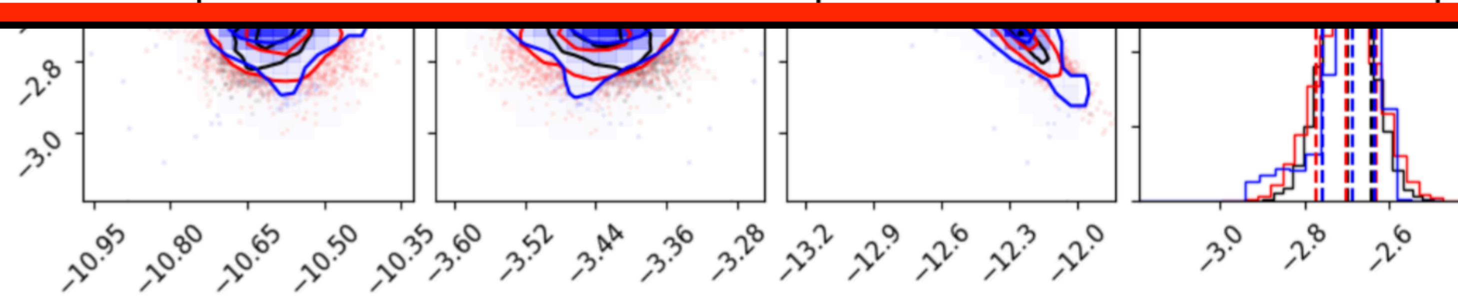

GW Spectroscopy

In reality …

Monomial Models: $\Omega^{(o)}_{\rm GW} \sim 10^{-9}\,,$ $f_o \sim 10^8 - 10^9~\rm Hz$

Large amplitude ! **… at high Frequency !**

GW Spectroscopy

In reality …

Monomial Models: $\Omega_{\rm GW}^{(o)} \sim 10^{-9}\,,$ $f_o \sim 10^8 - 10^9~{\rm Hz}$

Large amplitude ! … at high Frequency !

Very unfortunate… no detectors there !

GW Spectroscopy

In reality …

Monomial Models: $\Omega^{(o)}_{\rm GW} \sim 10^{-9}\,,\qquad f_o \sim 10^8 - 10^9~{\rm Hz}$

Large amplitude ! … at high Frequency !

Our example serves as proof of principle !

Very unfortunate… no detectors there !

GW Spectroscopy

Hope for other scenarios …

Our example serves as proof of principle !

GW Spectroscopy

Hope for other scenarios …

Multi-peak Stairway signatures expected at:
low scale (p)reheating
phase transitions
…..

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GW Spectroscopy

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Multi-peak Stairway signatures expected at:
low scale (p)reheating
phase transitions
.....

Our example serves as proof of principle !

Possible new door to particle physics using gravitational wave backgrounds !

GW Spectroscopy

Hope for other scenarios …

Multi-peak Stairway
signatures expected at:
low scale (p)reheating
phase transitions
.....

High-Freq GW Detection ?

Our example serves as proof of principle !

Possible new door to particle physics using gravitational wave backgrounds !

Example II

(Non-Linear)
Gauge Field Dynamics
@ Axion-Inflation

with

J. Lizarraga, A. Urio and J. Urrestilla

Work in progress

INFLATIONARY MODELS

Axion-Inflation

Freese, Frieman, Olinto '90; . . .

Shift symmetry $\phi \to \phi + const.$

$$V(\phi) + \frac{\phi}{f} F_{\mu\nu} \tilde{F}^{\mu\nu}$$

inflaton ϕ = pseudo-scalar axion

[J. Cook, L. Sorbo (arXiv:1109.0022)]

[N. Barnaby, E. Pajer, M. Peloso (arXiv:1110.3327)]

$\boxed{\text{Not the QCD axion;}}$

INFLATIONARY MODELS

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Freese, Frieman, Olinto '90; . . .

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inflaton ϕ = pseudo-scalar axion

[J. Cook, L. Sorbo (arXiv:1109.0022)] [N. Barnaby, E. Pajer, M. Peloso (arXiv:1110.3327)]

Photon:
2 helicities

$$\left[\frac{\partial^2}{\partial \tau^2} + k^2 \pm \frac{2k\xi}{\tau}\right] A_{\pm}(\tau, k) = 0,$$

$$\boxed{\xi \equiv \frac{\dot{\phi}}{2fH}}$$

Chiral instability

$$A_+ \propto e^{\pi\xi}\,, \quad |A_-| \ll |A_+|$$

A+ exponentially amplified,

INFLATIONARY MODELS

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$$V(\phi) + \frac{\phi}{f} F_{\mu\nu} \tilde{F}^{\mu\nu}$$

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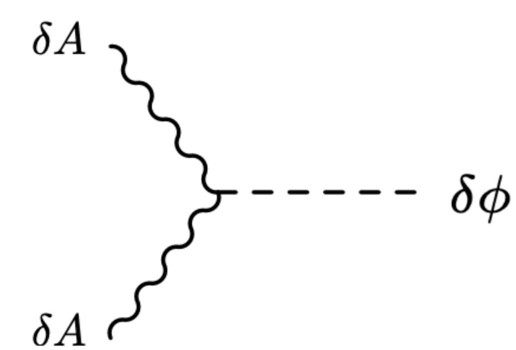
[J. Cook, L. Sorbo (arXiv:1109.0022)] [N. Barnaby, E. Pajer, M. Peloso (arXiv:1110.3327)]

Non-Gaussian scalar perturbations

The excited A_+ modes source inflaton perturbations $\delta\phi$ through <u>inverse decay</u>. These modes are highly non-Gaussian



This imposes $f \gtrsim 10^{16}$ GeV $\left(\text{recall } \mathcal{L} \supset -\frac{\phi F \tilde{F}}{f} \right)$

Barnaby, Peloso '10
Planck '15

INFLATIONARY MODELS

Axion-Inflation

Freese, Frieman, Olinto '90; . . .

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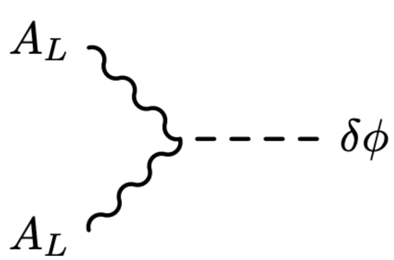
Scalar Pert.

Amplitude must be bounded, preventing overproduction of density perturbations



INFLATIONARY MODELS

Axion-Inflation

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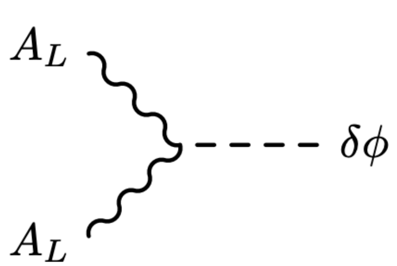
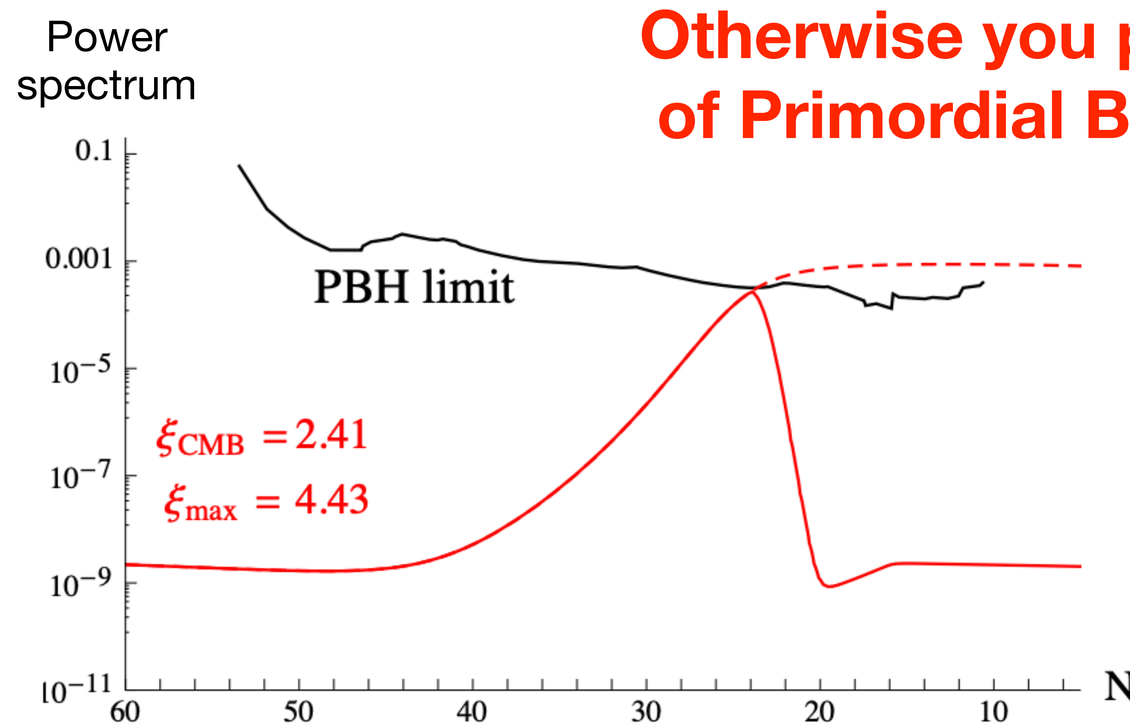
Scalar Pert.



Amplitude must be bounded, preventing overproduction of density perturbations

Otherwise you produce an excess of Primordial Black Holes (PBH) !

Linde, Mooij, Pajer '13



INFLATIONARY MODELS

Axion-Inflation Shift symmetry $\phi \to \phi + const.$

Freese, Frieman, Olinto '90; . . .

$$V(\phi) + \frac{\phi}{f} F_{\mu\nu}\tilde{F}^{\mu\nu}$$

inflaton ϕ = pseudo-scalar axion

[J. Cook, L. Sorbo (arXiv:1109.0022)] [N. Barnaby, E. Pajer, M. Peloso (arXiv:1110.3327)]

Chiral GWs !

$$h''_{ij} + 2\mathcal{H}h'_{ij} - \nabla^2 h_{ij} = 16\pi G \Pi^{\mathrm{TT}}_{ij} \propto \{E_i E_j + B_i B_j\}^{TT}$$

GW left-chirality only ! ⟸ A_μ **Chiral**

INFLATIONARY MODELS

Axion-Inflation

Freese, Frieman, Olinto '90; . . .

Shift symmetry $\phi \to \phi + const.$

$$V(\phi) + \frac{\phi}{f} F_{\mu\nu} \tilde{F}^{\mu\nu}$$

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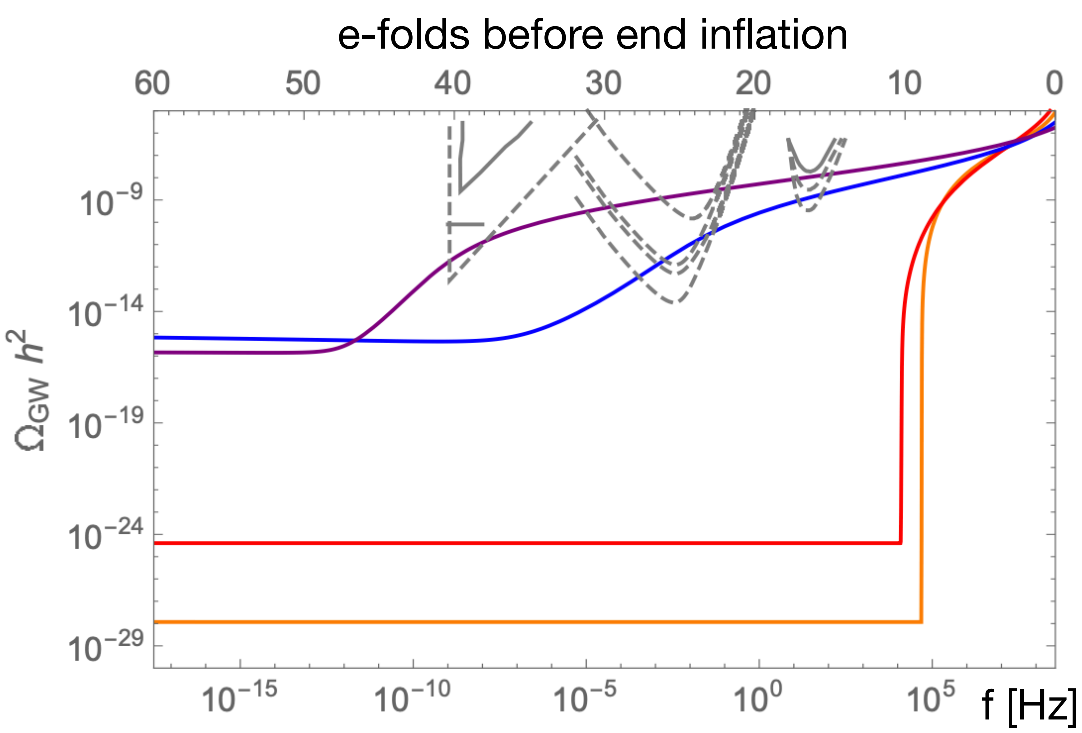

$$V(\phi) = \frac{1}{2} m^2 \phi^2$$

$$V(\phi) = V_0 \left(1 - e^{-\sqrt{\frac{2}{3}}\phi}\right)^2$$

$$V(\phi) = V_0 \left[1 - \left(\frac{\phi}{v}\right)^4\right]^2$$

$$V(\phi) = V_0 \left[1 - \left(\frac{\phi}{v}\right)^3\right]^2$$

Domcke, Pieroni, Binétruy '16

INFLATIONARY MODELS

Axion-Inflation

Freese, Frieman, Olinto '90; . . .

Shift symmetry $\phi \to \phi + const.$

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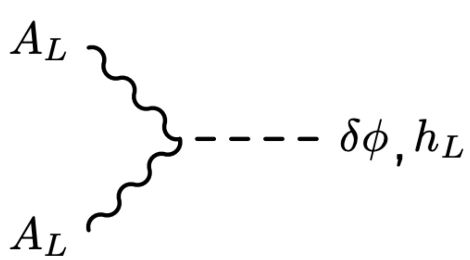
NG + GW !

GW production constrained by preventing overproduction of density perturbations



INFLATIONARY MODELS

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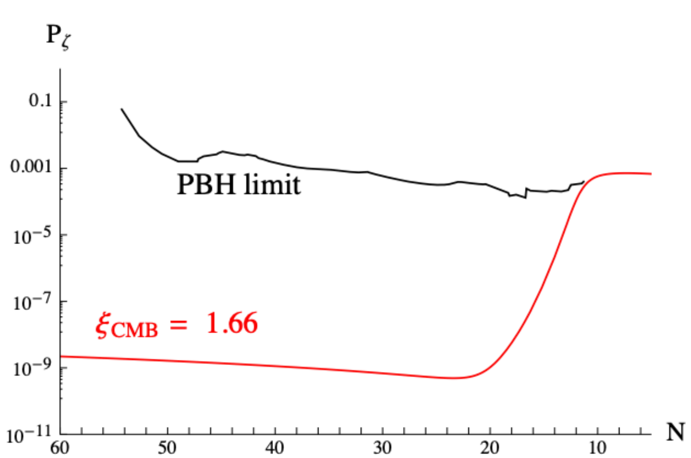
NG + GW !

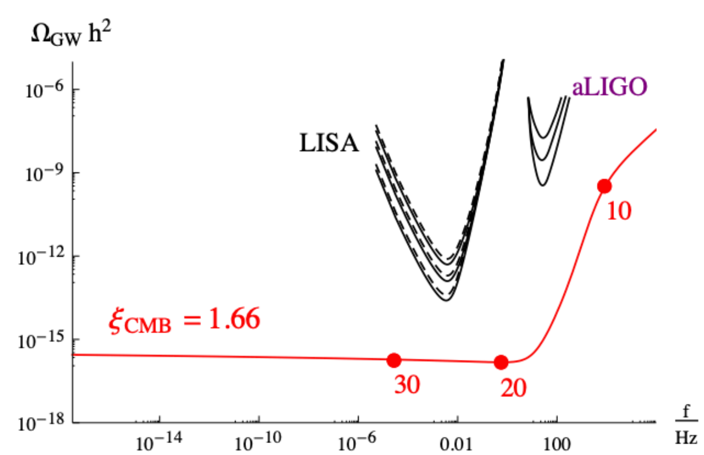
GW production constrained by preventing overproduction of density perturbations

For a monomial $V(\phi)$, PBH bounds prevent GW from being observable at aLIGO and LISA

Linde, Mooij, Pajer '13

Ga-Bellido, Peloso, Unal '16



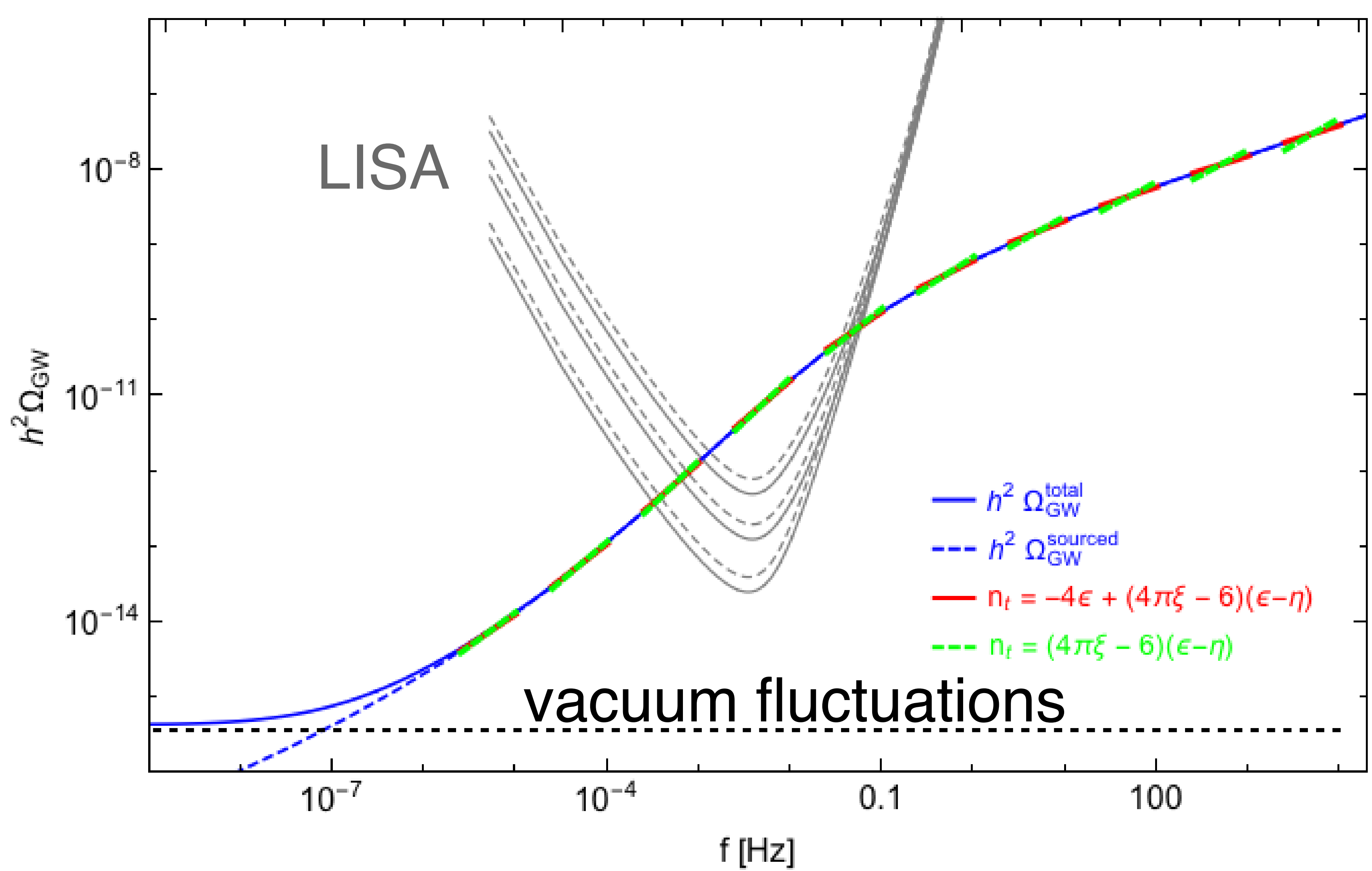


$$N \sim 15 - \ln\left(\frac{f}{100\,\mathrm{Hz}}\right)$$

INFLATIONARY MODELS

Axion-Inflation

GW energy spectrum today



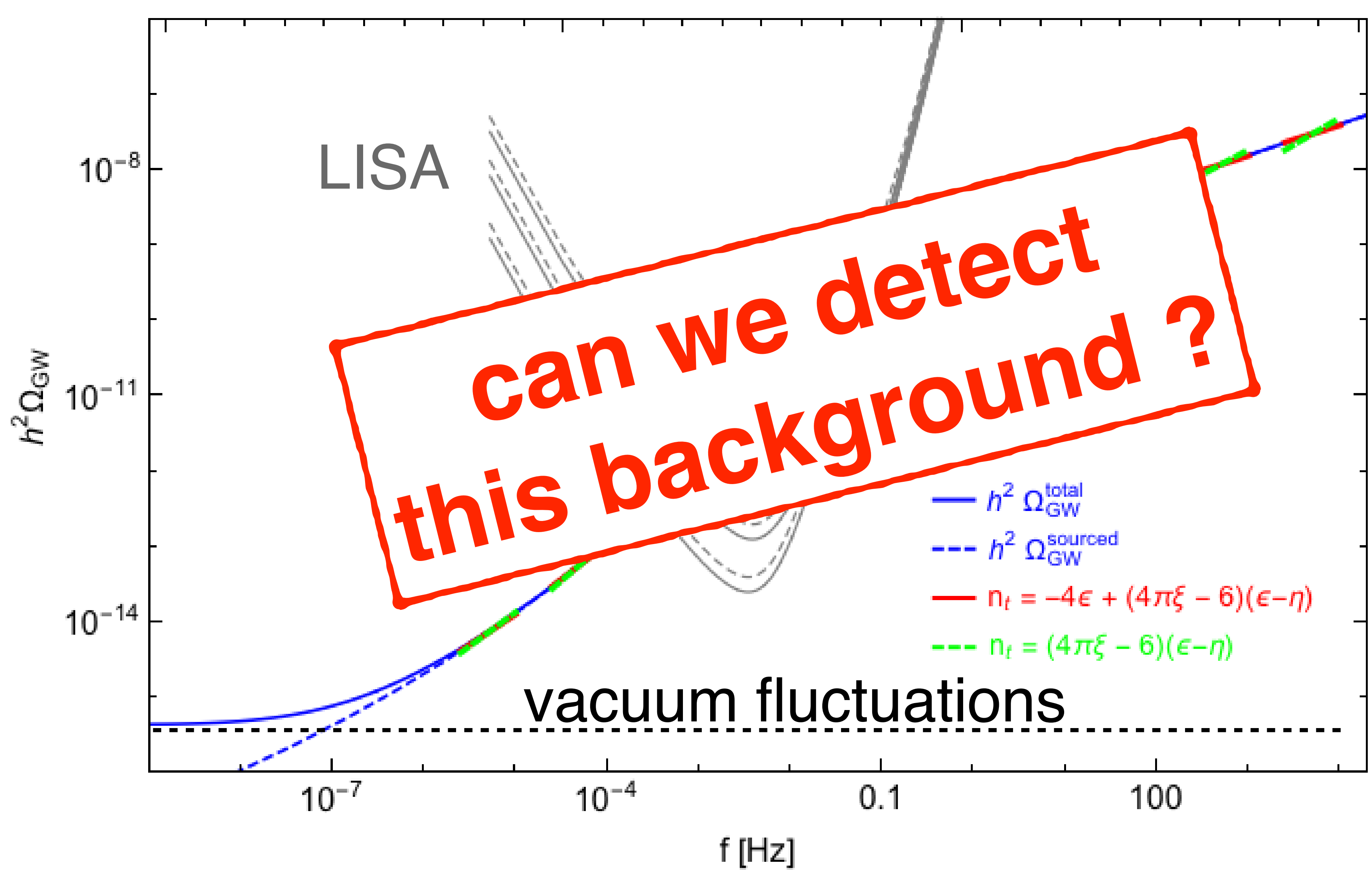
Gauge fields source a Blue-Tilted + Chiral + Non-G GW background

Bartolo et al '16, 1610.06481

INFLATIONARY MODELS

Axion-Inflation

GW energy spectrum today



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Axion-Inflation

PROBLEM: PNG, GW and PBH $\longrightarrow$ **Analytical approximations !**

Axion-Inflation

PROBLEM: PNG, GW and PBH → **Analytical approximations !**

Let's have a look to the full problem !

Axion-Inflation

PROBLEM: PNG, GW and PBH → **Analytical approximations !**

$$\pi_\phi \equiv \dot{\phi}\ , \quad E_i \equiv \dot{A}_i\ , \quad B_i \equiv \epsilon_{ijk}\partial_j A_k$$

$$\tilde{\pi}_\phi = a^3 \pi_\phi\ , \quad \tilde{\vec{E}} = a\vec{E}\ , \ \ \pi_a \equiv \dot{a}$$

Let's have a look to the full problem !

Axion-Inflation

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EoM

Local EoM $(\vec{\mathbf{x}}, \mathbf{t})$

$$\begin{cases} \dot{\tilde{\pi}}_\phi = a\vec{\nabla}^2\phi - a^3 m^2 \phi + \dfrac{1}{a\Lambda}\tilde{\vec{E}}\cdot\vec{B}\ , \\[2em] \dot{\tilde{\vec{E}}} = -\dfrac{1}{a}\vec{\nabla}\times\vec{B} - \dfrac{1}{a^3\Lambda}\tilde{\pi}_\phi\vec{B} + \dfrac{1}{a\Lambda}\vec{\nabla}\phi\times\tilde{\vec{E}}\ . \end{cases}$$

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$$\pi_\phi \equiv \dot{\phi}\ , \quad E_i \equiv \dot{A}_i\ , \quad B_i \equiv \epsilon_{ijk}\partial_j A_k$$

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EoM

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Hom. EoM $(\mathbf{t})$

$$\dot{\pi}_a = -\frac{a}{6m_{pl}^2}(\rho + 3p) = \frac{a}{3m_{pl}^2}\Big\langle \underbrace{-2K_\phi}_{\text{(Kin)}} + \underbrace{V}_{\text{(Pot)}} - \underbrace{K_A}_{\text{(Elec)}} - \underbrace{G_A}_{\text{(Mag)}} \Big\rangle$$

$$\left(K_\phi \equiv \frac{1}{2}\pi_\phi^2 = \frac{1}{2a^6}\tilde{\pi}_\phi^2\ , \quad G_\phi \equiv \frac{1}{2a^2}\sum(\partial_i\phi)^2\ , \quad V \equiv \frac{1}{2}m^2\phi^2\ , \right.$$
$$\left. K_A \equiv \frac{1}{2}\sum_i \frac{E_i^2}{a^2} = \frac{1}{2}\sum_i \frac{\tilde{E}_i^2}{a^4}\ , \quad G_A \equiv \frac{1}{2}\sum_i \frac{B_i^2}{a^4} \right)$$

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EoM

Local EoM $(\vec{\mathbf{x}}, \mathbf{t})$

$$\begin{cases} \dot{\tilde{\pi}}_\phi = a\vec{\nabla}^2\phi - a^3 m^2\phi + \dfrac{1}{a\Lambda}\tilde{\vec{E}}\cdot\vec{B}\ , \\[2ex] \dot{\tilde{\vec{E}}} = -\dfrac{1}{a}\vec{\nabla}\times\vec{B} - \dfrac{1}{a^3\Lambda}\tilde{\pi}_\phi\vec{B} + \dfrac{1}{a\Lambda}\vec{\nabla}\phi\times\tilde{\vec{E}}\ . \end{cases}$$

Non-Linear dynamics

Hom. EoM $(\mathbf{t})$

$$\dot{\pi}_a = -\frac{a}{6m_{pl}^2}(\rho + 3p) = \frac{a}{3m_{pl}^2}\langle -2\underbrace{K_\phi}_{\text{(Kin)}} + \underbrace{V}_{\text{(Pot)}} - \underbrace{K_A}_{\text{(Elec)}} - \underbrace{G_A}_{\text{(Mag)}}\rangle$$

$$\left(K_\phi \equiv \frac{1}{2}\pi_\phi^2 = \frac{1}{2a^6}\tilde{\pi}_\phi^2\ , \quad G_\phi \equiv \frac{1}{2a^2}\sum(\partial_i\phi)^2\ , \quad V \equiv \frac{1}{2}m^2\phi^2\ , \quad K_A \equiv \frac{1}{2}\sum_i\frac{E_i^2}{a^2} = \frac{1}{2}\sum_i\frac{\tilde{E}_i^2}{a^4}\ , \quad G_A \equiv \frac{1}{2}\sum_i\frac{B_i^2}{a^4} \right)$$

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$$\pi_\phi \equiv \dot{\phi} \ , \quad E_i \equiv \dot{A}_i \ , \quad B_i \equiv \epsilon_{ijk}\partial_j A_k$$

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EoM

Linear Regime

Local EoM $(\vec{\mathbf{x}}, \mathbf{t})$

$$\begin{cases} \dot{\tilde{\pi}}_\phi = \cancel{a\vec{\nabla}^2\phi} - a^3 m^2 \phi \cancel{+ \dfrac{1}{a\Lambda}\vec{\tilde{E}}\cdot\vec{B}} \ , \\ \dot{\vec{\tilde{E}}} = -\dfrac{1}{a}\vec{\nabla}\times\vec{B} - \dfrac{1}{a^3\Lambda}\tilde{\pi}_\phi\vec{B} + \cancel{\dfrac{1}{a\Lambda}\vec{\nabla}\phi\times\vec{\tilde{E}}} \ . \end{cases}$$

Interaction

Hom. EoM $(\mathbf{t})$

$$\dot{\pi}_a = -\frac{a}{6m_{pl}^2}(\rho + 3p) = \frac{a}{3m_{pl}^2}\langle -2K_\phi + V - K_A - G_A \rangle$$

(Kin) (Pot) (Elec) (Mag)

$$\left(K_\phi \equiv \frac{1}{2}\pi_\phi^2 = \frac{1}{2a^6}\tilde{\pi}_\phi^2 \ , \quad G_\phi \equiv \frac{1}{2a^2}\sum(\partial_i\phi)^2 \ , \quad V \equiv \frac{1}{2}m^2\phi^2 \ , \quad K_A \equiv \frac{1}{2}\sum_i \frac{E_i^2}{a^2} = \frac{1}{2}\sum_i \frac{\tilde{E}_i^2}{a^4} \ , \quad G_A \equiv \frac{1}{2}\sum_i \frac{B_i^2}{a^4} \right)$$

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EoM

Linear Regime

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Hom. EoM $(\mathbf{t})$

$$\dot{\pi}_a = -\frac{a}{6m_{pl}^2}(\rho + 3p) = \frac{a}{3m_{pl}^2}\langle \underbrace{-2K_\phi}_{\text{(Kin)}} + \underbrace{V}_{\text{(Pot)}} \cancel{\underbrace{-K_A}_{\text{(Elec)}} \underbrace{-G_A}_{\text{(Mag)}}} \rangle$$

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EoM

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$$\begin{cases} \dot{\tilde\pi}_\phi = \cancel{a\vec\nabla^2\phi} - a^3 m^2\phi \cancel{+ \frac{1}{a\Lambda}\vec{\tilde E}\cdot\vec B} \,, \\ \dot{\vec{\tilde E}} = -\frac{1}{a}\vec\nabla\times\vec B - \frac{1}{a^3\Lambda}\tilde\pi_\phi\vec B \cancel{+ \frac{1}{a\Lambda}\vec\nabla\phi\times\vec{\tilde E}} \,. \end{cases}$$

Interaction

Hom. EoM $(\mathbf{t})$

$$\dot\pi_a = -\frac{a}{6m_{pl}^2}(\rho+3p) = \frac{a}{3m_{pl}^2}\Big\langle \underset{\text{(Kin)}}{-2K_\phi} + \underset{\text{(Pot)}}{V} \cancel{\underset{\text{(Elec)}}{-K_A} \underset{\text{(Mag)}}{-G_A}} \Big\rangle$$

$$\left(\begin{array}{l} \pi_\phi \equiv \dot\phi \,, \quad E_i \equiv \dot A_i \,, \quad B_i \equiv \epsilon_{ijk}\partial_j A_k \\ \tilde\pi_\phi = a^3\pi_\phi \,, \quad \vec{\tilde E} = a\vec E \,, \quad \pi_a \equiv \dot a \end{array} \right)$$

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Axion-Inflation

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EoM

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Axion-Inflation

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EoM

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$$\begin{cases} \dot{\tilde{\pi}}_\phi = \cancel{a\vec{\nabla}^2\phi} - a^3 m^2 \phi + \dfrac{1}{a\Lambda}\vec{\tilde{E}}\cdot\vec{B}\,, \\[2ex] \dot{\vec{\tilde{E}}} = -\dfrac{1}{a}\vec{\nabla}\times\vec{B} - \dfrac{1}{a^3\Lambda}\tilde{\pi}_\phi\vec{B} + \cancel{\dfrac{1}{a\Lambda}\vec{\nabla}\phi\times\vec{\tilde{E}}}\,. \end{cases}$$

Back-Reaction

Hom. EoM $(\mathbf{t})$

$$\dot{\pi}_a = -\frac{a}{6m_{pl}^2}(\rho + 3p) = \frac{a}{3m_{pl}^2}\Big\langle \underset{\text{(Kin)}}{-2K_\phi} + \underset{\text{(Pot)}}{V} - \underset{\text{(Elec)}}{K_A} - \underset{\text{(Mag)}}{G_A} \Big\rangle$$

$$\left(\begin{array}{l} \pi_\phi \equiv \dot{\phi}\,, \quad E_i \equiv \dot{A}_i\,, \quad B_i \equiv \epsilon_{ijk}\partial_j A_k \\ \tilde{\pi}_\phi = a^3\pi_\phi\,, \quad \vec{\tilde{E}} = a\vec{E}\,, \quad \pi_a \equiv \dot{a} \end{array} \right)$$

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Axion-Inflation

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EoM

Local EoM $(\vec{\mathbf{x}}, \mathbf{t})$

$$\begin{cases} \dot{\tilde{\pi}}_\phi = \cancel{a\vec{\nabla}^2\phi} - a^3 m^2 \phi + \dfrac{1}{a\Lambda}\langle \tilde{\vec{E}} \cdot \vec{B} \,\rangle \\ \dot{\tilde{\vec{E}}} = -\dfrac{1}{a}\vec{\nabla} \times \vec{B} - \dfrac{1}{a^3\Lambda}\tilde{\pi}_\phi \vec{B} + \cancel{\dfrac{1}{a\Lambda}\vec{\nabla}\phi \times \tilde{\vec{E}}}\,. \end{cases}$$

Hom. (t) Approx.

Back-Reaction (Homog. Approx.)

Hom. EoM $(\mathbf{t})$

$$\dot{\pi}_a = -\frac{a}{6m_{pl}^2}(\rho + 3p) = \frac{a}{3m_{pl}^2}\Big(\big\langle \underset{\text{(Kin)}}{-2K_\phi} + \underset{\text{(Pot)}}{V} \big\rangle - \big\langle \underset{\text{(Elec)}}{K_A} + \underset{\text{(Mag)}}{G_A} \big\rangle\Big)$$

$$\begin{pmatrix} \pi_\phi \equiv \dot{\phi}\,, \quad E_i \equiv \dot{A}_i\,, \quad B_i \equiv \epsilon_{ijk}\partial_j A_k \\ \tilde{\pi}_\phi = a^3\pi_\phi\,, \quad \tilde{\vec{E}} = a\vec{E}\,, \quad \pi_a \equiv \dot{a} \end{pmatrix}$$

$$\begin{pmatrix} K_\phi \equiv \frac{1}{2}\pi_\phi^2 = \frac{1}{2a^6}\tilde{\pi}_\phi^2\,, \quad G_\phi \equiv \frac{1}{2a^2}\sum(\partial_i\phi)^2\,, \quad V \equiv \frac{1}{2}m^2\phi^2\,, \\ K_A \equiv \frac{1}{2}\sum_i \frac{E_i^2}{a^2} = \frac{1}{2}\sum_i \frac{\tilde{E}_i^2}{a^4}\,, \quad G_A \equiv \frac{1}{2}\sum_i \frac{B_i^2}{a^4} \end{pmatrix}$$

Axion-Inflation

PROBLEM: PNG, GW and PBH $\longrightarrow$ **Analytical approximations !**

$$\langle \vec{E}\vec{B}\rangle = -\frac{\lambda}{a^4}\int \frac{dk}{4\pi}k^3\frac{d}{d\tau}\left|A_{-\lambda}(\tau,\vec{k})\right|^2 \quad ;$$

$$\langle K_A + G_A\rangle \equiv \left\langle \frac{E^2+B^2}{2}\right\rangle = \frac{1}{a^4}\int\frac{dk}{4\pi^2}k^2\left(\left|\frac{dA_{-\lambda}(\tau,\vec{k})}{d\tau}\right|^2 + k^2\left|A_{-\lambda}(\tau,\vec{k})\right|^2\right) \quad ;$$

$$\lambda = \pm \quad \begin{cases} \lambda = +\,,\ \text{if}\,\dot\phi > 0 \\ \lambda = -\,,\ \text{if}\,\dot\phi < 0 \end{cases}$$

EoM

Local EoM $(\vec{\mathbf{x}},\mathbf{t})$

$$\begin{cases} \dot{\tilde{\pi}}_\phi = \cancel{a\vec{\nabla}^2\phi} - a^3m^2\phi + \dfrac{1}{a\Lambda}\langle\tilde{\vec{E}}\cdot\vec{B}\,\rangle \\[2ex] \dot{\tilde{\vec{E}}} = -\dfrac{1}{a}\vec{\nabla}\times\vec{B} - \dfrac{1}{a^3\Lambda}\tilde{\pi}_\phi\vec{B} + \cancel{\dfrac{1}{a\Lambda}\vec{\nabla}\phi\times\tilde{\vec{E}}}\,. \end{cases}$$

Hom. (t) Approx.

Back-Reaction (Homog. Approx.)

Hom. EoM $(\mathbf{t})$

$$\dot{\pi}_a = -\frac{a}{6m_{pl}^2}(\rho+3p) = \frac{a}{3m_{pl}^2}\Big(\big\langle -2K_\phi + V\big\rangle - \big\langle K_A + G_A\big\rangle\Big)$$

(Kin) (Pot) (Elec) (Mag)

$$\left(\begin{array}{l} \pi_\phi \equiv \dot\phi\,, \quad E_i \equiv \dot{A}_i\,, \quad B_i \equiv \epsilon_{ijk}\partial_j A_k \\ \tilde{\pi}_\phi = a^3\pi_\phi\,, \quad \tilde{\vec{E}} = a\vec{E}\,, \quad \pi_a \equiv \dot{a} \end{array}\right)$$

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Axion-Inflation

PROBLEM: PNG, GW and PBH ⟶ **Analytical approximations !**

$$\langle \vec{E}\vec{B} \rangle = -\frac{\lambda}{a^4}\int \frac{dk}{4\pi} k^3 \frac{d}{d\tau}\left|A_{-\lambda}(\tau,\vec{k})\right|^2 \quad ;$$

$$\lambda = \pm \;\begin{cases} \lambda = +\,,\ \text{if}\,\phi > 0 \\ \lambda = -\,,\ \text{if}\,\phi < 0 \end{cases}$$

$$\langle K_A + G_A \rangle \equiv \left\langle \frac{E^2+B^2}{2} \right\rangle = \frac{1}{a^4}\int \frac{dk}{4\pi^2} k^2 \left(\left|\frac{dA_{-\lambda}(\tau,\vec{k})}{d\tau}\right|^2 + k^2 \left|A_{-\lambda}(\tau,\vec{k})\right|^2 \right) \quad ;$$

EoM

Local EoM $(\vec{\mathbf{x}},\mathbf{t})$

$$\begin{cases} \dot{\tilde{\pi}}_\phi = \cancel{a\vec{\nabla}^2\phi} - a^3 m^2 \phi + \dfrac{1}{a\Lambda}\langle \tilde{\vec{E}} \cdot \vec{B} \rangle \\[2ex] \dfrac{d^2 A_\pm(\tau,\vec{k})}{d\tau^2} + \left[k^2 \pm 2\lambda\xi k a H\right] A_\pm(\tau,\vec{k}) = 0 \; \cancel{\dfrac{1}{a\Lambda}\vec{\nabla}\phi \times \tilde{\vec{E}}}\,. \end{cases}$$

Hom. (t) Approx.

Back-Reaction (Homog. Approx.)

Hom. EoM $(\mathbf{t})$

$$\dot{\pi}_a = -\frac{a}{6m_{pl}^2}(\rho + 3p) = \frac{a}{3m_{pl}^2}\Big(\langle -2K_\phi + V \rangle - \langle K_A + G_A \rangle\Big)$$

(Kin) (Pot) (Elec) (Mag)

$$\left(\begin{array}{l} \pi_\phi \equiv \dot{\phi}\,, \quad E_i \equiv \dot{A}_i\,, \quad B_i \equiv \epsilon_{ijk}\partial_j A_k \\ \tilde{\pi}_\phi = a^3 \pi_\phi\,, \quad \tilde{\vec{E}} = a\vec{E}\,, \quad \pi_a \equiv \dot{a} \end{array} \right)$$

$$\left(\begin{array}{c} K_\phi \equiv \frac{1}{2}\pi_\phi^2 = \frac{1}{2a^6}\tilde{\pi}_\phi^2\,, \quad G_\phi \equiv \frac{1}{2a^2}\sum(\partial_i\phi)^2\,, \quad V \equiv \frac{1}{2}m^2\phi^2\,, \\ K_A \equiv \frac{1}{2}\sum_i \frac{E_i^2}{a^2} = \frac{1}{2}\sum_i \frac{\tilde{E}_i^2}{a^4}\,, \quad G_A \equiv \frac{1}{2}\sum_i \frac{B_i^2}{a^4} \end{array} \right)$$

Axion-Inflation

PROBLEM: PNG, GW and PBH $\longrightarrow$ **Analytical approximations !**

$$\langle\vec{E}\vec{B}\rangle = -\frac{\lambda}{a^4}\int\frac{dk}{4\pi}k^3\frac{d}{d\tau}\left|A_{-\lambda}(\tau,\vec{k})\right|^2 \quad ;$$

$$\langle K_A + G_A\rangle \equiv \left\langle\frac{E^2+B^2}{2}\right\rangle = \frac{1}{a^4}\int\frac{dk}{4\pi^2}k^2\left(\left|\frac{dA_{-\lambda}(\tau,\vec{k})}{d\tau}\right|^2 + k^2\left|A_{-\lambda}(\tau,\vec{k})\right|^2\right) \quad ;$$

$\lambda = \pm$ → $\lambda = +\,,\ \text{if}\,\phi > 0$; $\lambda = -\,,\ \text{if}\,\phi < 0$

EoM

Local EoM $(\vec{\mathbf{x}},\mathbf{t})$

$$\begin{cases} \dot{\tilde{\pi}}_\phi = \cancel{a\vec{\nabla}^2\phi} - a^3m^2\phi + \dfrac{1}{a\Lambda}\langle\tilde{\vec{E}}\cdot\vec{B}\,\rangle \\[2ex] \dfrac{d^2A_\pm(\tau,\vec{k})}{d\tau^2} + \left[k^2 \pm 2\lambda\xi kaH\right]A_\pm(\tau,\vec{k}) = 0 \; \cancel{\dfrac{1}{a\Lambda}\vec{\nabla}\phi\times\tilde{\vec{E}}}\,. \end{cases}$$

Hom. (t) Approx.

Back-Reaction (Homog. Approx.)

Hom. EoM $(\mathbf{t})$

$$\dot{\pi}_a = -\frac{a}{6m_{pl}^2}(\rho+3p) = \frac{a}{3m_{pl}^2}\Big(\big\langle -2\underbrace{K_\phi}_{\text{(Kin)}} + \underbrace{V}_{\text{(Pot)}}\big\rangle - \big\langle\underbrace{K_A}_{\text{(Elec)}} + \underbrace{G_A}_{\text{(Mag)}}\big\rangle\Big)$$

DallAgata et al 2019, Domcke et 2020 $\longrightarrow$ **Elaborated Iterative scheme !**

Axion-Inflation

PROBLEM: PNG, GW and PBH → **Analytical approximations !**

$$\langle \vec{E}\vec{B} \rangle = -\frac{\lambda}{a^4}\int \frac{dk}{4\pi}k^3\frac{d}{d\tau}\left|A_{-\lambda}(\tau,\vec{k})\right|^2 \quad ;$$

$$\langle K_A + G_A \rangle \equiv \left\langle \frac{E^2+B^2}{2}\right\rangle = \frac{1}{a^4}\int \frac{dk}{4\pi^2}k^2\left(\left|\frac{dA_{-\lambda}(\tau,\vec{k})}{d\tau}\right|^2 + k^2\left|A_{-\lambda}(\tau,\vec{k})\right|^2\right) \quad ;$$

$\lambda = \pm$ → $\lambda = +\,,\text{if}\,\dot\phi > 0$; $\lambda = -\,,\text{if}\,\dot\phi < 0$

EoM

Local EoM $(\vec{\mathbf{x}},\mathbf{t})$

$$\begin{cases} \dot{\tilde{\pi}}_\phi = \cancel{a\vec{\nabla}^2\phi} - a^3 m^2\phi + \dfrac{1}{a\Lambda}\langle\tilde{\vec{E}}\cdot\vec{B}\,\rangle \\[2ex] \dfrac{d^2A_\pm(\tau,\vec{k})}{d\tau^2} + \left[k^2 \pm 2\lambda\xi k a H\right]A_\pm(\tau,\vec{k}) = 0 \;\cancel{\dfrac{1}{a\Lambda}\vec{\nabla}\phi\times\tilde{\vec{E}}}\,. \end{cases}$$

Hom. (t) Approx.

Back-Reaction (Homog. Approx.)

Hom. EoM $(\mathbf{t})$

$$\dot{\pi}_a = -\frac{a}{6m_{pl}^2}(\rho+3p) = \frac{a}{3m_{pl}^2}\Big(\langle -2K_\phi + V\rangle - \langle K_A + G_A\rangle\Big)$$

(Kin) (Pot) (Elec) (Mag)

Gorbar et al 2021 → **Correlator Gradient Expansion**

Axion-Inflation

PROBLEM: PNG, GW and PBH $\longrightarrow$ **Analytical approximations !**

Can we do better than homogeneous backreaction ?

EoM

Local EoM $(\vec{\mathbf{x}}, \mathbf{t})$

$$
\begin{cases}
\dot{\tilde{\pi}}_\phi = \cancel{a\vec{\nabla}^2\phi} - a^3 m^2\phi + \dfrac{1}{a\Lambda}\langle \tilde{\vec{E}}\cdot\vec{B}\,\rangle \\[2ex]
\dfrac{d^2 A_\pm(\tau,\vec{k})}{d\tau^2} + \left[k^2 \pm 2\lambda\xi k a H\right] A_\pm(\tau,\vec{k}) = 0 \,\cancel{\dfrac{1}{a\Lambda}\vec{\nabla}\phi\times\tilde{\vec{E}}}\,.
\end{cases}
$$

Hom. (t) Approx.

Back-Reaction (Homog. Approx.)

Hom. EoM $(\mathbf{t})$

$$
\dot{\pi}_a = -\frac{a}{6m_{pl}^2}(\rho + 3p) = \frac{a}{3m_{pl}^2}\Big(\big\langle \underbrace{-2K_\phi}_{\text{(Kin)}} + \underbrace{V}_{\text{(Pot)}}\big\rangle - \big\langle \underbrace{K_A}_{\text{(Elec)}} + \underbrace{G_A}_{\text{(Mag)}}\big\rangle\Big)
$$

Axion-Inflation

PROBLEM: PNG, GW and PBH → **Analytical approximations !**

Yes, we need a full lattice approach

EoM

Local EoM $(\vec{\mathbf{x}}, \mathbf{t})$

$$\begin{cases} \dot{\tilde{\pi}}_\phi = \cancel{a\vec{\nabla}^2\phi} - a^3 m^2 \phi + \dfrac{1}{a\Lambda}\tilde{\vec{E}}\cdot\vec{B} \\ \dfrac{d^2 A_\pm(\tau,\vec{k})}{d\tau^2} + \left[k^2 \pm 2\lambda\xi k a H\right] A_\pm(\tau,\vec{k}) = 0 \;\; \cancel{\dfrac{1}{a\Lambda}\vec{\nabla}\phi\times\tilde{\vec{E}}}\,. \end{cases}$$

Back-Reaction (Source InHom.)

Hom. EoM $(\mathbf{t})$

$$\dot{\pi}_a = -\frac{a}{6m_{pl}^2}(\rho+3p) = \frac{a}{3m_{pl}^2}\Big\langle \underbrace{-2K_\phi}_{\text{(Kin)}} + \underbrace{V}_{\text{(Pot)}} - \underbrace{K_A}_{\text{(Elec)}} - \underbrace{G_A}_{\text{(Mag)}} \Big\rangle$$

Axion-Inflation

PROBLEM: PNG, GW and PBH → **Analytical approximations !**

Yes, we need a full lattice approach

EoM

Local EoM $(\vec{\mathbf{x}}, \mathbf{t})$

$$\begin{cases} \dot{\tilde{\pi}}_\phi = \cancel{a\vec{\nabla}^2\phi} - a^3 m^2 \phi + \dfrac{1}{a\Lambda}\vec{\tilde{E}} \cdot \vec{B} \\ \dot{\vec{\tilde{E}}} = -\dfrac{1}{a}\vec{\nabla} \times \vec{B} - \dfrac{1}{a^3\Lambda}\tilde{\pi}_\phi \vec{B} + \cancel{\dfrac{1}{a\Lambda}\vec{\nabla}\phi \times \vec{\tilde{E}}}\,. \end{cases}$$

Back-Reaction (Source InHom.)

Hom. EoM $(\mathbf{t})$

$$\dot{\pi}_a = -\frac{a}{6m_{pl}^2}(\rho + 3p) = \frac{a}{3m_{pl}^2}\Big\langle \underset{\text{(Kin)}}{-2K_\phi} + \underset{\text{(Pot)}}{V} - \underset{\text{(Elec)}}{K_A} - \underset{\text{(Mag)}}{G_A} \Big\rangle$$

Axion-Inflation

PROBLEM: PNG, GW and PBH → **Analytical approximations !**

Yes, we need a full lattice approach

EoM

Local EoM $(\vec{\mathbf{x}}, \mathbf{t})$

$$\begin{cases} \dot{\tilde{\pi}}_\phi = a\vec{\nabla}^2\phi - a^3 m^2 \phi + \dfrac{1}{a\Lambda}\tilde{\vec{E}} \cdot \vec{B} \\[2ex] \dot{\tilde{\vec{E}}} = -\dfrac{1}{a}\vec{\nabla} \times \vec{B} - \dfrac{1}{a^3\Lambda}\tilde{\pi}_\phi \vec{B} + \dfrac{1}{a\Lambda}\vec{\nabla}\phi \times \tilde{\vec{E}} \,. \end{cases}$$

Back-Reaction (Fully InHom.)

Hom. EoM $(\mathbf{t})$

$$\dot{\pi}_a = -\frac{a}{6m_{pl}^2}(\rho + 3p) = \frac{a}{3m_{pl}^2}\Big\langle \underbrace{-2K_\phi}_{\text{(Kin)}} + \underbrace{V}_{\text{(Pot)}} - (\underbrace{K_A}_{\text{(Elec)}} + \underbrace{G_A}_{\text{(Mag)}})\Big\rangle$$

Axion-Inflation

PROBLEM: PNG, GW and PBH $\longrightarrow$ **Analytical approximations !**

How to "latticize" this system of EOM ?

EoM

Local EoM $(\vec{\mathbf{x}}, \mathbf{t})$

$$\begin{cases} \dot{\tilde{\pi}}_\phi = a\vec{\nabla}^2\phi - a^3 m^2\phi + \dfrac{1}{a\Lambda}\vec{\tilde{E}}\cdot\vec{B} \\ \dot{\vec{\tilde{E}}} = -\dfrac{1}{a}\vec{\nabla}\times\vec{B} - \dfrac{1}{a^3\Lambda}\tilde{\pi}_\phi\vec{B} + \dfrac{1}{a\Lambda}\vec{\nabla}\phi\times\vec{\tilde{E}}\,. \end{cases}$$

Back-Reaction (Fully InHom.)

Hom. EoM $(\mathbf{t})$

$$\dot{\pi}_a = -\frac{a}{6m_{pl}^2}(\rho + 3p) = \frac{a}{3m_{pl}^2}\langle -2K_\phi + V - (K_A + G_A)\rangle$$

(Kin) (Pot) (Elec) (Mag)

LATTICE FORMULATION of $\phi F\tilde{F}$

EoM

Inflaton EoM

$$\Delta_0^+\left(a^3\pi_\phi\right) = a_{+\frac{\hat{0}}{2}}\sum_i \Delta_i^-\Delta_i^+\phi_{+\frac{\hat{0}}{2}} - a^3_{+\frac{\hat{0}}{2}} m^2\phi_{+\frac{\hat{0}}{2}} + \frac{1}{\Lambda}\sum_i \frac{1}{2}E^{(2)}_{i,+\frac{\hat{0}}{2}}\left(B_i^{(4)} + B^{(4)}_{i,+\hat{0}}\right)$$

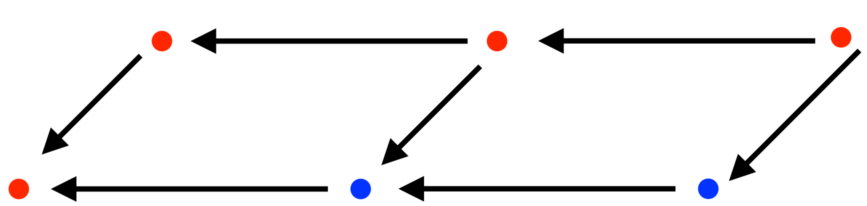
Gauge Fld EoM

$$\Delta_0^-\left(a_{+\frac{\hat{0}}{2}}E_{i,+\frac{\hat{0}}{2}}\right) = -\frac{1}{a}\sum_{j,k}\epsilon_{ijk}\Delta_j^- B_k - \frac{1}{2\Lambda}\left(\pi_\phi B_i^{(4)} + \pi_{\phi,+i}B^{(4)}_{i,+i}\right)$$
$$+\frac{1}{8\Lambda}(2 + dx\Delta_i^+)\sum_\pm\sum_{j,k}\left\{\epsilon_{ijk}[(\Delta_j^\pm\phi)E^{(2)}_{k,\pm j}]_{+\frac{\hat{0}}{2}} + [(\Delta_j^\pm\phi)E^{(2)}_{k,\pm j}]_{-\frac{\hat{0}}{2}}\right\}$$

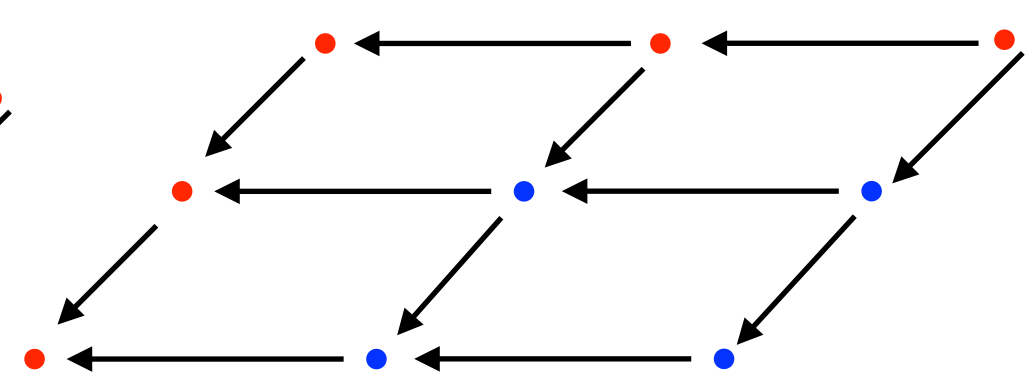
$$a_{+\frac{\hat{0}}{2}}\sum_i \Delta_i^- E_{i,+\frac{\hat{0}}{2}} = -\frac{1}{4\Lambda}\sum_\pm\sum_i\left(\Delta_i^\pm\phi_{+\frac{\hat{0}}{2}}\right)\left(B_i^{(4)} + B^{(4)}_{i,+\hat{0}}\right)_{\pm i} \quad , \quad \text{(Gauss Law)}$$

DGF, Shaposhnikov 2017
Canivete, DGF 2018

$E_i^{(2)}$



$B_i^{(4)}$



Lattice gauge techniques

LATTICE FORMULATION of $\phi F\tilde{F}$

EoM

Inflaton EoM

$$\Delta_0^+\left(a^3\pi_\phi\right) = a_{+\frac{\hat{0}}{2}}\sum_i \Delta_i^-\Delta_i^+\phi_{+\frac{\hat{0}}{2}} - a^3_{+\frac{\hat{0}}{2}}m^2\phi_{+\frac{\hat{0}}{2}} + \frac{1}{\Lambda}\sum_i \frac{1}{2}E^{(2)}_{i,+\frac{\hat{0}}{2}}\left(B_i^{(4)} + B^{(4)}_{i,+\hat{0}}\right)$$

Gauge Fld EoM

$$\Delta_0^-\left(a_{+\frac{\hat{0}}{2}}E_{i,+\frac{\hat{0}}{2}}\right) = -\frac{1}{a}\sum_{j,k}\epsilon_{ijk}\Delta_j^-B_k - \frac{1}{2\Lambda}\left(\pi_\phi B_i^{(4)} + \pi_{\phi,+i}B^{(4)}_{i,+i}\right)$$
$$+\frac{1}{8\Lambda}(2+dx\Delta_i^+)\sum_\pm\sum_{j,k}\left\{\epsilon_{ijk}[(\Delta_j^\pm\phi)E^{(2)}_{k,\pm j}]_{+\frac{\hat{0}}{2}} + [(\Delta_j^\pm\phi)E^{(2)}_{k,\pm j}]_{-\frac{\hat{0}}{2}}\right\}$$

$$a_{+\frac{\hat{0}}{2}}\sum_i \Delta_i^-E_{i,+\frac{\hat{0}}{2}} = -\frac{1}{4\Lambda}\sum_\pm\sum_i\left(\Delta_i^\pm\phi_{+\frac{\hat{0}}{2}}\right)\left(B_i^{(4)} + B^{(4)}_{i,+\hat{0}}\right)_{\pm i}\ ,$$ (Gauss Law)

DGF, Shaposhnikov 2017
Canivete, DGF 2018

1. Lattice Gauge Inv: $A_\mu \longrightarrow A_\mu + \Delta_\mu^+\alpha$
2. Cont. Limit to $\mathcal{O}(dx^2)$
3. Lattice Bianchi Identities: $\Delta_i^-(B_i^{(4)} + B^{(4)}_{i,+\hat{0}}) = 0\,, \ldots$
4. Topological Term: $(F_{\mu\nu}\tilde{F}^{\mu\nu})_L = \Delta_\mu^+K^\mu$ (**CS current**)
   $[\,F_{\mu\nu}\tilde{F}^{\mu\nu} = \partial_\mu K^\mu\,]$

Exact Shift Sym. on the lattice !

LATTICE FORMULATION of $\phi F\tilde{F}$

EoM

Inflaton EoM

$$\Delta_0^+\left(a^3\pi_\phi\right) = a_{+\frac{\hat{0}}{2}}\sum_i \Delta_i^-\Delta_i^+\phi_{+\frac{\hat{0}}{2}} - a^3_{+\frac{\hat{0}}{2}}m^2\phi_{+\frac{\hat{0}}{2}} + \frac{1}{\Lambda}\sum_i \frac{1}{2}E^{(2)}_{i,+\frac{\hat{0}}{2}}\left(B_i^{(4)} + B_{i,+\hat{0}}^{(4)}\right)$$

Gauge Fld EoM

$$\Delta_0^-\left(a_{+\frac{\hat{0}}{2}}E_{i,+\frac{\hat{0}}{2}}\right) = -\frac{1}{a}\sum_{j,k}\epsilon_{ijk}\Delta_j^-B_k - \frac{1}{2\Lambda}\left(\pi_\phi B_i^{(4)} + \pi_{\phi,+i}B_{i,+i}^{(4)}\right)$$
$$+\frac{1}{8\Lambda}(2+dx\Delta_i^+)\sum_\pm\sum_{j,k}\left\{\epsilon_{ijk}[(\Delta_j^\pm\phi)E^{(2)}_{k,\pm j}]_{+\frac{\hat{0}}{2}} + [(\Delta_j^\pm\phi)E^{(2)}_{k,\pm j}]_{-\frac{\hat{0}}{2}}\right\}$$

$$a_{+\frac{\hat{0}}{2}}\sum_i \Delta_i^- E_{i,+\frac{\hat{0}}{2}} = -\frac{1}{4\Lambda}\sum_\pm\sum_i\left(\Delta_i^\pm\phi_{+\frac{\hat{0}}{2}}\right)\left(B_i^{(4)} + B_{i,+\hat{0}}^{(4)}\right)_{\pm i}\,,\quad \text{(Gauss Law)}$$

DGF, Shaposhnikov 2017
Canivete, DGF 2018

$$\dot{\pi}_\phi = -3H\pi_\phi + \frac{1}{a^2}\vec{\nabla}^2\phi - m^2\phi + \frac{1}{a^3\Lambda}\vec{E}\cdot\vec{B},$$

$$\dot{\vec{E}} = -H\vec{E} - \frac{1}{a^2}\vec{\nabla}\times\vec{B} - \frac{1}{a\Lambda}\pi_\phi\vec{B} + \frac{1}{a\Lambda}\vec{\nabla}\phi\times\vec{E}$$

$$\vec{\nabla}\cdot\vec{E} = -\frac{1}{a\Lambda}\vec{\nabla}\phi\cdot\vec{B}\ \text{(Gauss Law)}$$

EoM Continuum

LATTICE FORMULATION of $\phi F\tilde{F}$

EoM

Inflaton EoM

$$\Delta_0^+\left(a^3\pi_\phi\right) = a_{+\frac{\hat{0}}{2}}\sum_i \Delta_i^-\Delta_i^+\phi_{+\frac{\hat{0}}{2}} - a^3_{+\frac{\hat{0}}{2}}m^2\phi_{+\frac{\hat{0}}{2}} + \frac{1}{\Lambda}\sum_i \frac{1}{2}E^{(2)}_{i,+\frac{\hat{0}}{2}}\left(B_i^{(4)} + B^{(4)}_{i,+\hat{0}}\right)$$

Gauge Fld EoM

$$\Delta_0^-\left(a_{+\frac{\hat{0}}{2}}E_{i,+\frac{\hat{0}}{2}}\right) = -\frac{1}{a}\sum_{j,k}\epsilon_{ijk}\Delta_j^- B_k - \frac{1}{2\Lambda}\left(\pi_\phi B_i^{(4)} + \pi_{\phi,+i}B^{(4)}_{i,+i}\right)$$
$$+\frac{1}{8\Lambda}(2 + dx\Delta_i^+)\sum_{\pm}\sum_{j,k}\left\{\epsilon_{ijk}[(\Delta_j^\pm\phi)E^{(2)}_{k,\pm j}]_{+\frac{\hat{0}}{2}} + [(\Delta_j^\pm\phi)E^{(2)}_{k,\pm j}]_{-\frac{\hat{0}}{2}}\right\}$$

$$a_{+\frac{\hat{0}}{2}}\sum_i \Delta_i^- E_{i,+\frac{\hat{0}}{2}} = -\frac{1}{4\Lambda}\sum_{\pm}\sum_i\left(\Delta_i^\pm\phi_{+\frac{\hat{0}}{2}}\right)\left(B_i^{(4)} + B^{(4)}_{i,+\hat{0}}\right)_{\pm i} \quad ,$$ (Gauss Law)

DGF, Shaposhnikov 2017
Canivete, DGF 2018

$$\dot{\pi}_\phi = -3H\pi_\phi + \frac{1}{a^2}\vec{\nabla}^2\phi - m^2\phi + \frac{1}{a^3\Lambda}\vec{E}\cdot\vec{B},$$

$$\dot{\vec{E}} = -H\vec{E} - \frac{1}{a^2}\vec{\nabla}\times\vec{B} - \frac{1}{a\Lambda}\pi_\phi\vec{B} + \frac{1}{a\Lambda}\vec{\nabla}\phi\times\vec{E}$$

$$\vec{\nabla}\cdot\vec{E} = -\frac{1}{a\Lambda}\vec{\nabla}\phi\cdot\vec{B}$$ (Gauss Law)

EoM Continuum

LATTICE FORMULATION of $\phi F\tilde{F}$

EoM

Inflaton EoM

$$\Delta_0^+\left(a^3\pi_\phi\right) = a_{+\frac{\hat{0}}{2}}\sum_i \Delta_i^-\Delta_i^+\phi_{+\frac{\hat{0}}{2}} - a^3_{+\frac{\hat{0}}{2}}m^2\phi_{+\frac{\hat{0}}{2}} + \frac{1}{\Lambda}\sum_i \frac{1}{2}E^{(2)}_{i,+\frac{\hat{0}}{2}}\left(B_i^{(4)} + B^{(4)}_{i,+\hat{0}}\right)$$

Gauge Fld EoM

$$\Delta_0^-\left(a_{+\frac{\hat{0}}{2}}E_{i,+\frac{\hat{0}}{2}}\right) = -\frac{1}{a}\sum_{j,k}\epsilon_{ijk}\Delta_j^- B_k - \frac{1}{2\Lambda}\underline{\left(\pi_\phi B_i^{(4)} + \pi_{\phi,+i}B^{(4)}_{i,+i}\right)}$$

$$+\frac{1}{8\Lambda}(2 + dx\Delta_i^+)\sum_{\pm}\sum_{j,k}\left\{\epsilon_{ijk}[(\Delta_j^\pm\phi)E^{(2)}_{k,\pm j}]_{+\frac{\hat{0}}{2}} + [(\Delta_j^\pm\phi)E^{(2)}_{k,\pm j}]_{-\frac{\hat{0}}{2}}\right\}$$

$$a_{+\frac{\hat{0}}{2}}\sum_i \Delta_i^- E_{i,+\frac{\hat{0}}{2}} = -\frac{1}{4\Lambda}\sum_{\pm}\sum_i\left(\Delta_i^\pm\phi_{+\frac{\hat{0}}{2}}\right)\left(B_i^{(4)} + B^{(4)}_{i,+\hat{0}}\right)_{\pm i}\ ,$$ (Gauss Law)

DGF, Shaposhnikov 2017
Canivete, DGF 2018

$$\dot{\pi}_\phi = -3H\pi_\phi + \frac{1}{a^2}\vec{\nabla}^2\phi - m^2\phi + \frac{1}{a^3\Lambda}\vec{E}\cdot\vec{B},$$

$$\dot{\vec{E}} = -H\vec{E} - \frac{1}{a^2}\vec{\nabla}\times\vec{B} - \underline{\frac{1}{a\Lambda}\pi_\phi\vec{B}} + \frac{1}{a\Lambda}\vec{\nabla}\phi\times\vec{E}$$

$$\vec{\nabla}\cdot\vec{E} = -\frac{1}{a\Lambda}\vec{\nabla}\phi\cdot\vec{B}$$ (Gauss Law)

EoM Continuum

LATTICE FORMULATION of $\phi F\tilde{F}$

EoM

Inflaton EoM

$$\Delta_0^+\left(a^3\pi_\phi\right) = a_{+\frac{\hat{0}}{2}}\sum_i \Delta_i^-\Delta_i^+\phi_{+\frac{\hat{0}}{2}} - a^3_{+\frac{\hat{0}}{2}}m^2\phi_{+\frac{\hat{0}}{2}} + \frac{1}{\Lambda}\sum_i \frac{1}{2}E^{(2)}_{i,+\frac{\hat{0}}{2}}\left(B_i^{(4)} + B^{(4)}_{i,+\hat{0}}\right)$$

Gauge Fld EoM

$$\Delta_0^-\left(a_{+\frac{\hat{0}}{2}}E_{i,+\frac{\hat{0}}{2}}\right) = -\frac{1}{a}\sum_{j,k}\epsilon_{ijk}\Delta_j^- B_k - \frac{1}{2\Lambda}\left(\pi_\phi B_i^{(4)} + \pi_{\phi,+i}B^{(4)}_{i,+i}\right)$$
$$+\frac{1}{8\Lambda}(2 + dx\Delta_i^+)\sum_\pm\sum_{j,k}\left\{\epsilon_{ijk}[(\Delta_j^\pm\phi)E^{(2)}_{k,\pm j}]_{+\frac{\hat{0}}{2}} + [(\Delta_j^\pm\phi)E^{(2)}_{k,\pm j}]_{-\frac{\hat{0}}{2}}\right\}$$

$$a_{+\frac{\hat{0}}{2}}\sum_i \Delta_i^- E_{i,+\frac{\hat{0}}{2}} = -\frac{1}{4\Lambda}\sum_\pm\sum_i\left(\Delta_i^\pm\phi_{+\frac{\hat{0}}{2}}\right)\left(B_i^{(4)} + B^{(4)}_{i,+\hat{0}}\right)_{\pm i} \;,$$ (Gauss Law)

DGF, Shaposhnikov 2017
Canivete, DGF 2018

$$\dot{\pi}_\phi = -3H\pi_\phi + \frac{1}{a^2}\vec{\nabla}^2\phi - m^2\phi + \frac{1}{a^3\Lambda}\vec{E}\cdot\vec{B},$$
$$\dot{\vec{E}} = -H\vec{E} - \frac{1}{a^2}\vec{\nabla}\times\vec{B} - \frac{1}{a\Lambda}\pi_\phi\vec{B} + \frac{1}{a\Lambda}\vec{\nabla}\phi\times\vec{E}$$

$$\vec{\nabla}\cdot\vec{E} = -\frac{1}{a\Lambda}\vec{\nabla}\phi\cdot\vec{B}$$ (Gauss Law)

EoM Continuum

LATTICE FORMULATION of $\phi F \tilde{F}$

Lattice Formulation

EoM

$$
\begin{aligned}
\Delta_0^+\left(a^3\pi_\phi\right) &= a_{+\frac{\hat{0}}{2}}\sum_i \Delta_i^-\Delta_i^+\phi_{+\frac{\hat{0}}{2}} - a^3_{+\frac{\hat{0}}{2}} m^2 \phi_{+\frac{\hat{0}}{2}} \\
&\quad + \frac{1}{\Lambda}\sum_i \frac{1}{2} E^{(2)}_{i,+\frac{\hat{0}}{2}}\left(B_i^{(4)} + B^{(4)}_{i,+\hat{0}}\right), \\
\Delta_0^-\left(a_{+\frac{\hat{0}}{2}} E_{i,+\frac{\hat{0}}{2}}\right) &= -\frac{1}{a}\sum_{j,k}\epsilon_{ijk}\Delta_j^- B_k - \frac{1}{2\Lambda}\left(\pi_\phi B_i^{(4)} + \pi_{\phi,+i} B^{(4)}_{i,+i}\right) \\
&\quad + \frac{1}{8\Lambda}(2 + dx\Delta_i^+)\sum_{\pm}\sum_{j,k}\left\{\epsilon_{ijk}[(\Delta_j^\pm\phi)E^{(2)}_{k,\pm j}]_{+\frac{\hat{0}}{2}} + [(\Delta_j^\pm\phi)E^{(2)}_{k,\pm j}]_{-\frac{\hat{0}}{2}}\right\} \\
a_{+\frac{\hat{0}}{2}}\sum_i \Delta_i^- E_{i,+\frac{\hat{0}}{2}} &= -\frac{1}{4\Lambda}\sum_{\pm}\sum_i\left(\Delta_i^\pm\phi_{+\frac{\hat{0}}{2}}\right)\left(B_i^{(4)} + B^{(4)}_{i,+\hat{0}}\right)_{\pm i}, \quad \text{(Gauss Law)}
\end{aligned}
$$

Expansion

$$
\left(\Delta_0^+ a_{-\hat{0}/2}\right)^2 = \frac{a^2}{3m_{\rm pl}^2}\rho_L\,,
$$

$$
\Delta_0^-\Delta_0^+ a_{+\hat{0}/2} = -\frac{a_{+\hat{0}/2}}{6m_{\rm pl}^2}(\rho_L + 3p_L)_{+\hat{0}/2}
$$

LATTICE FORMULATION of $\phi F\tilde{F}$

Lattice Formulation

EoM

$$\begin{aligned}
\Delta_0^+\left(a^3\pi_\phi\right) &= a_{+\frac{\hat{0}}{2}}\sum_i \Delta_i^-\Delta_i^+\phi_{+\frac{\hat{0}}{2}} - a^3_{+\frac{\hat{0}}{2}} m^2\phi_{+\frac{\hat{0}}{2}} \\
&\quad + \frac{1}{\Lambda}\sum_i \frac{1}{2}E^{(2)}_{i,+\frac{\hat{0}}{2}}\left(B_i^{(4)} + B^{(4)}_{i,+\hat{0}}\right), \\
\Delta_0^-\left(a_{+\frac{\hat{0}}{2}}E_{i,+\frac{\hat{0}}{2}}\right) &= -\frac{1}{a}\sum_{j,k}\epsilon_{ijk}\Delta_j^- B_k - \frac{1}{2\Lambda}\left(\pi_\phi B_i^{(4)} + \pi_{\phi,+i}B^{(4)}_{i,+i}\right) \\
&\quad + \frac{1}{8\Lambda}(2 + dx\Delta_i^+)\sum_\pm\sum_{j,k}\left\{\epsilon_{ijk}[(\Delta_j^\pm\phi)E^{(2)}_{k,\pm j}]_{+\frac{\hat{0}}{2}} + [(\Delta_j^\pm\phi)E^{(2)}_{k,\pm j}]_{-\frac{\hat{0}}{2}}\right\} \\
a_{+\frac{\hat{0}}{2}}\sum_i \Delta_i^- E_{i,+\frac{\hat{0}}{2}} &= -\frac{1}{4\Lambda}\sum_\pm\sum_i\left(\Delta_i^\pm\phi_{+\frac{\hat{0}}{2}}\right)\left(B_i^{(4)} + B^{(4)}_{i,+\hat{0}}\right)_{\pm i}, \quad \text{(Gauss Law)}
\end{aligned}$$

Expansion

$$\left(\Delta_0^+ a_{-\hat{0}/2}\right)^2 = \frac{a^2}{3m_{\rm pl}^2}\rho_L\,,$$

$$\Delta_0^-\Delta_0^+ a_{+\hat{0}/2} = -\frac{a_{+\hat{0}/2}}{6m_{\rm pl}^2}(\rho_L + 3p_L)_{+\hat{0}/2}$$

$$\rho_L = \bar{H}^{\rm kin} + \frac{1}{a^2}\frac{1}{2}(\bar{H}^{\rm grad}_{-\hat{0}/2} + \bar{H}^{\rm grad}_{+\hat{0}/2}) + \frac{1}{2}(\bar{H}^{\rm pot}_{-\hat{0}/2} + \bar{H}^{\rm pot}_{+\hat{0}/2}) + \frac{1}{a^2}\frac{1}{2}(\bar{H}^{E}_{-\hat{0}/2} + \bar{H}^{E}_{+\hat{0}/2}) + \frac{1}{a^4}\bar{H}^B\,,$$

$$(\rho_L + 3p_L)_{+\hat{0}/2} = 2(\bar{H}^{\rm kin} + \bar{H}^{\rm kin}_{+\hat{0}}) - 2\bar{H}^{\rm pot}_{+\hat{0}/2} + \frac{2}{a^2_{+\hat{0}/2}}\bar{H}^E + \frac{1}{a^4_{+\hat{0}/2}}(\bar{H}^B + \bar{H}^B_{+\hat{0}})\,,$$

$$\left(\begin{aligned}
&\bar{H}^{\rm kin} = \left\langle\frac{1}{N^3}\sum_{\vec{n}}\frac{\pi_\phi^2}{2}\right\rangle \quad \bar{H}^{\rm grad} = \left\langle\frac{1}{N^3}\sum_{\vec{n}}\frac{1}{2}\sum_i(\Delta_i^+\phi_{+\frac{\hat{0}}{2}})^2\right\rangle, \quad \bar{H}^{\rm pot} = \left\langle\frac{1}{N^3}\sum_{\vec{n}}\frac{1}{2}m^2\phi^2_{+\frac{\hat{0}}{2}}\right\rangle \\
&\bar{H}^{E} = \left\langle\frac{1}{N^3}\sum_{\vec{n}}\sum_i\frac{1}{2}E^2_{i,+\frac{\hat{0}}{2}}\right\rangle \quad \bar{H}^{B} = \left\langle\frac{1}{N^3}\sum_{\vec{n}}\sum_{i,j}\frac{1}{4}(\Delta_i^+A_j - \Delta_j^+A_i)^2\right\rangle
\end{aligned}\right)$$

LATTICE FORMULATION of $\phi F\tilde{F}$

Lattice Formulation

EoM

$$
\begin{aligned}
\Delta_0^+\left(a^3\pi_\phi\right) &= a_{+\frac{\hat{0}}{2}}\sum_i \Delta_i^-\Delta_i^+\phi_{+\frac{\hat{0}}{2}} - a^3_{+\frac{\hat{0}}{2}}m^2\phi_{+\frac{\hat{0}}{2}} \\
&\quad +\frac{1}{\Lambda}\sum_i \frac{1}{2}E^{(2)}_{i,+\frac{\hat{0}}{2}}\left(B_i^{(4)}+B^{(4)}_{i,+\hat{0}}\right), \\
\Delta_0^-\left(a_{+\frac{\hat{0}}{2}}E_{i,+\frac{\hat{0}}{2}}\right) &= -\frac{1}{a}\sum_{j,k}\epsilon_{ijk}\Delta_j^-B_k - \frac{1}{2\Lambda}\left(\pi_\phi B_i^{(4)} + \pi_{\phi,+i}B^{(4)}_{i,+i}\right) \\
&\quad +\frac{1}{8\Lambda}(2+dx\Delta_i^+)\sum_{\pm}\sum_{j,k}\left\{\epsilon_{ijk}[(\Delta_j^\pm\phi)E^{(2)}_{k,\pm j}]_{+\frac{\hat{0}}{2}} + [(\Delta_j^\pm\phi)E^{(2)}_{k,\pm j}]_{-\frac{\hat{0}}{2}}\right\} \\
a_{+\frac{\hat{0}}{2}}\sum_i \Delta_i^- E_{i,+\frac{\hat{0}}{2}} &= -\frac{1}{4\Lambda}\sum_{\pm}\sum_i\left(\Delta_i^\pm\phi_{+\frac{\hat{0}}{2}}\right)\left(B_i^{(4)}+B^{(4)}_{i,+\hat{0}}\right)_{\pm i}, \quad \text{(Gauss Law)}
\end{aligned}
$$

Expansion

$$
\begin{aligned}
\left(\Delta_0^+ a_{-\hat{0}/2}\right)^2 &= \frac{a^2}{3m_{\rm pl}^2}\rho_L\,, \\
\Delta_0^-\Delta_0^+ a_{+\hat{0}/2} &= -\frac{a_{+\hat{0}/2}}{6m_{\rm pl}^2}(\rho_L+3p_L)_{+\hat{0}/2}
\end{aligned}
$$

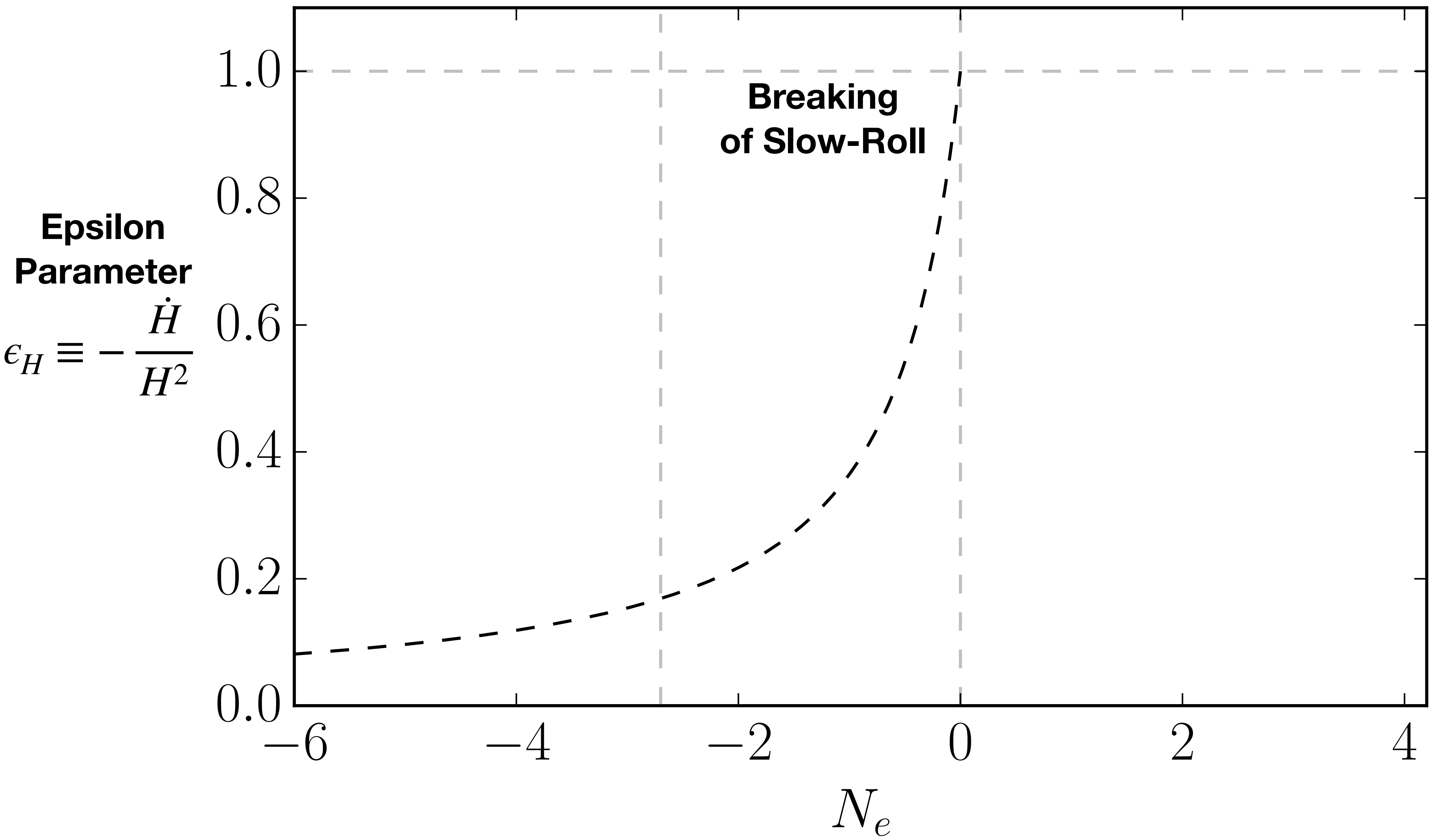
* Now I will show you our **work in progress** from
DGF, Lizarraga, Urio & Urrestilla 2022

* Alternative Lattice formulation (not shift-symmetric): Caravano et al 2022

$$V(\phi) = \frac{1}{2}m^2\phi^2 \ ; \ \frac{\phi}{4\Lambda}F\tilde{F} \ ; \ \Lambda = \frac{m_p}{20}$$

Axion-Inflation($V(\phi) = \frac{1}{2}m^2\phi^2$; $\frac{\phi}{4\Lambda}F\tilde{F}$; $\Lambda = \frac{m_p}{20}$)

Linear regime (– – –)



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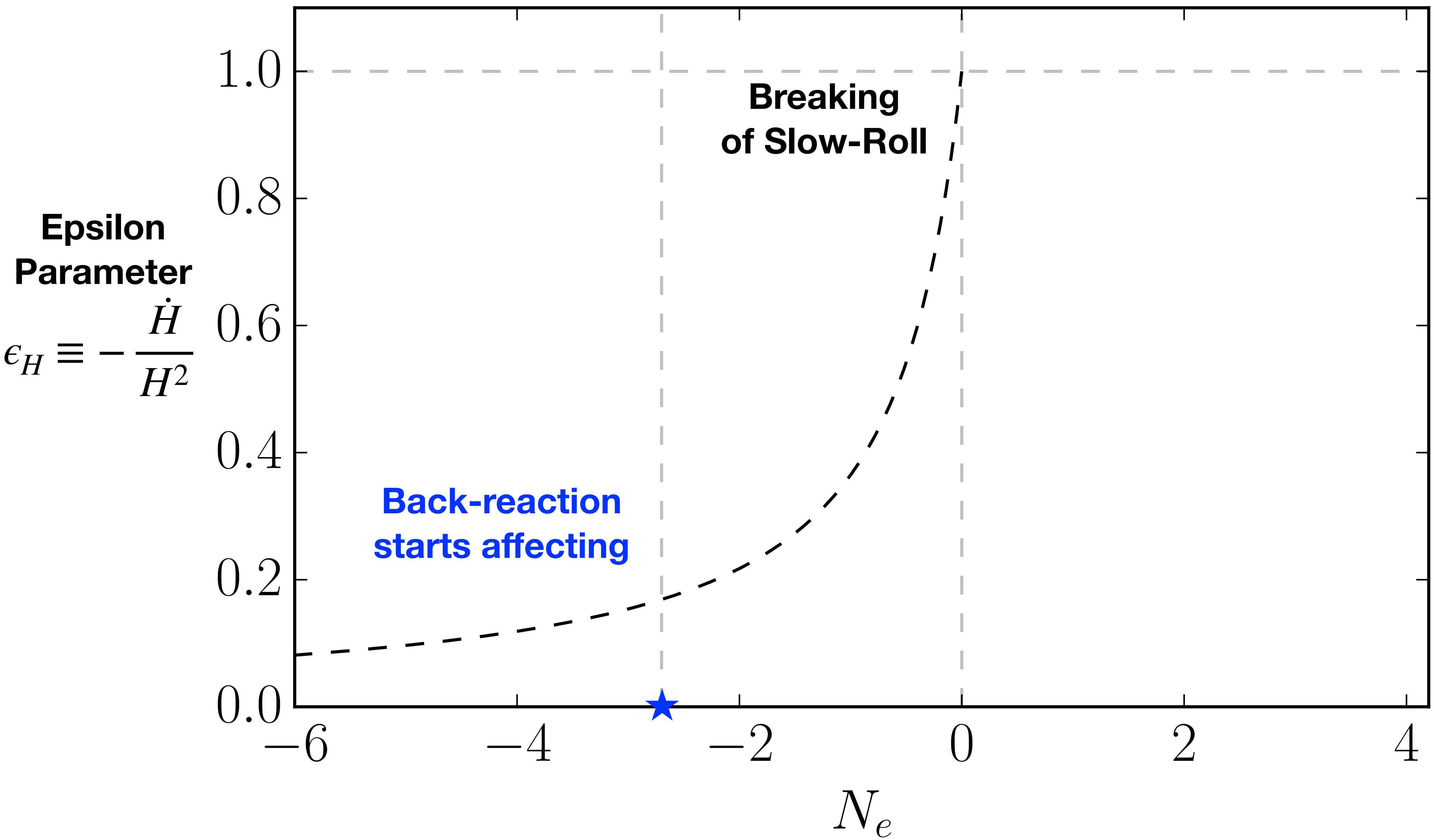
Linear regime (– – –)

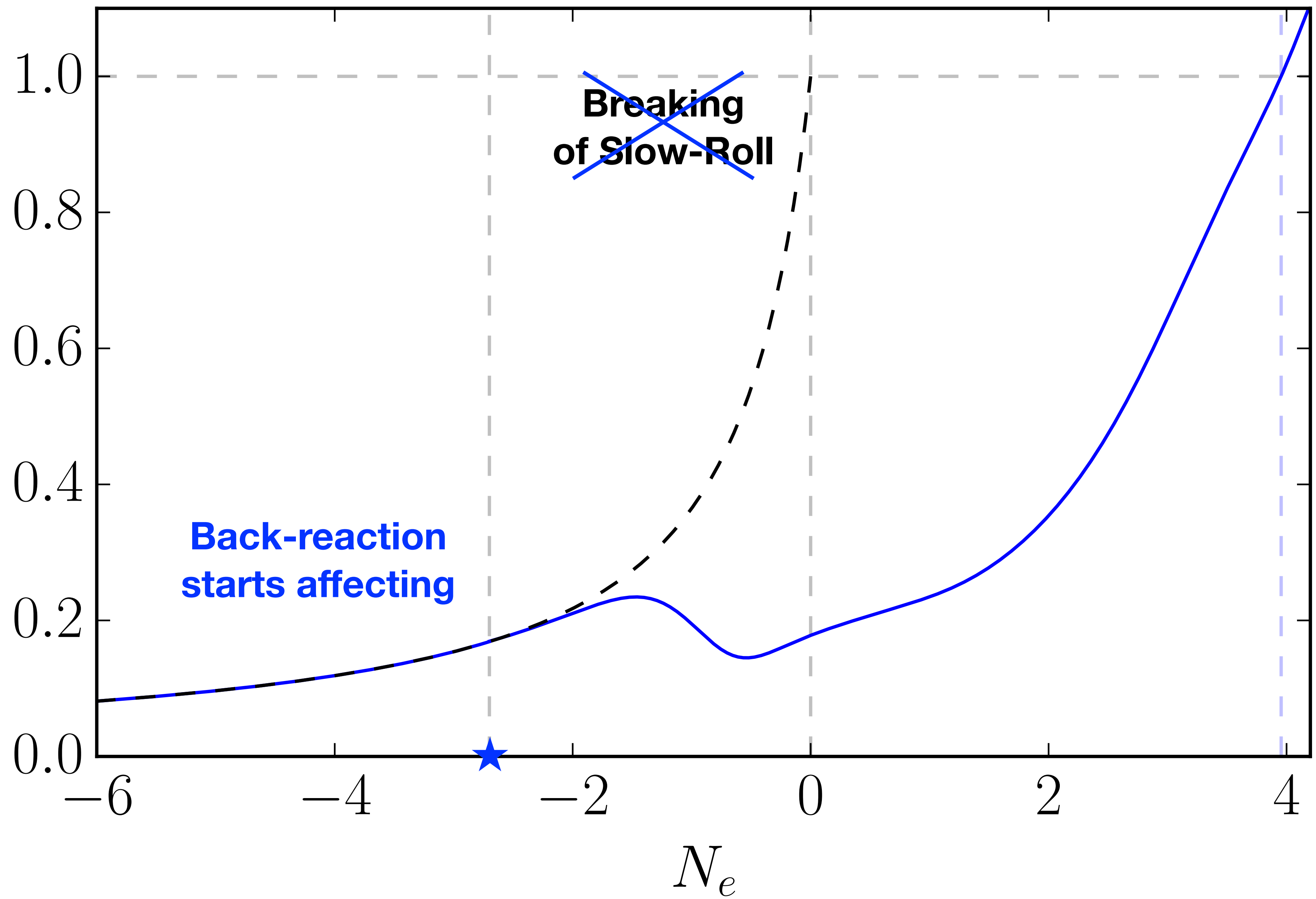


Axion-Inflation($V(\phi) = \frac{1}{2}m^2\phi^2\ ;\ \frac{\phi}{4\Lambda}F\tilde{F}\ ;\ \Lambda = \frac{m_p}{20}$)

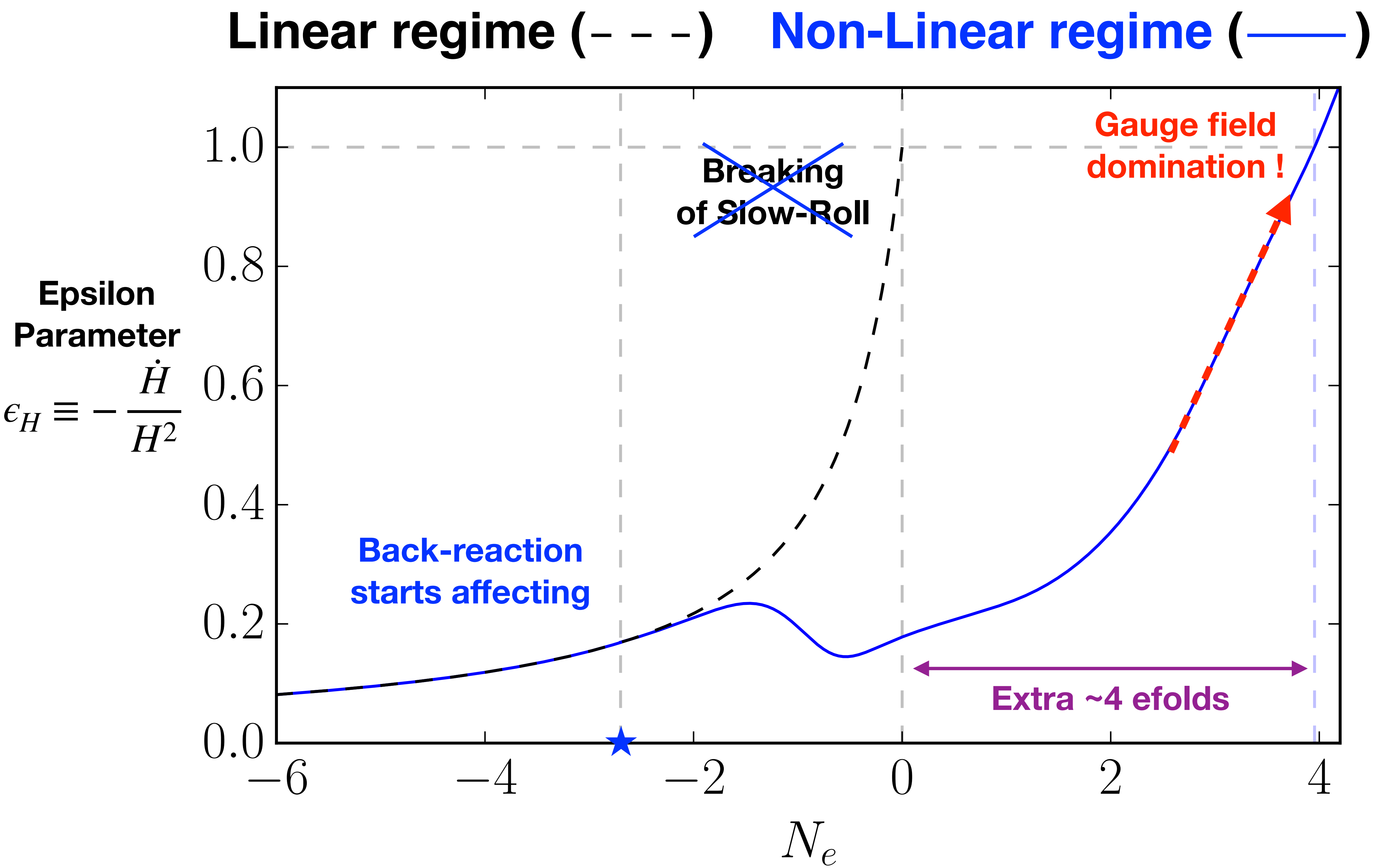
Linear regime (- - -) Non-Linear regime (——)

Epsilon Parameter

$\epsilon_H \equiv -\frac{\dot{H}}{H^2}$



Axion-Inflation($V(\phi) = \frac{1}{2}m^2\phi^2 \; ; \; \frac{\phi}{4\Lambda}F\tilde{F} \; ; \; \Lambda = \frac{m_p}{20}$)



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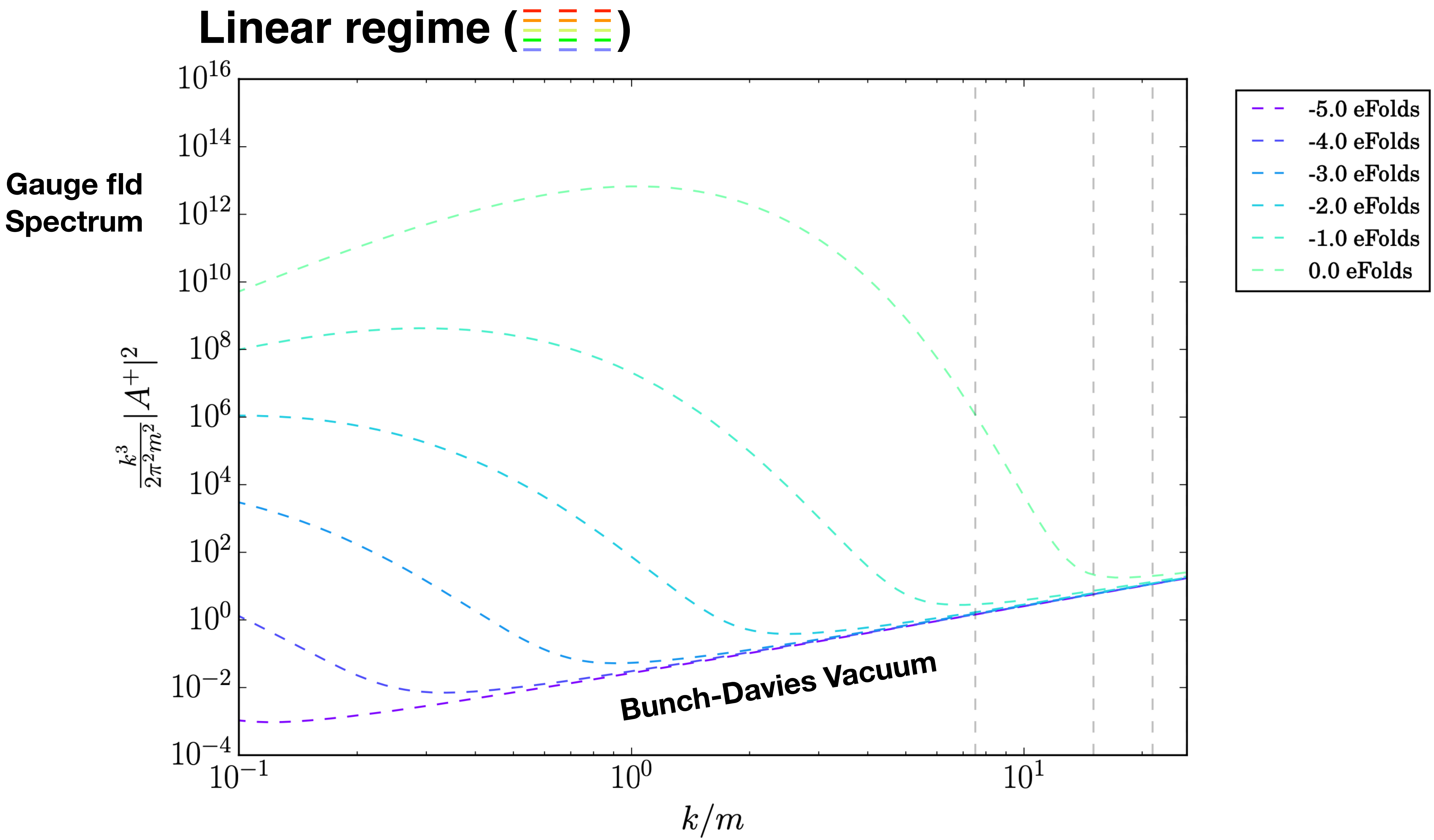

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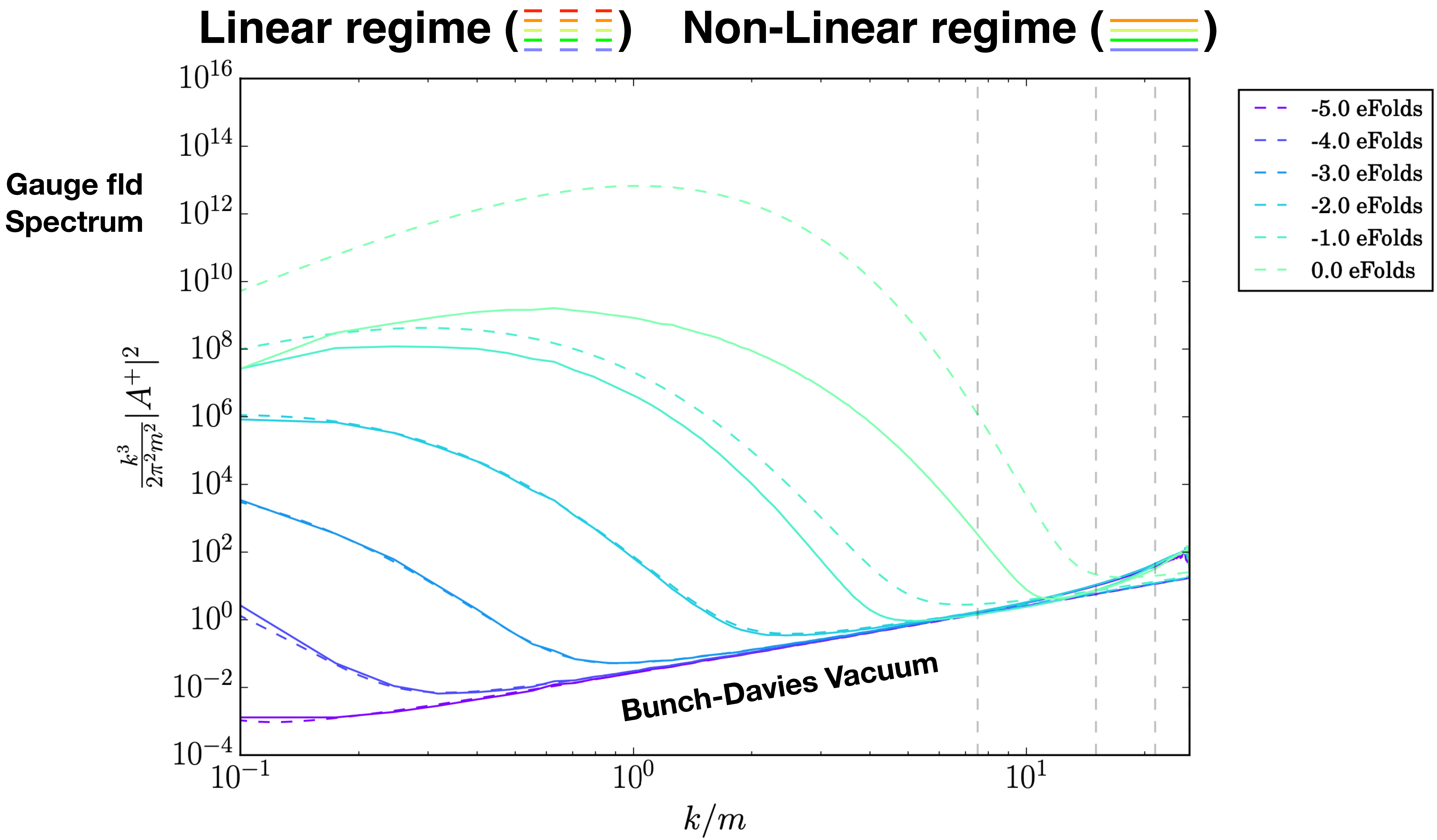

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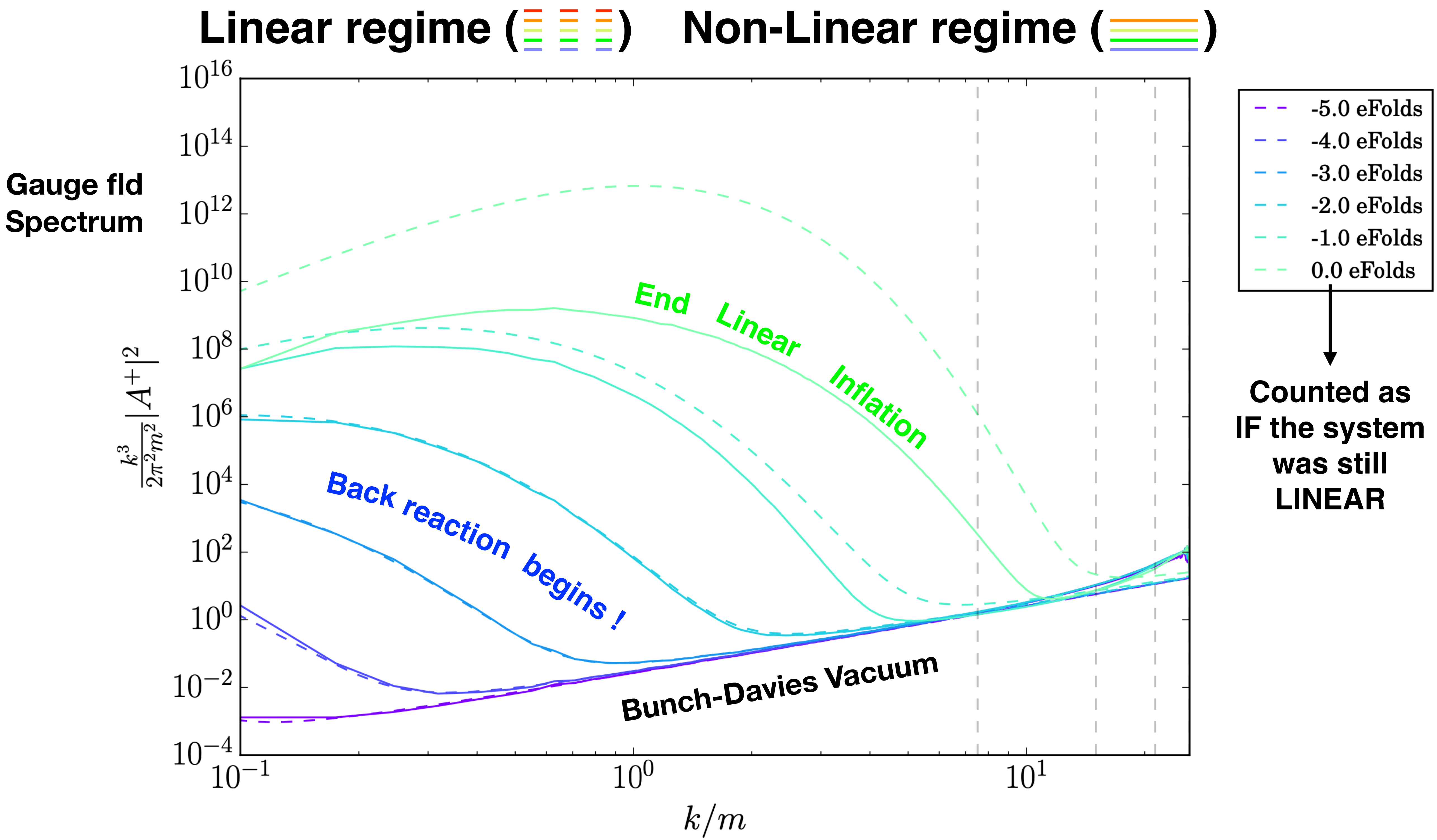

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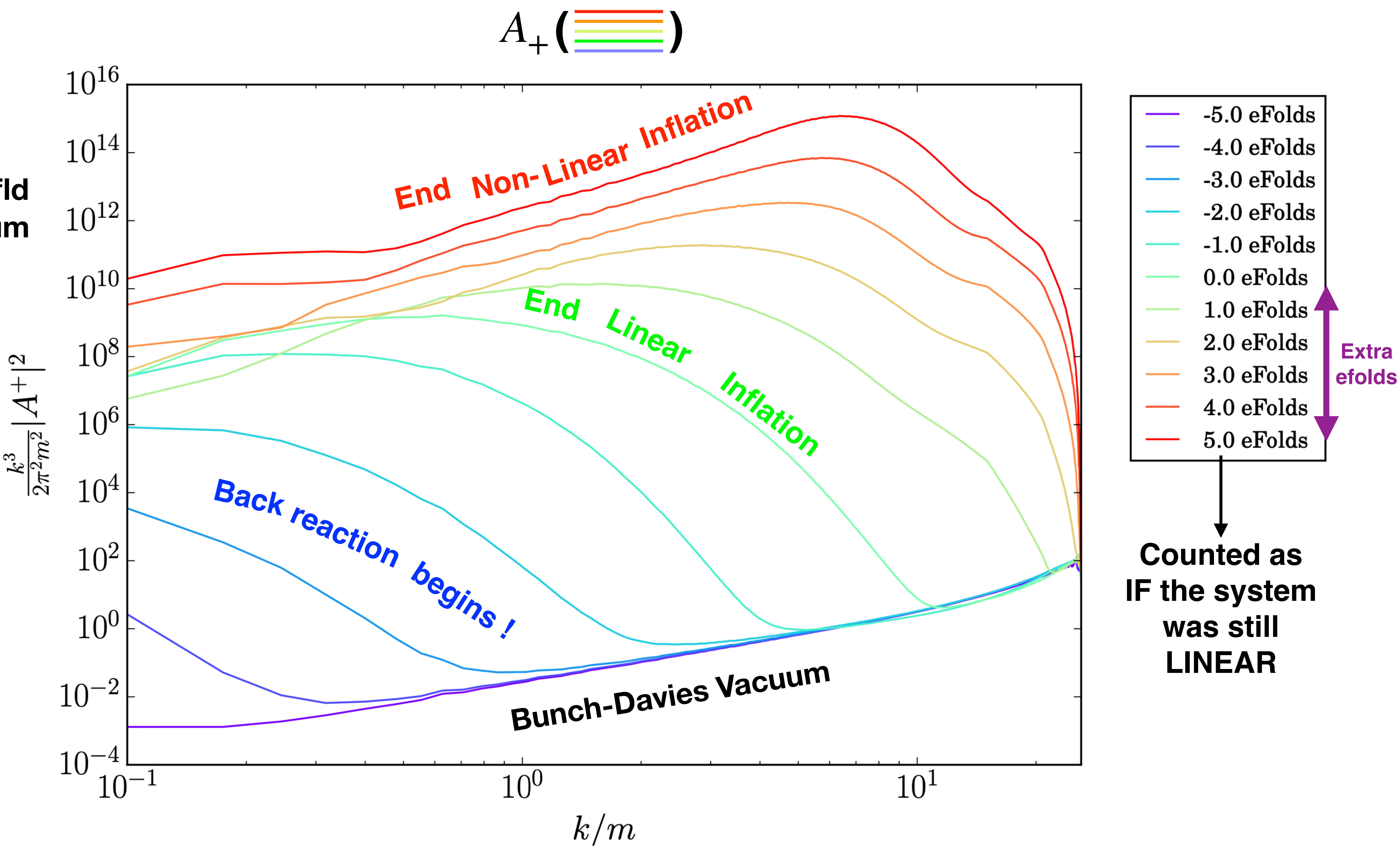
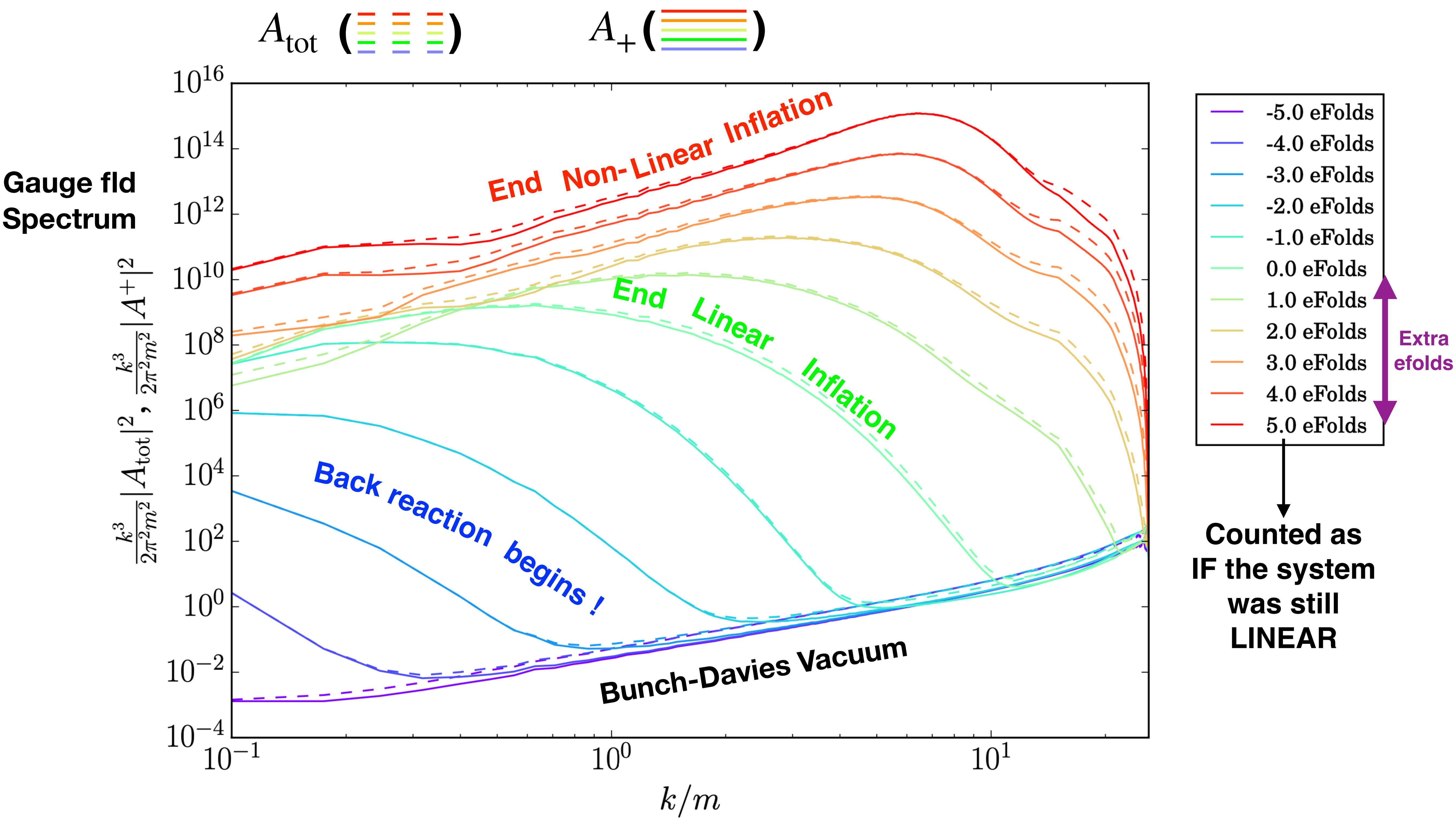
Gauge fld Spectrum



Counted as IF the system was still LINEAR

Axion-Inflation($V(\phi) = \frac{1}{2}m^2\phi^2 \; ; \; \frac{\phi}{4\Lambda}F\tilde{F} \; ; \; \Lambda = \frac{m_p}{20}$)



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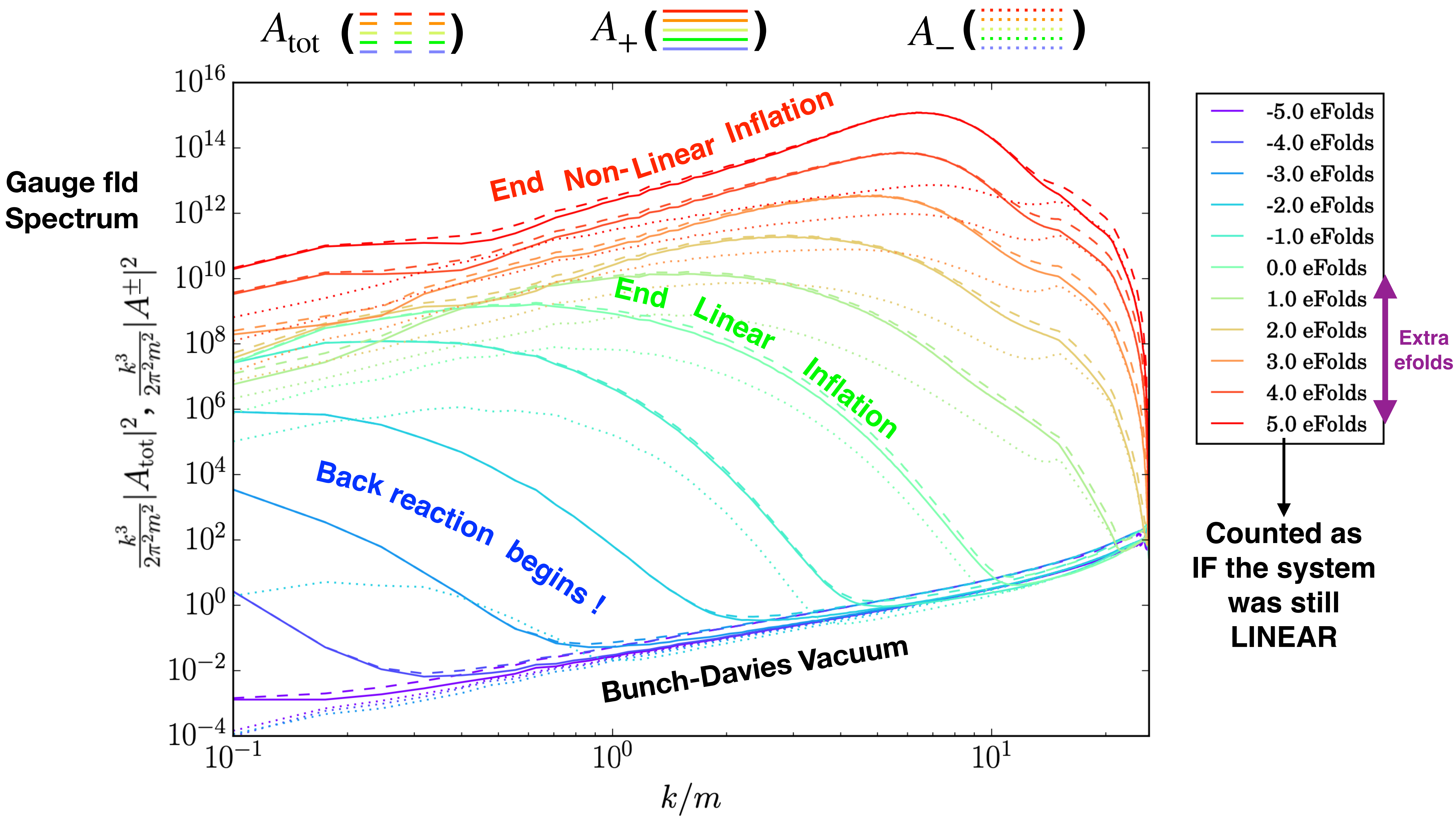

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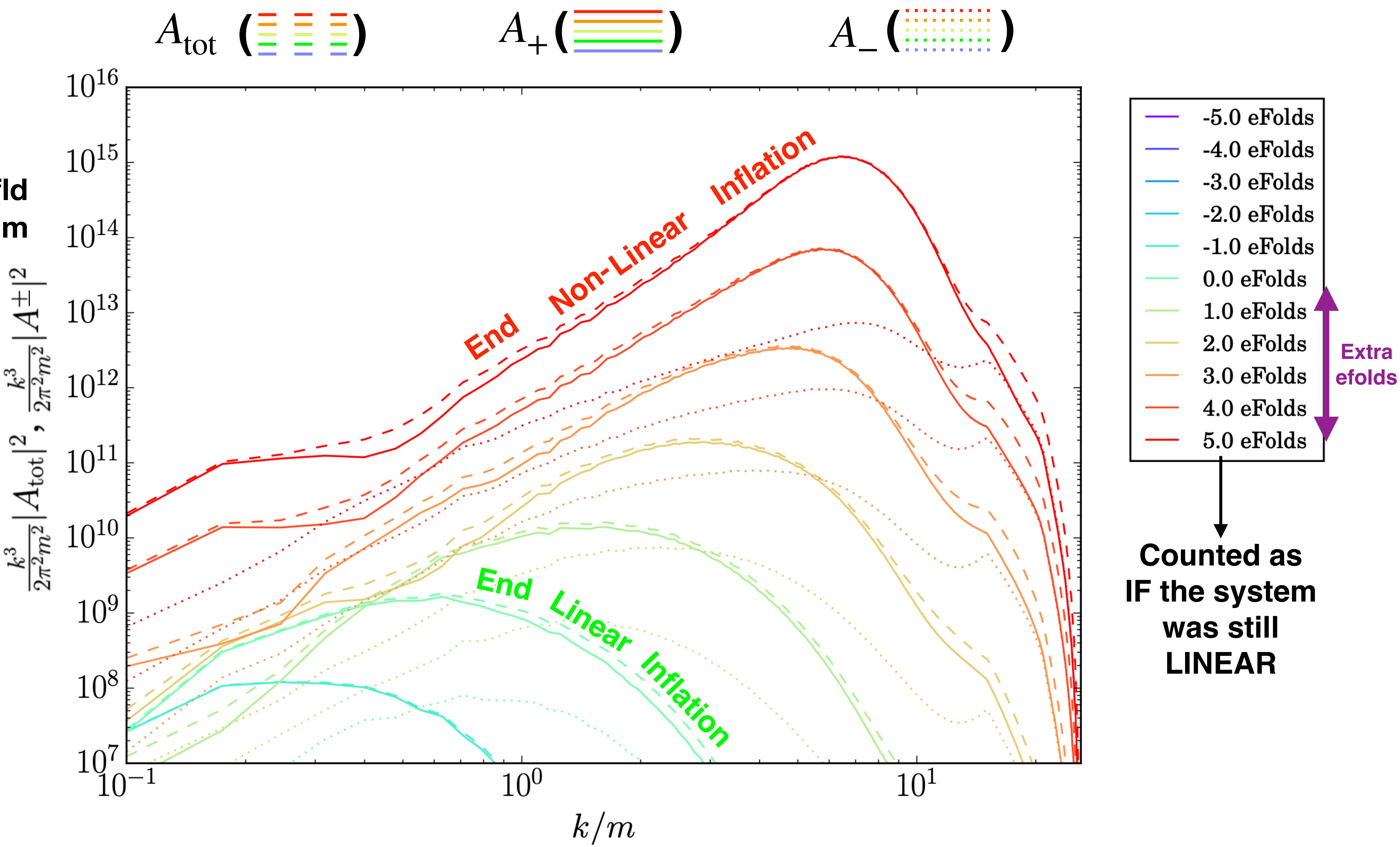
Gauge fld Spectrum



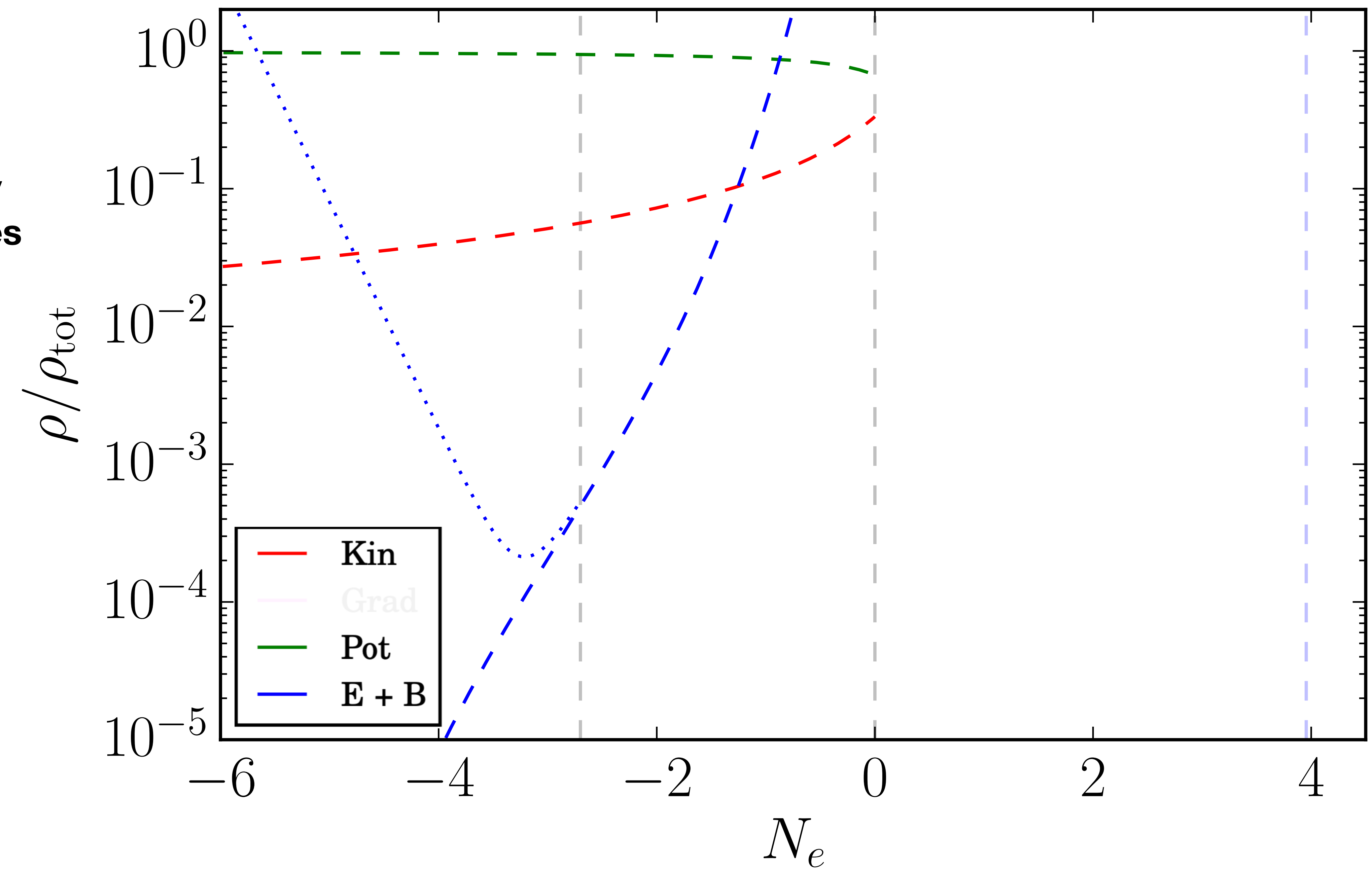
Counted as IF the system was still LINEAR

Zoom

Axion-Inflation($V(\phi) = \frac{1}{2}m^2\phi^2$; $\frac{\phi}{4\Lambda}F\tilde{F}$; $\Lambda = \frac{m_p}{20}$)

Linear (– – –)

Energy Densities



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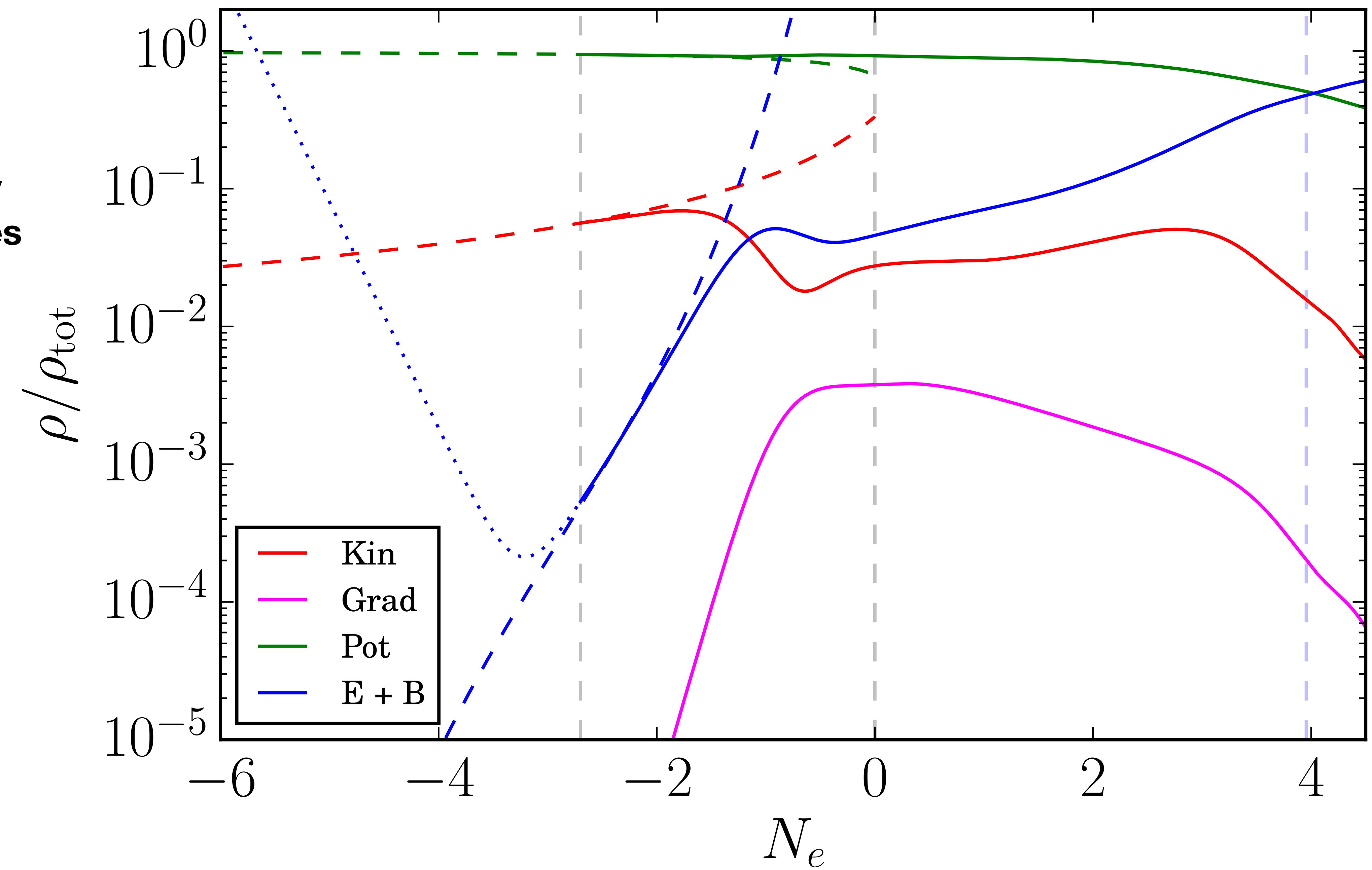
Linear (– – –) **Non-Linear (——)**

Energy Densities



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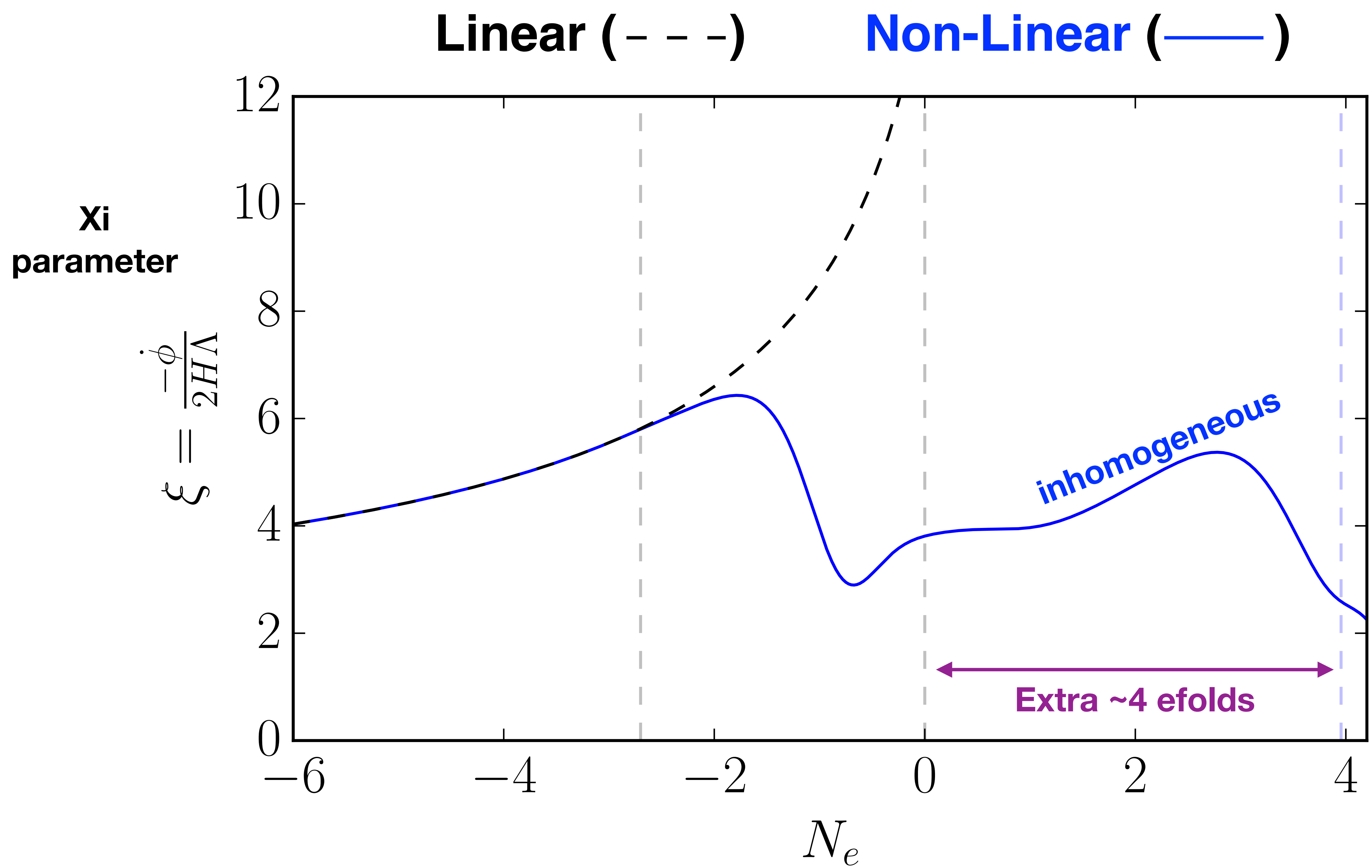

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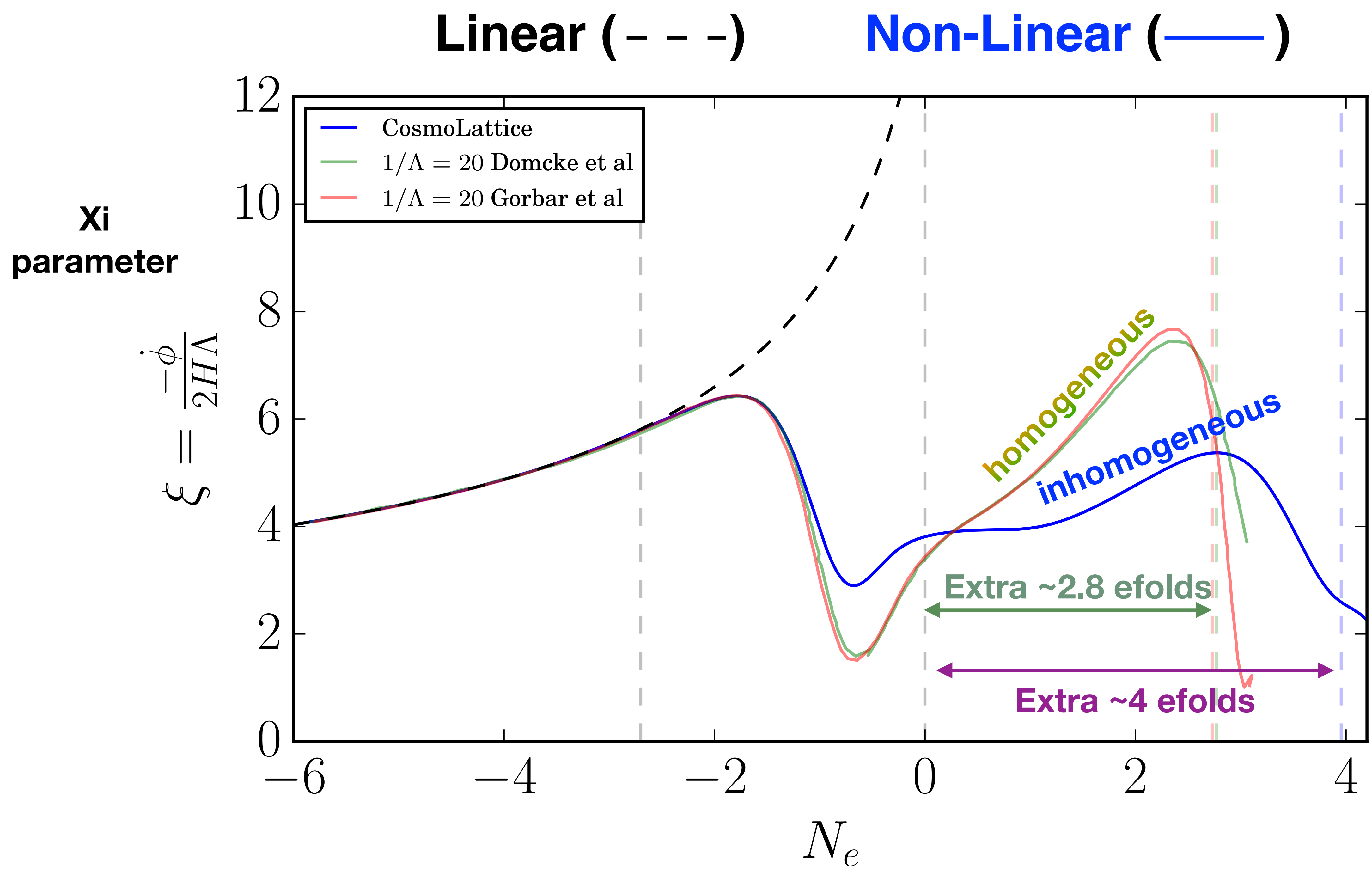
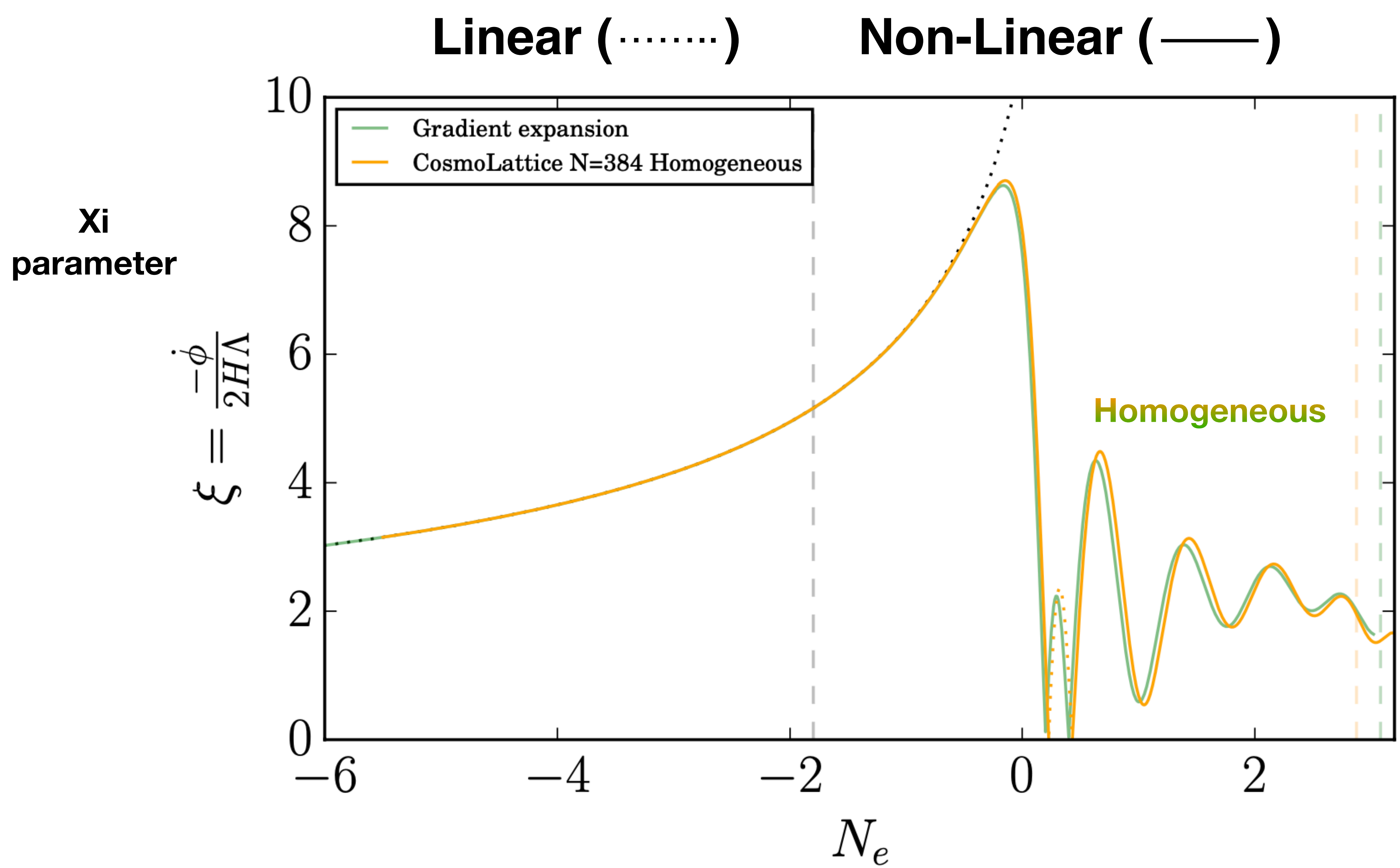


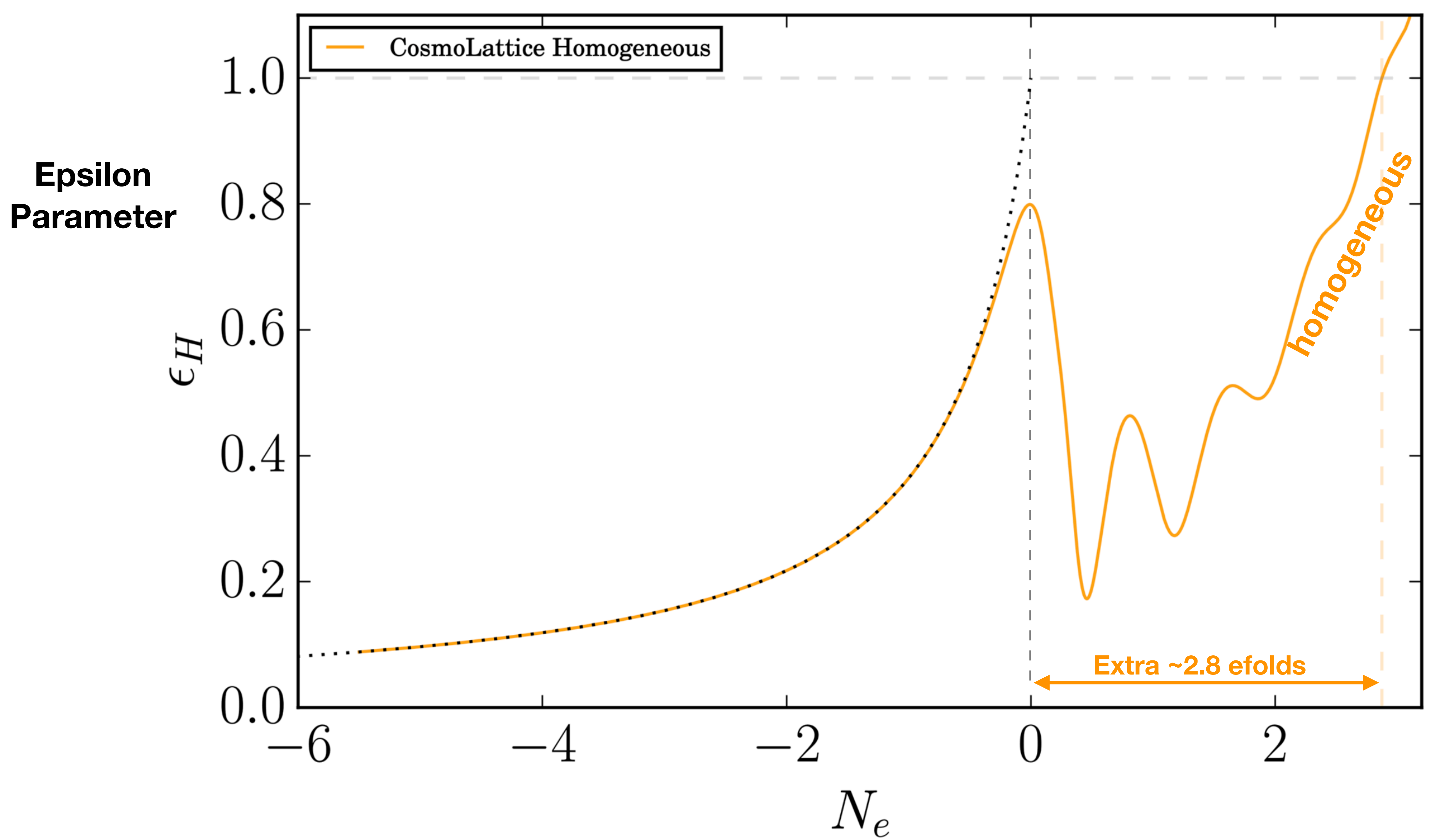
$$V(\phi) = \frac{1}{2}m^2\phi^2 \; ; \; \frac{\phi}{4\Lambda}F\tilde{F} \; ; \boxed{\Lambda = \frac{m_p}{15}}$$

Axion-Inflation($V(\phi) = \frac{1}{2}m^2\phi^2$; $\frac{\phi}{4\Lambda}F\tilde{F}$; $\Lambda = \frac{m_p}{15}$)



Axion-Inflation($V(\phi) = \frac{1}{2}m^2\phi^2$; $\frac{\phi}{4\Lambda}F\tilde{F}$; $\Lambda = \frac{m_p}{15}$)

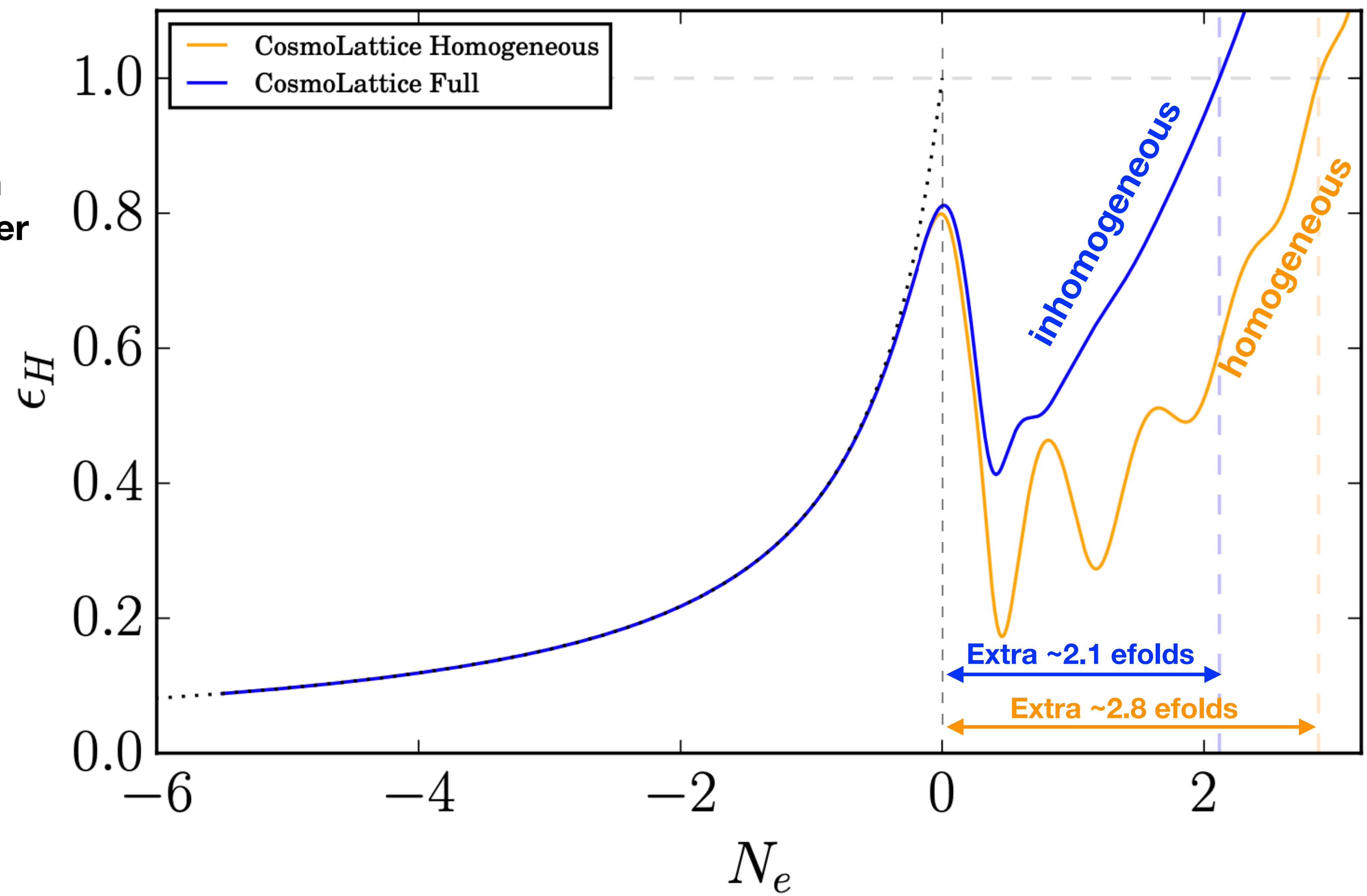
Linear (········) **Non-Linear (——)**



Axion-Inflation$\left(V(\phi) = \frac{1}{2}m^2\phi^2 \;;\; \frac{\phi}{4\Lambda}F\tilde{F} \;;\; \Lambda = \frac{m_p}{15}\right)$

Linear (········) **Non-Linear (——)**

Epsilon Parameter



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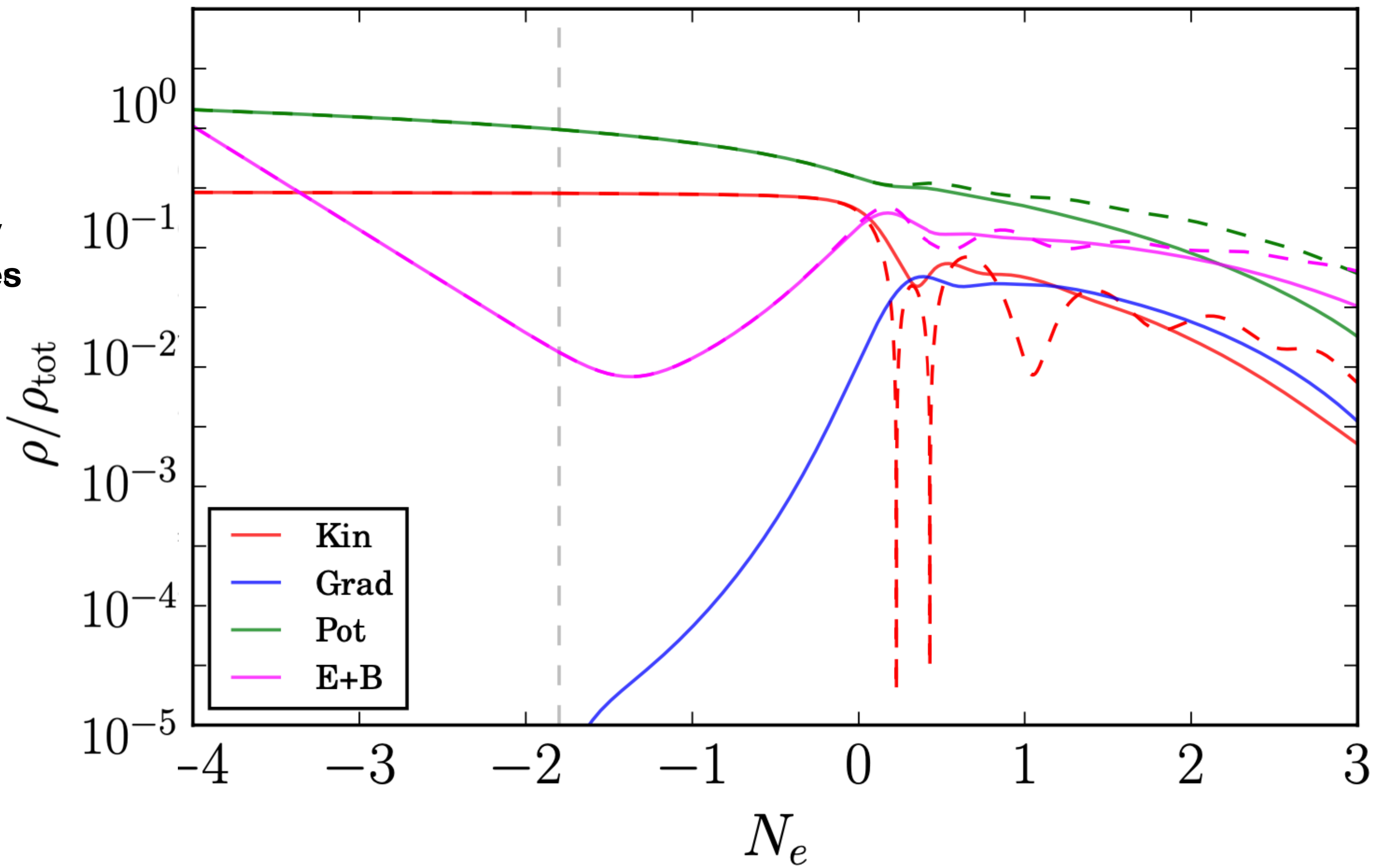
Homogeneous backreaction (- - -) **Inhomogeneous backreaction** (——)

Energy Densities



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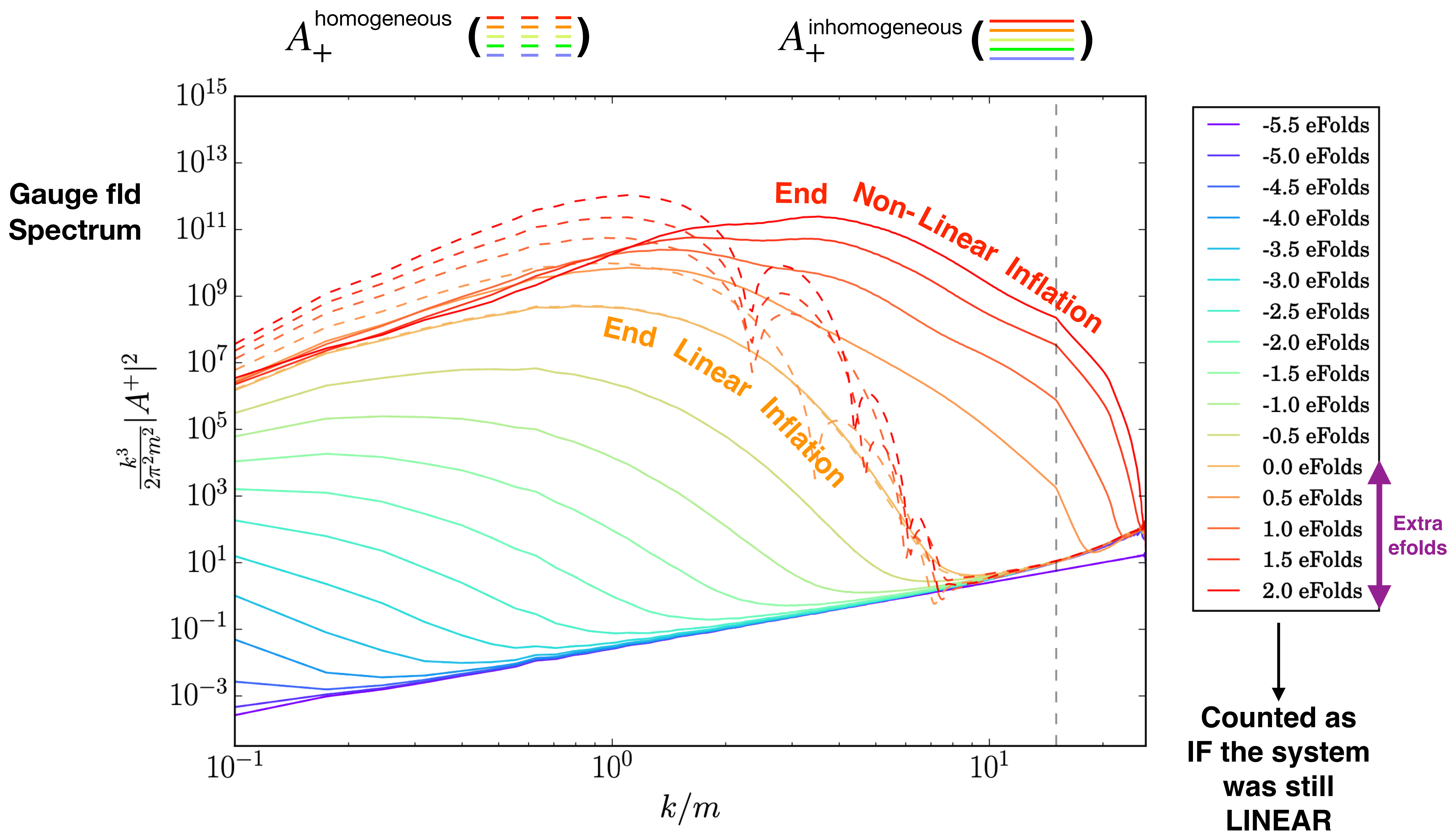

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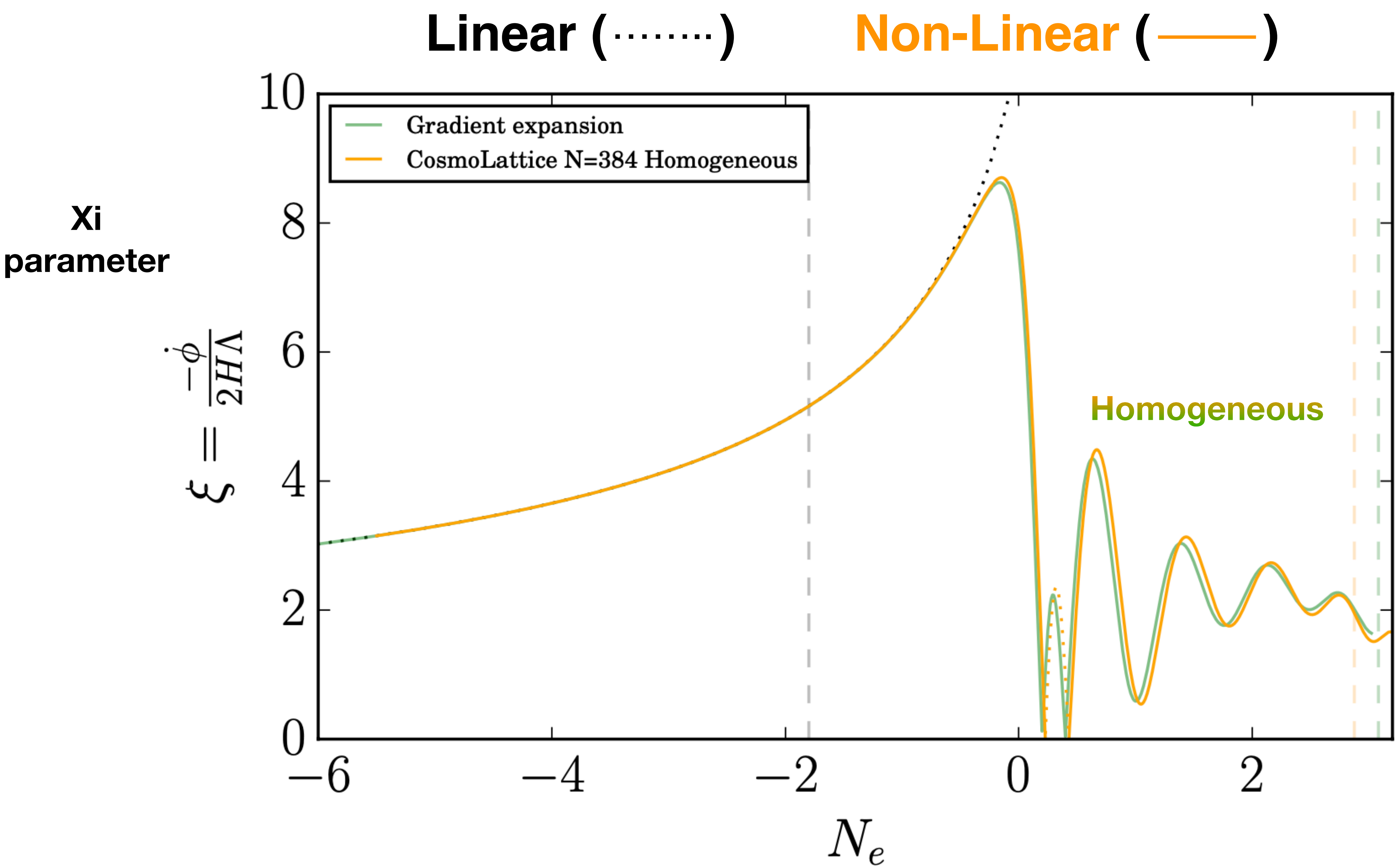
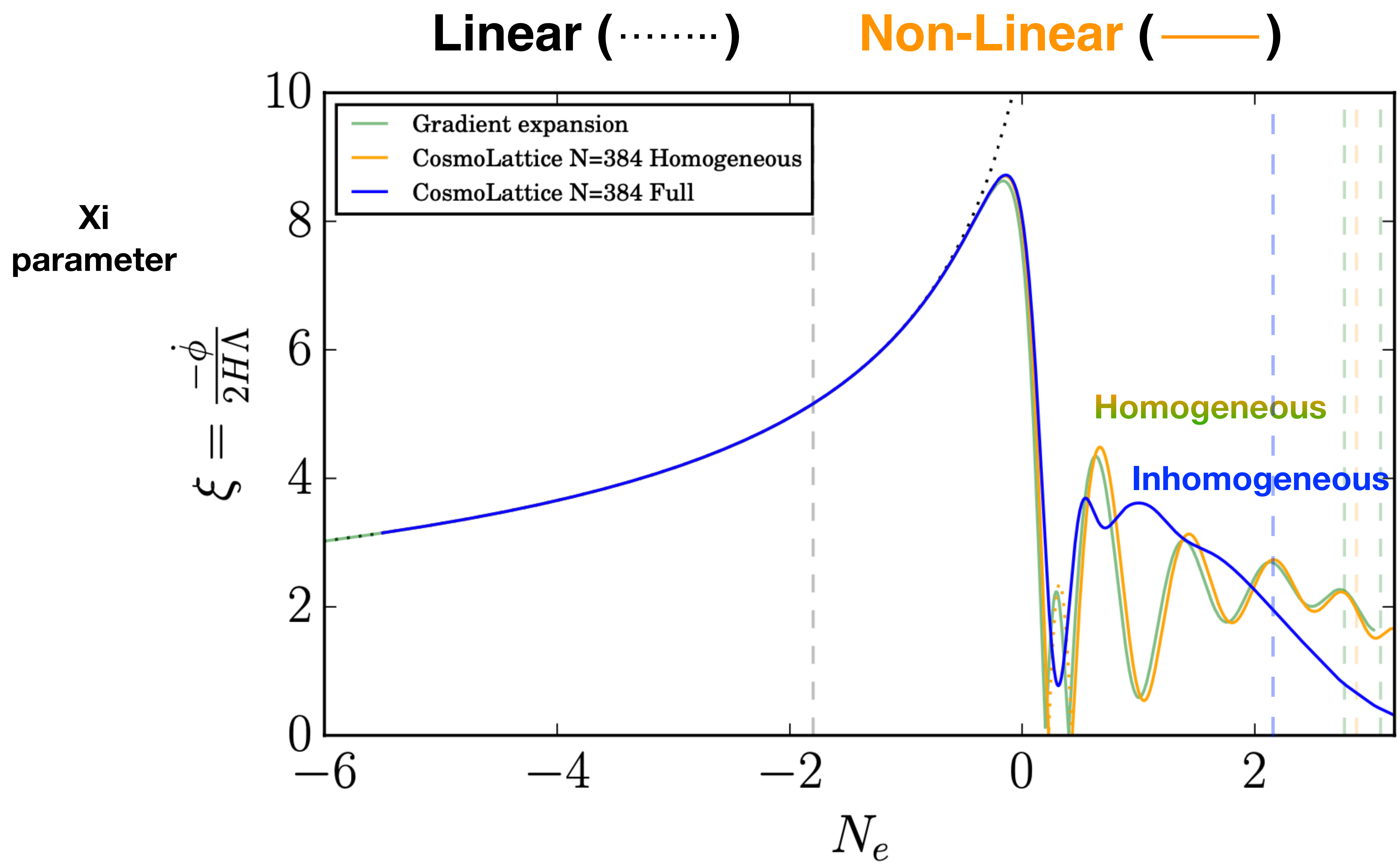


Axion-Inflation($V(\phi) = \frac{1}{2}m^2\phi^2$; $\frac{\phi}{4\Lambda}F\tilde{F}$; $\Lambda = \frac{m_p}{15}$)



Axion-Inflation($V(\phi)=\frac{1}{2}m^2\phi^2\ ;\ \frac{\phi}{4\Lambda}F\tilde{F}\ ;\ \Lambda=\frac{m_p}{X}$)

$(X=15,20,25)$

Summary

* ξ Controls the Gauge field excitation

* Linear change in ξ : exponential response in A_μ

* Predictions/constraints (PNG, PBH and GWs) depend crucially on ξ : we will re-assess real observability !

* Adding Schwinger pair production easy via $\overrightarrow{J}=\sigma\overrightarrow{E}$

* Other phenomena: BAU, Magnetogenesis, …

Axion-Inflation($V(\phi) = \frac{1}{2}m^2\phi^2$; $\frac{\phi}{4\Lambda}F\tilde{F}$; $\Lambda = \frac{m_p}{X}$)

$(X = 15, 20, 25)$

Summary

Soon in the ArXiv !

* ξ Controls the Gauge field excitation

* Linear change in ξ : exponential response in A_μ

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* Adding Schwinger pair production easy via $\overrightarrow{J} = \sigma \overrightarrow{E}$

* Other phenomena: BAU, Magnetogenesis, …

Example III

String Loop Dynamics (+ GW emission)

with

J. Baeza-Ballesteros, E. Copeland & J. Lizarraga

Work in progress

String Loop Dynamics + GW emission

GOAL

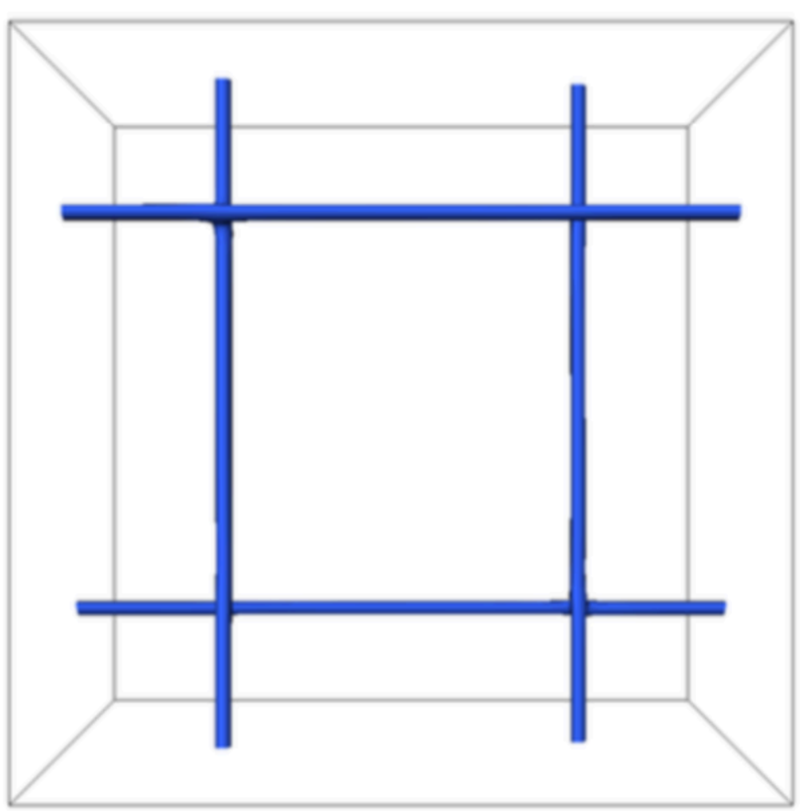
Dynamics of an isolated loop and its GW emission

String Loop Dynamics + GW emission

GOAL

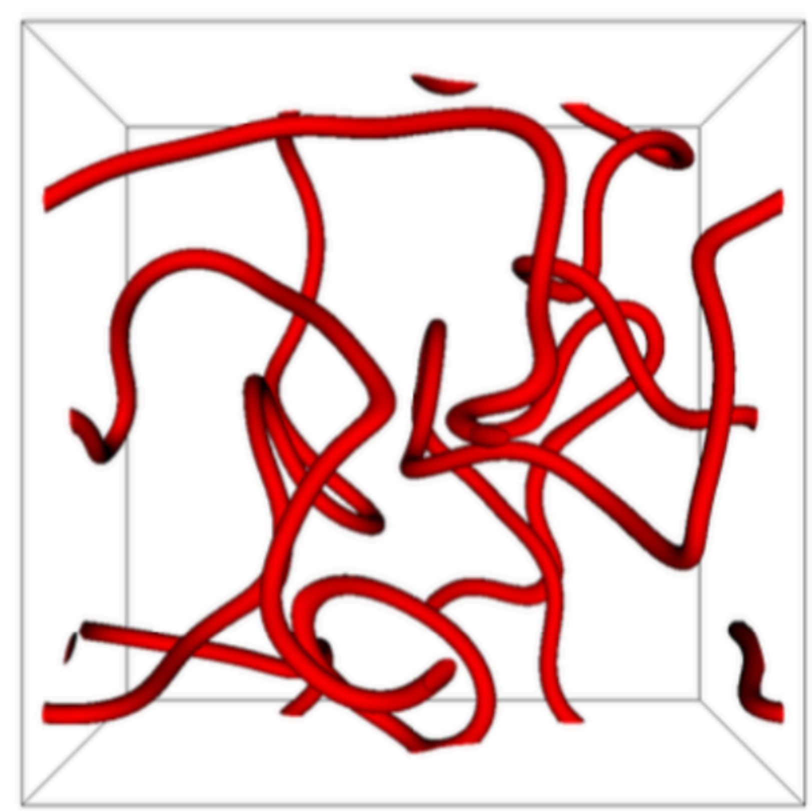
Dynamics of an isolated loop and its GW emission

Case I : Nielsen-Olesen



(following Vachaspati et al 2020)

Case II : Network



(following Lizarraga et al 2020/21)

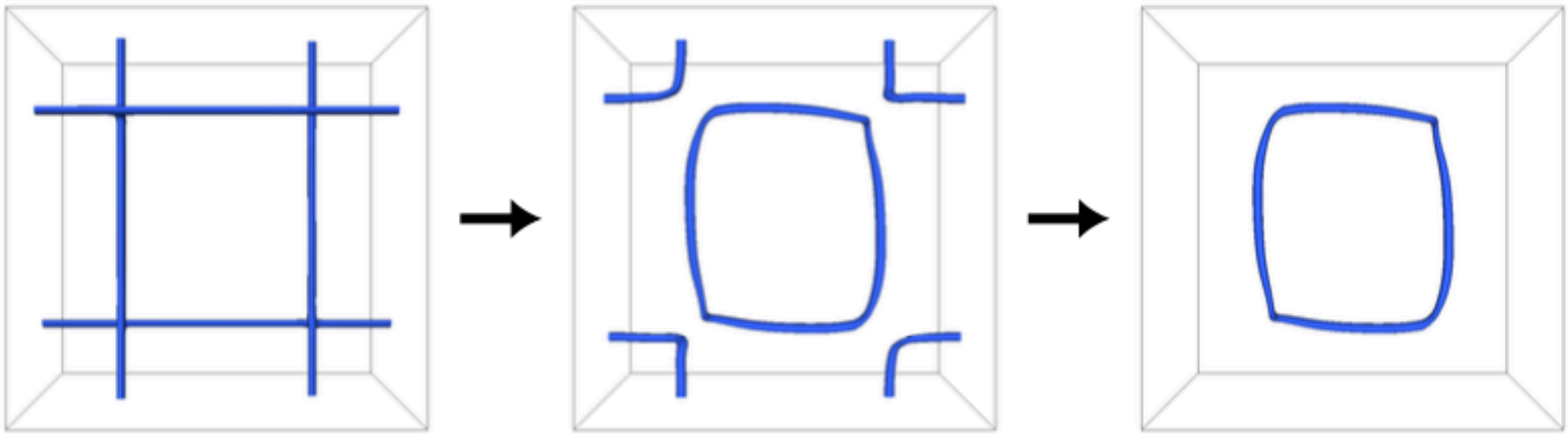
String Loop Dynamics + GW emission

GOAL

Dynamics of an isolated loop and its GW emission

Case I

— Isolate the inner loop —



String Loop Dynamics + GW emission

GOAL

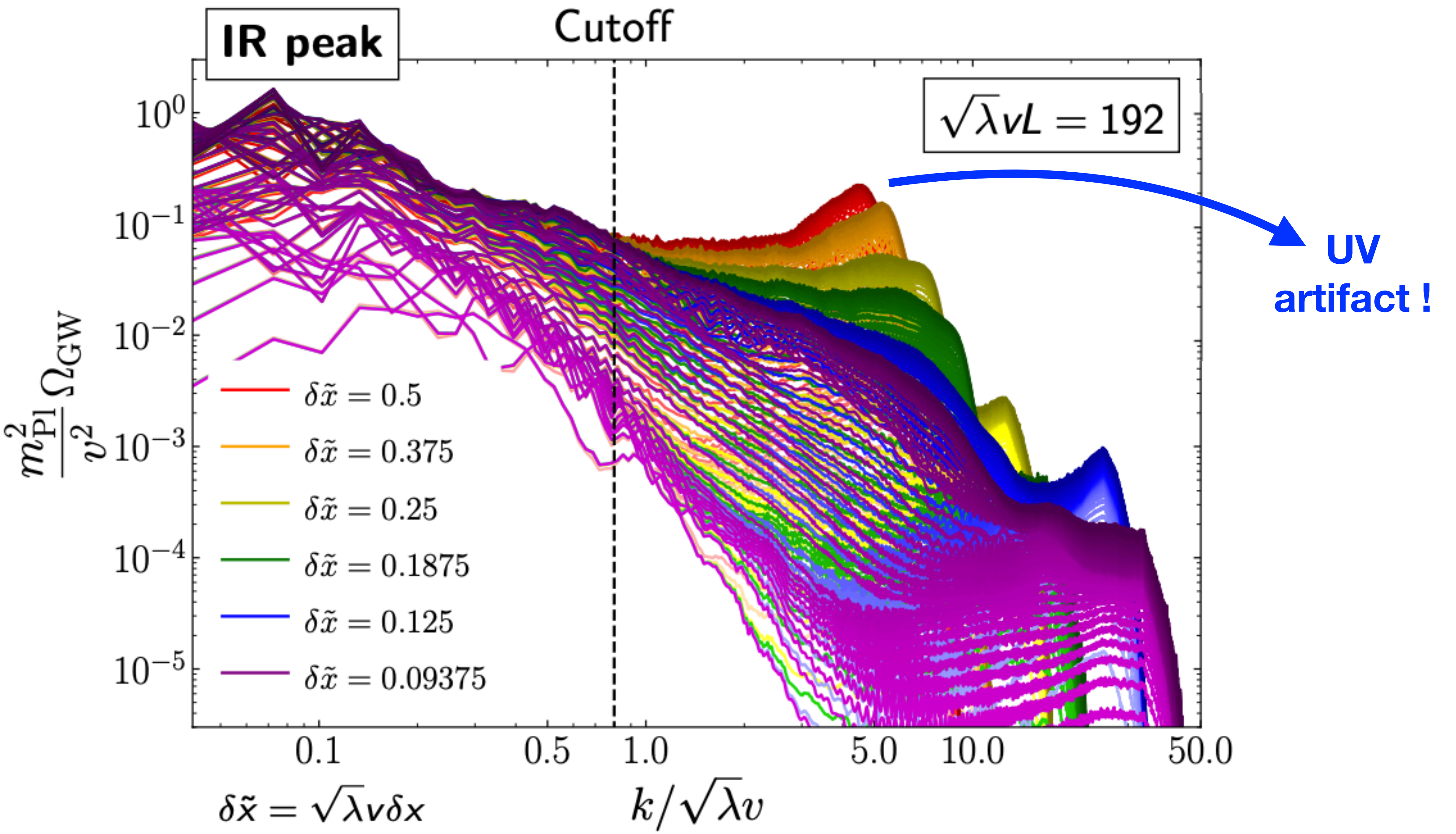
Dynamics of an isolated loop and its GW emission

Case II

— Only one loop remains eventually —

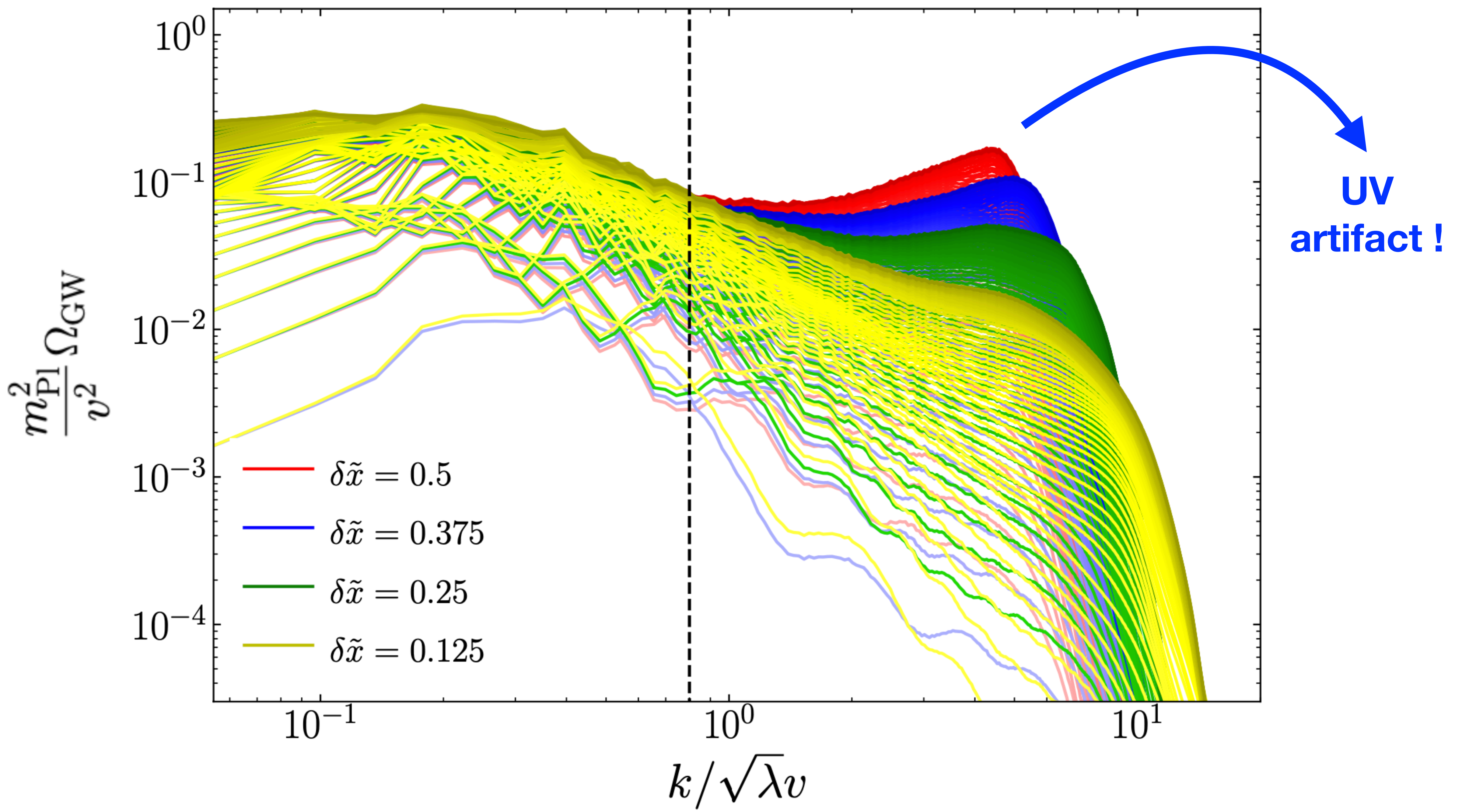
String Loop Dynamics + GW emission

GW energy density power spectrum (Case I)



String Loop Dynamics + GW emission

GW energy density power spectrum (Case II)



String Loop Dynamics + GW emission

Impact of our Study

Re-evaluation of GW emission from cosmic string network

Implications for

DM Axion string network
AH local string network
Comparison with NG
...

To know more…

CosmoLattice

http://www.cosmolattice.net/

Lattice Theory: arXiv: 2006.15122

Code Manual: arXiv: 2102.01031

Thanks for your attention !

If you want to learn how to
"latticesize" your problems ...

CosmoLattice

School 2022: Sept 5-8

@Valencia:

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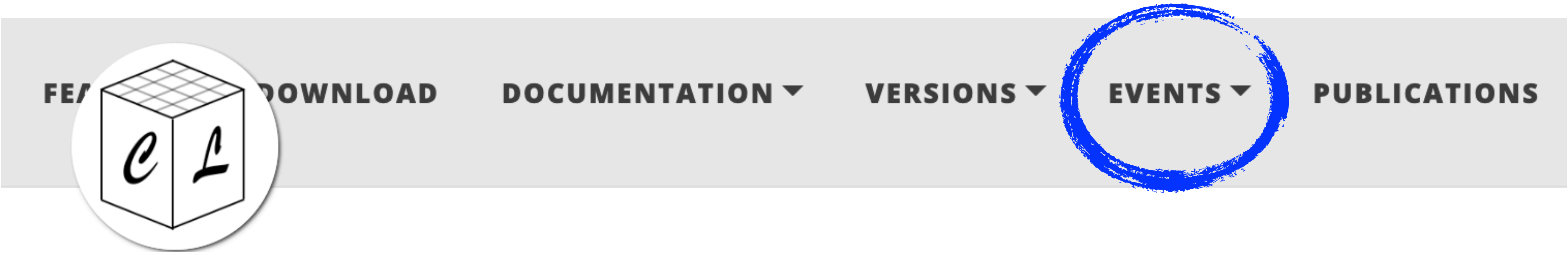
Keep an eye for CL School 2023:

https://cosmolattice.net



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Thanks for your attention !

Back Slides

CosmoLattice

What Field theory ?

- **Matter content:**

$$\boxed{S = -\int d^4x\sqrt{-g}\left\{\frac{1}{2}\partial_\mu\phi\partial^\mu\phi + (D_\mu^A\varphi)^*(D_A^\mu\varphi) + \frac{1}{4}F_{\mu\nu}F^{\mu\nu} + (D_\mu\Phi)^\dagger(D^\mu\Phi) + \frac{1}{2}\mathrm{Tr}\{G_{\mu\nu}G^{\mu\nu}\} + V(\phi,|\varphi|,|\Phi|)\right\}}$$

What Field theory ?

- **Matter content:**

$$S = -\int d^4x\sqrt{-g}\left\{\frac{1}{2}\partial_\mu\phi\partial^\mu\phi + \underbrace{(D^A_\mu\varphi)^*(D^\mu_A\varphi) + \frac{1}{4}F_{\mu\nu}F^{\mu\nu}}_{} + \underbrace{(D_\mu\Phi)^\dagger(D^\mu\Phi) + \frac{1}{2}\mathrm{Tr}\{G_{\mu\nu}G^{\mu\nu}\}}_{} + V(\phi,|\varphi|,|\Phi|)\right\}$$

$$\boxed{\phi \in \mathscr{R}e}$$

Scalar sector

$$\varphi \equiv \frac{1}{\sqrt{2}}(\varphi_0 + i\varphi_1)$$

$$D^{\mathrm{A}}_\mu \equiv \partial_\mu - iQ^{(\varphi)}_{\mathrm{A}} g_{_A} A_\mu$$

$$F_{\mu\nu} \equiv \partial_\mu A_\nu - \partial_\nu A_\mu$$

U(1) gauge sector

$$\Phi = \frac{1}{\sqrt{2}}\begin{pmatrix}\varphi_0 + i\varphi_1 \\ \varphi_2 + i\varphi_3\end{pmatrix}$$

$$D_\mu \equiv \mathcal{I}D^{\mathrm{A}}_\mu - ig_B Q_B B^a_\mu T_a$$

$$G_{\mu\nu} \equiv \partial_\mu B_\nu - \partial_\nu B_\mu - i[B_\mu, B_\nu]$$

SU(2) gauge sector

What Field theory ?

- **Matter content:**

$$S = -\int d^4x\sqrt{-g}\left\{\frac{1}{2}\partial_\mu\phi\partial^\mu\phi + \underbrace{(D^A_\mu\varphi)^*(D^\mu_A\varphi) + \frac{1}{4}F_{\mu\nu}F^{\mu\nu}} + \underbrace{(D_\mu\Phi)^\dagger(D^\mu\Phi) + \frac{1}{2}\mathrm{Tr}\{G_{\mu\nu}G^{\mu\nu}\}} + V(\phi,|\varphi|,|\Phi|)\right\}$$

$$\boxed{\phi \in \mathscr{Re}}$$

Scalar sector

$$\varphi \equiv \frac{1}{\sqrt{2}}(\varphi_0 + i\varphi_1)$$
$$D^{\mathrm{A}}_\mu \equiv \partial_\mu - iQ^{(\varphi)}_{\mathrm{A}} g_{_A} A_\mu$$
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U(1) gauge sector

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$$G_{\mu\nu} \equiv \partial_\mu B_\nu - \partial_\nu B_\mu - i[B_\mu, B_\nu]$$

SU(2) gauge sector

Scalar potential

What Field theory ?

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SU(2) gauge sector

Scalar potential

➤ **Background Metric:**

$$ds^2 = -\,dt^2 + a^2(t)\,\delta_{ij}\,dx^idx^j$$

- ➤ **Self-consistent expansion** (Friedmann equations)
- ➤ **Fixed power-law background** $a(t) \sim t^{\frac{2}{3(1+w)}}$

Lattice Equations

➤ **Hamiltonian scheme**: coupled first-order differential equations

➤ **Scalar fld example**

$$\phi'' - \frac{1}{a^2}\nabla^2\phi + \frac{3}{a}\frac{da}{dt}\frac{d\phi}{dt} = -\frac{\partial V}{\partial\phi} \xrightarrow{\pi_\phi \equiv \phi' a^{3-\alpha}}$$

KICK: $(\pi_\phi)' = -a^{3+\alpha}\frac{\partial V}{\partial\phi} + a^{1+\alpha}\nabla^2\phi$

DRIFT: $\phi' \equiv \pi_\phi a^{\alpha-3}$

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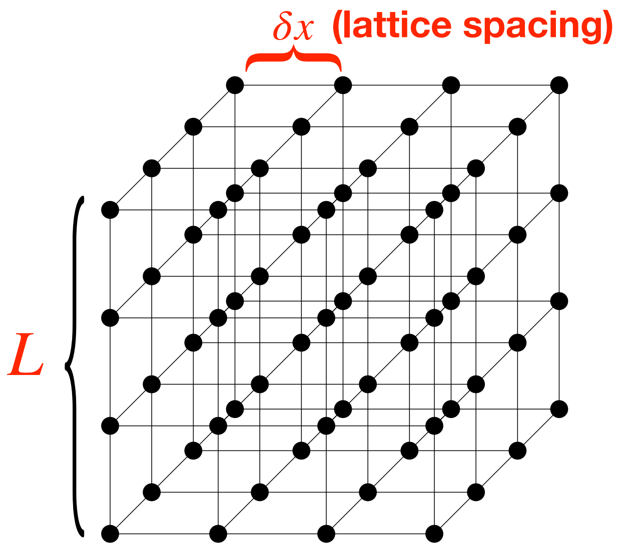
- **Scalar Fields and momenta** are defined in the **lattice sites**



N : number of points/dimension

$L = N \cdot \delta x$: length side

δt : time step

Minimum and maximum momenta:

$$k_{\rm min} = \frac{2\pi}{L} \qquad k_{\rm max} = \frac{\sqrt{3}}{2} N k_{\rm min}$$

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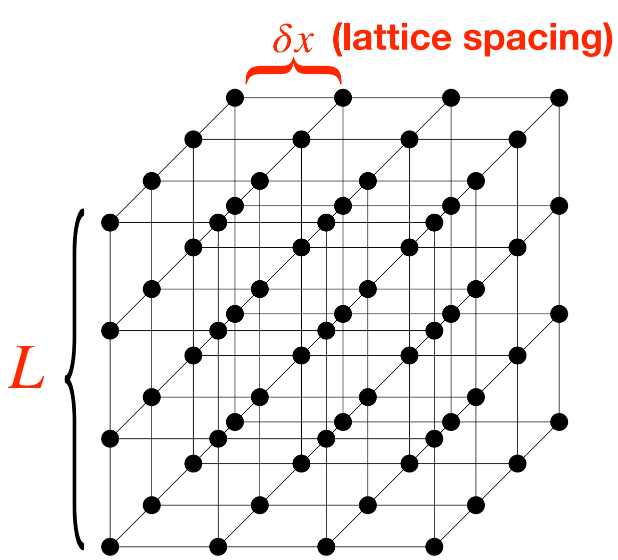

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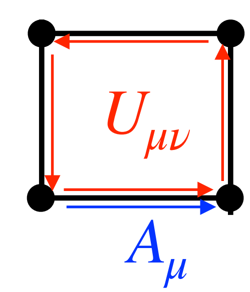
- **Gauge fields** introduced via **links** and **plaquettes** (like in **lattice-QCD**)



Writing a model

- Equations solved in (dimensionless) ***program variables***:

Choose:

$$\left\{\alpha, \omega_*, f_*\right\} \rightarrow$$

$$d\tilde{\eta} \equiv a^{-\alpha}\omega_* dt$$
$$d\tilde{x}^i \equiv \omega_* dx^i$$

Space and time

$$\tilde{\phi} = \frac{\phi}{f_*} \quad \tilde{\varphi} = \frac{\varphi}{f_*} \quad \tilde{\Phi} = \frac{\Phi}{f_*}$$

Scalar fields

$$\widetilde{A}_\mu = \frac{A_\mu}{\omega_*} \quad \widetilde{B}^a_\mu = \frac{B^a_\mu}{\omega_*}$$

Gauge fields

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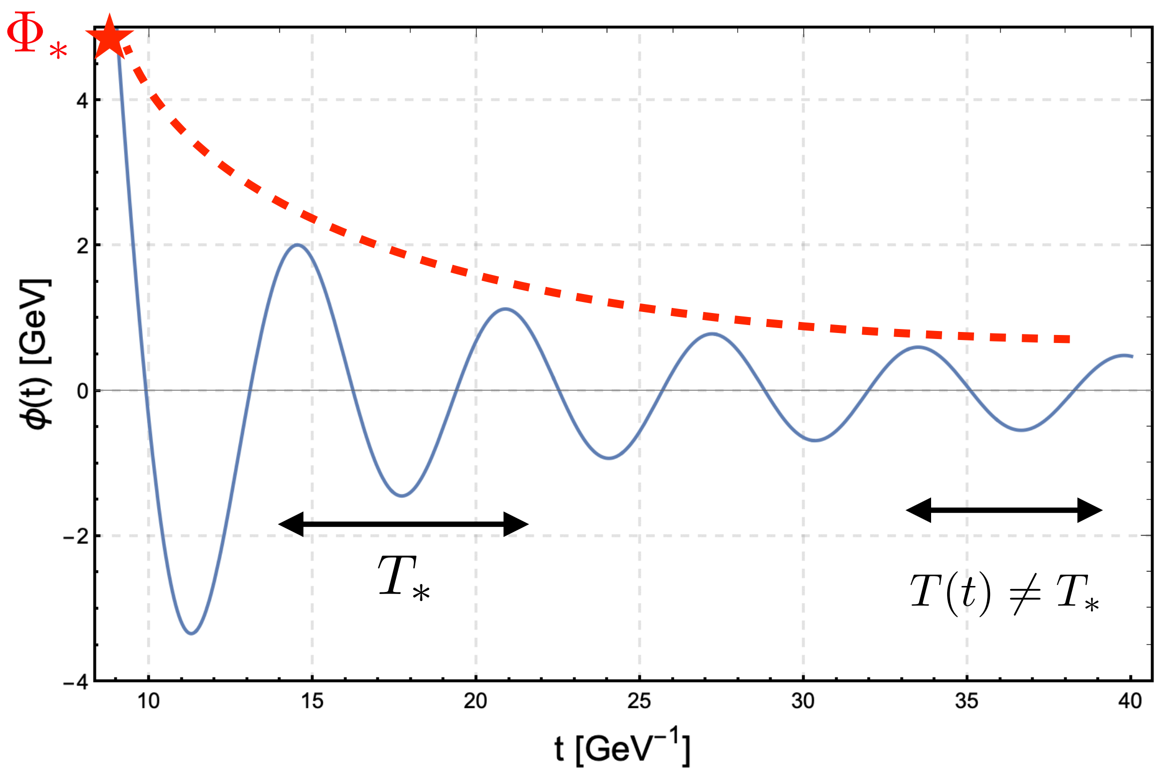
Gauge fields

Example: $\phi(t) \simeq \Phi_* \times f_{\rm osc}(t)$



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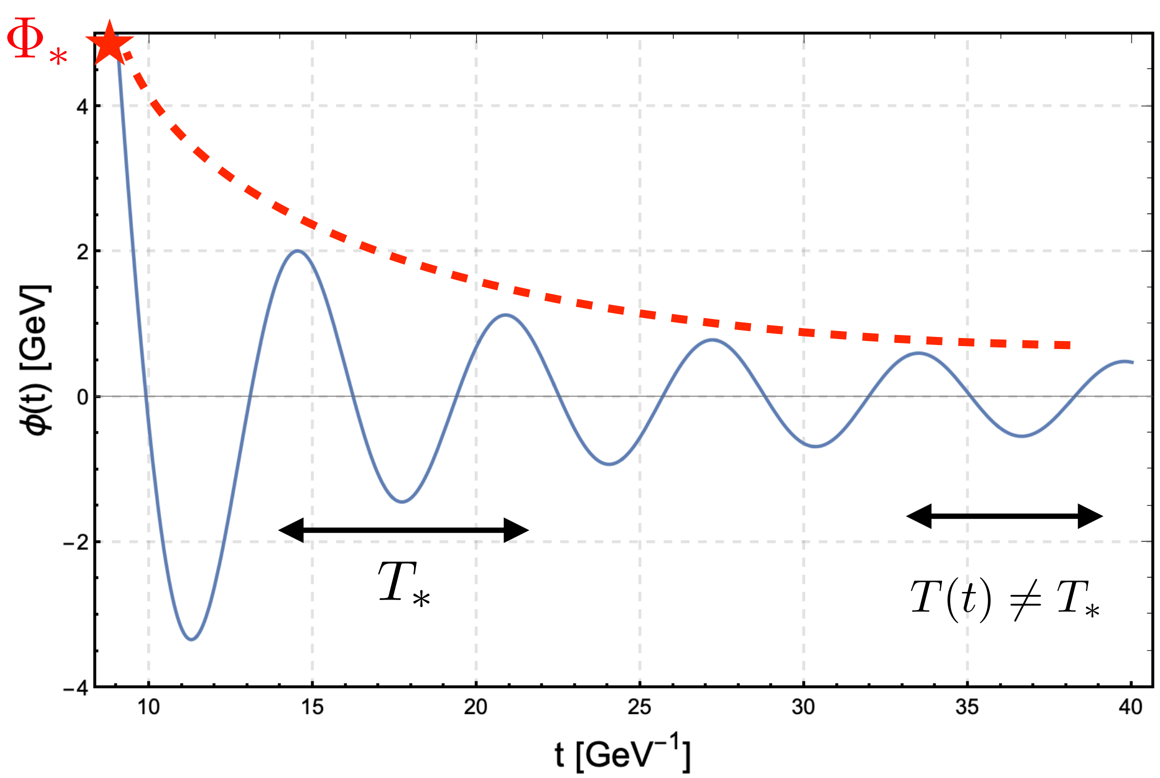

$$\begin{cases} f_* = \Phi_* \\ \omega_* = 1/T_* \\ \alpha \longrightarrow \text{Make period constant in } \tilde{\eta} \end{cases}$$

Writing a model

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Gauge fields

- Write **scalar potential** and first and second derivatives in **one file** (*model.h*)

$$\tilde{V}(\tilde{\phi}, |\tilde{\varphi}|, |\widetilde{\Phi}|) \equiv \frac{1}{f_*^2\omega_*^2} V(f_*\tilde{\phi}, f_*|\tilde{\varphi}|, f_*|\widetilde{\Phi}|) \rightarrow \frac{\partial\tilde{V}}{\partial\tilde{\phi}}, \frac{\partial\tilde{V}}{\partial|\tilde{\varphi}|}, \frac{\partial\tilde{V}}{\partial|\tilde{\Phi}|}, \frac{\partial^2\tilde{V}}{\partial\tilde{\phi}^2}, \frac{\partial^2\tilde{V}}{\partial|\tilde{\varphi}|^2}, \frac{\partial^2\tilde{V}}{\partial|\tilde{\Phi}|^2},$$

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$$\tilde{V}(\tilde{\phi}, |\tilde{\varphi}|, |\widetilde{\Phi}|) \equiv \frac{1}{f_*^2\omega_*^2} V(f_*\tilde{\phi}, f_*|\tilde{\varphi}|, f_*|\widetilde{\Phi}|)$$

$$\frac{\partial \tilde{V}}{\partial \tilde{\phi}}, \quad \frac{\partial \tilde{V}}{\partial |\tilde{\varphi}|}, \quad \frac{\partial \tilde{V}}{\partial |\tilde{\Phi}|}, \quad \frac{\partial^2 \tilde{V}}{\partial \tilde{\phi}^2}, \quad \frac{\partial^2 \tilde{V}}{\partial |\tilde{\varphi}|^2}, \quad \frac{\partial^2 \tilde{V}}{\partial |\tilde{\Phi}|^2},$$

- **Parameters** passed via **one file** (input.txt)

(no need to re-compile !)

```
#Output
outputfile = ./

#Evolution
expansion = true
evolver = VV2

#Lattice
N = 32
dt = 0.01
kIR = 0.75
nBinsSpectra = 55

#Times
tOutputFreq = 0.1
tOutputInfreq = 1
tMax = 300

#IC
kCutOff = 1.75
initial_amplitudes = 7.42675e18 0 # homogeneous amplitudes in GeV
initial_momenta = -6.2969e30 0    # homogeneous amplitudes in GeV2

#Model Parameters
lambda = 9e-14
q = 100
```

Self-consistent Expansion

- Algorithms use **second Friedmann equation** to **evolve the scale factor**.
- The **first Friedmann equation** is used to check the accuracy of the simulation.

$$\frac{a''}{a} = \frac{a^{2\alpha}}{3m_p^2}\Big\langle(\alpha-2)(K_\phi+K_\varphi+K_\Phi)+\alpha(G_\phi+G_\varphi+G_\Phi)+(\alpha+1)V + (\alpha-1)(K_{U(1)}+G_{U(1)}+K_{SU(2)}+G_{SU(2)})\Big\rangle$$

$$\left(\frac{a'}{a}\right)^2 = \frac{a^{2\alpha}}{3m_p^2}\left\langle K_\phi+K_\varphi+K_\Phi+G_\phi+G_\varphi+G_\Phi+K_{U(1)}+G_{U(1)}+K_{SU(2)}+G_{SU(2)}+V\right\rangle$$

$\langle ... \rangle$ represents volume averaging

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$\langle\ldots\rangle$ represents volume averaging

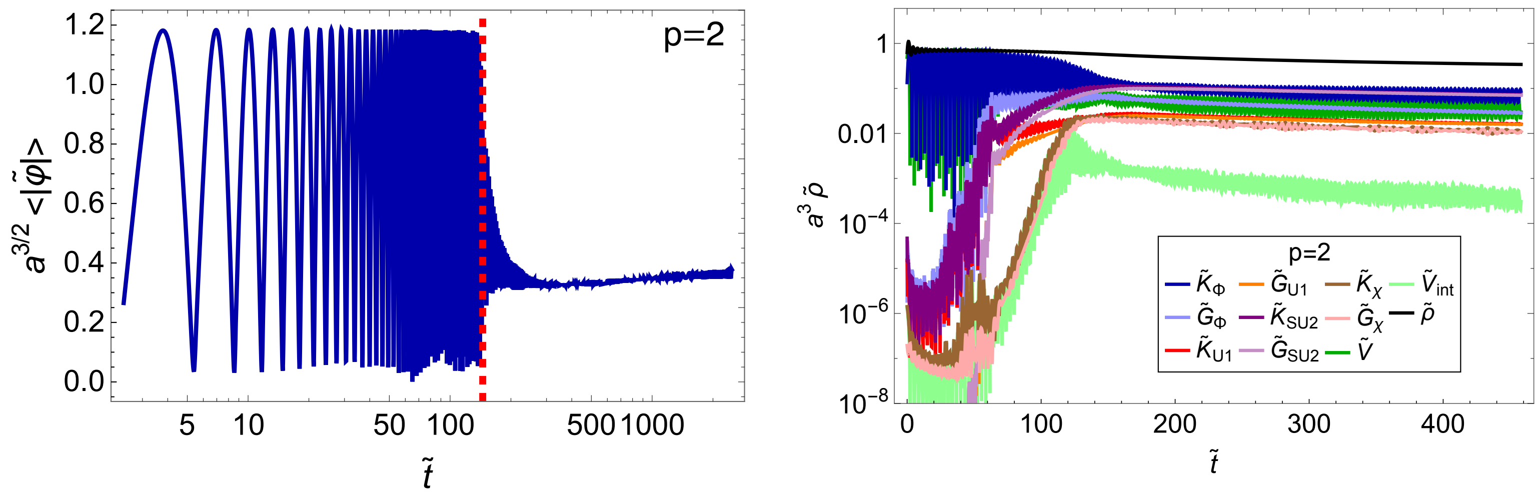
$$\begin{aligned} K_\phi &= \tfrac{1}{2a^{2\alpha}}\phi'^2 \\ K_\varphi &= \tfrac{1}{a^{2\alpha}}(D_0^A\varphi)^*(D_0^A\varphi) \\ K_\Phi &= \tfrac{1}{a^{2\alpha}}(D_0\Phi)^\dagger(D_0\Phi) \end{aligned} \;;\quad \begin{aligned} G_\phi &= \tfrac{1}{2a^2}\textstyle\sum_i(\partial_i\phi)^2 \\ G_\varphi &= \tfrac{1}{a^2}\textstyle\sum_i(D_i^A\varphi)^*(D_i^A\varphi) \\ G_\Phi &= \tfrac{1}{a^2}\textstyle\sum_i(D_i\Phi)^\dagger(D_i\Phi) \end{aligned} \;;\quad \begin{aligned} K_{U(1)} &= \tfrac{1}{2a^{2+2\alpha}}\textstyle\sum_i F_{0i}^2 \\ K_{SU(2)} &= \tfrac{1}{2a^{2+2\alpha}}\textstyle\sum_{a,i}(G_{0i}^a)^2 \\ G_{U(1)} &= \tfrac{1}{2a^4}\textstyle\sum_{i,j<i}F_{ij}^2 \\ G_{SU(2)} &= \tfrac{1}{2a^4}\textstyle\sum_{a,i,j<i}(G_{ij}^a)^2 \end{aligned}$$

(Kinetic-Scalar) (Gradient-Scalar) (Electric & Magnetic)

Output from your Run

Output Types

Volume averages: variance, energies, etc



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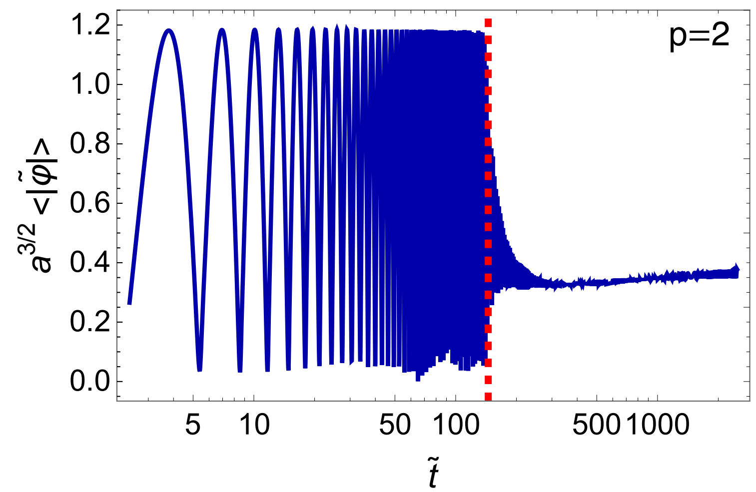
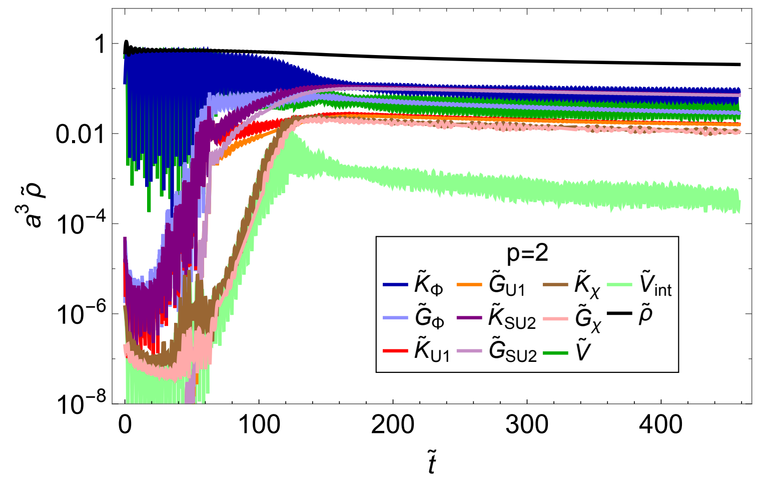
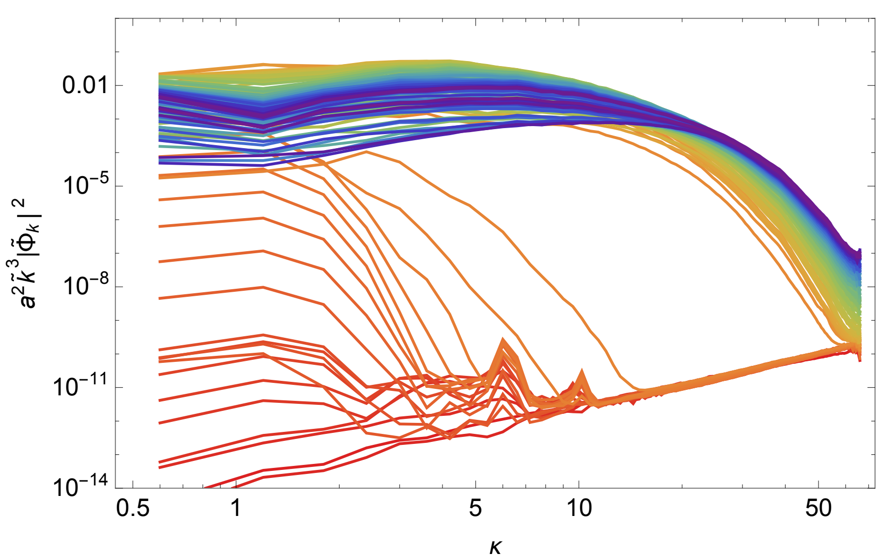




Fld Spectra: Raw/Binned



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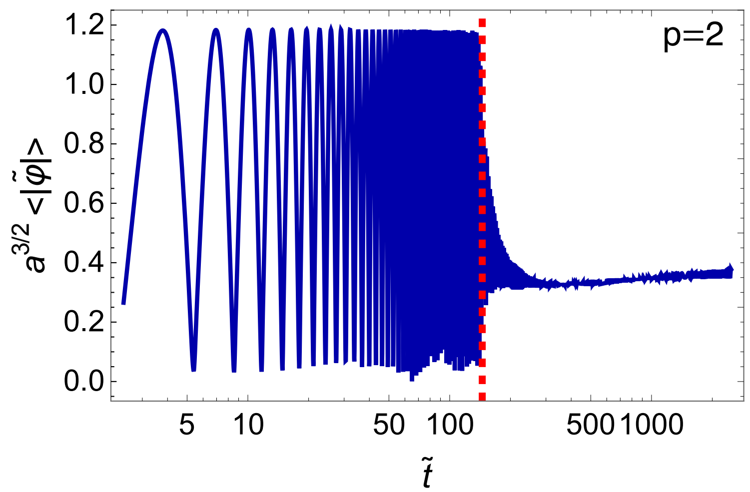

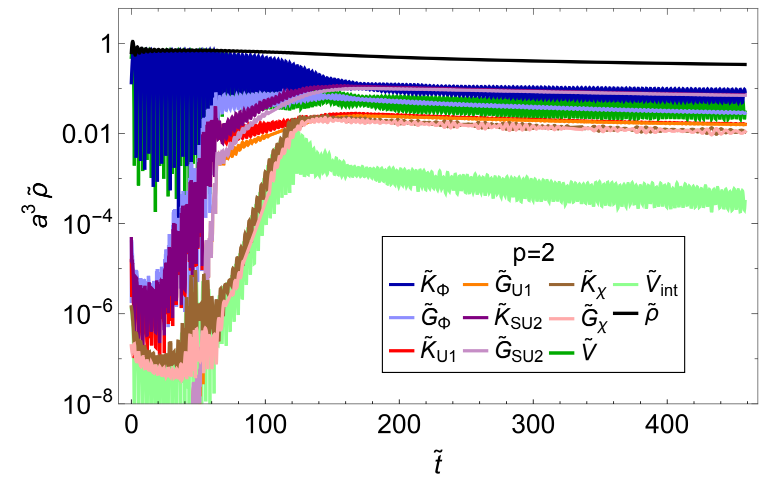

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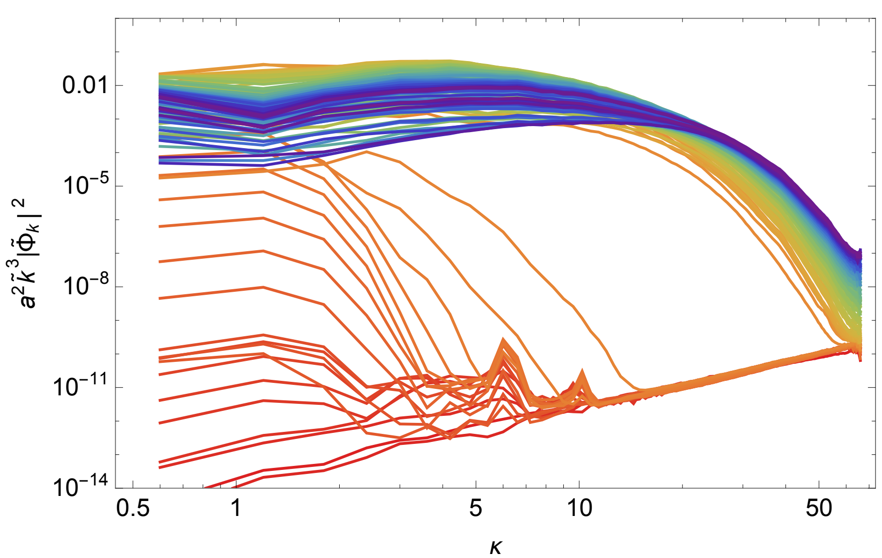
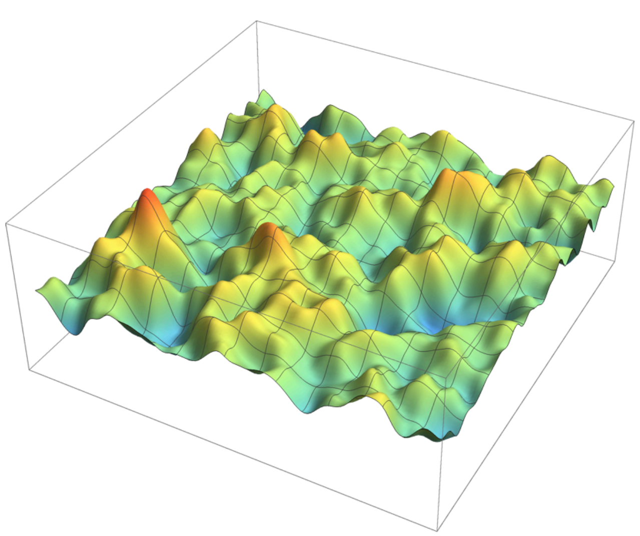
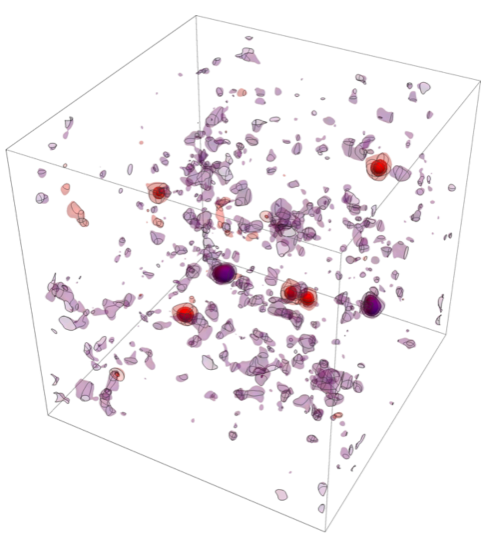


Snapshots: 2D/3D distribution





Constraints

Energy conservation

- The **first Friedmann equation** is used to check the accuracy of the simulation.

$$\boxed{\left(\frac{\dot{a}}{a}\right)^2 \simeq \frac{\rho}{3m_p^2}}$$

$$\Delta_e \equiv \frac{\langle \mathrm{LHS} - \mathrm{RHS} \rangle}{\langle \mathrm{LHS} + \mathrm{RHS} \rangle}$$

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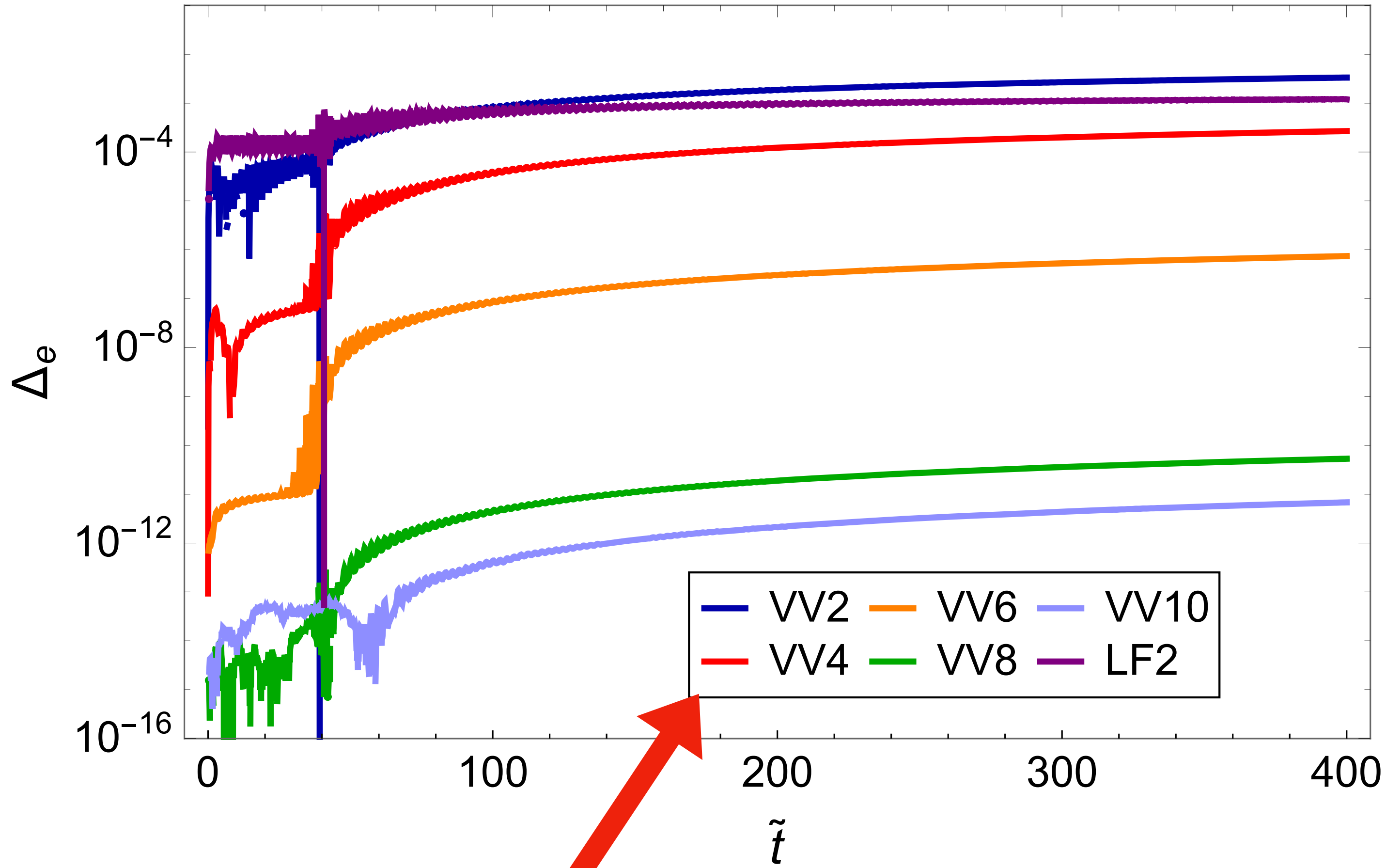

Evolution algorithms:

- **VVn:** Velocity-verlet of accuracy order O(dt^n)
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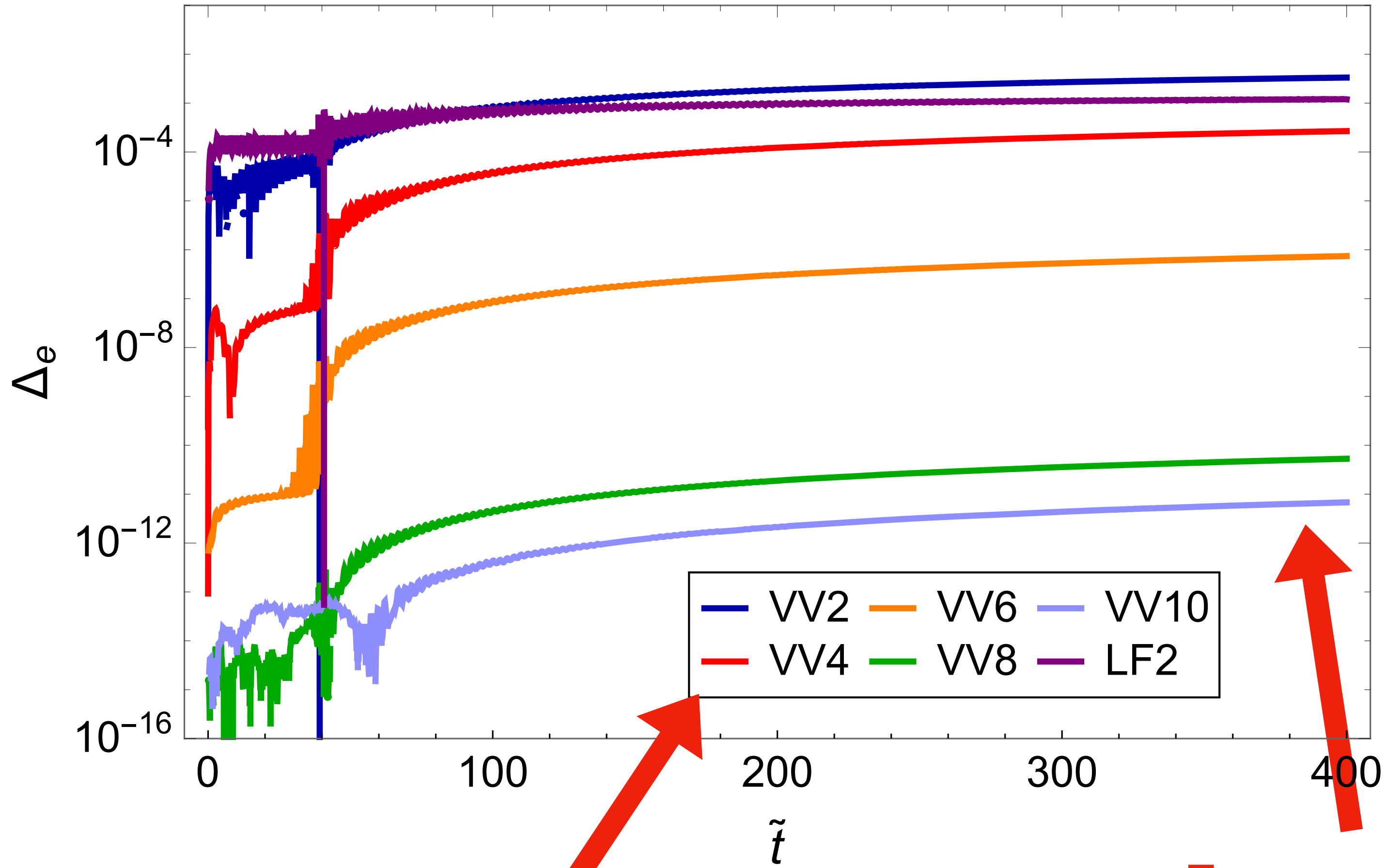

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Energy conserved up to machine precision for VV10!

Gauge theories: Gauss constraint

- Preservation of U(1) & SU(2) **Gauss constraints** (for all integrators!)

$$\partial_i F_{0i} = a^2 J_0^A$$
$$(\mathscr{D}_i)_{ab}(G_{0i})^b = a^2 (J_0)_a$$

Gauge charges

$$\Delta_g \equiv \frac{\langle\sqrt{(\mathrm{LHS}-\mathrm{RHS})^2}\rangle}{\langle\sqrt{(\mathrm{LHS}+\mathrm{RHS})^2}\rangle}$$

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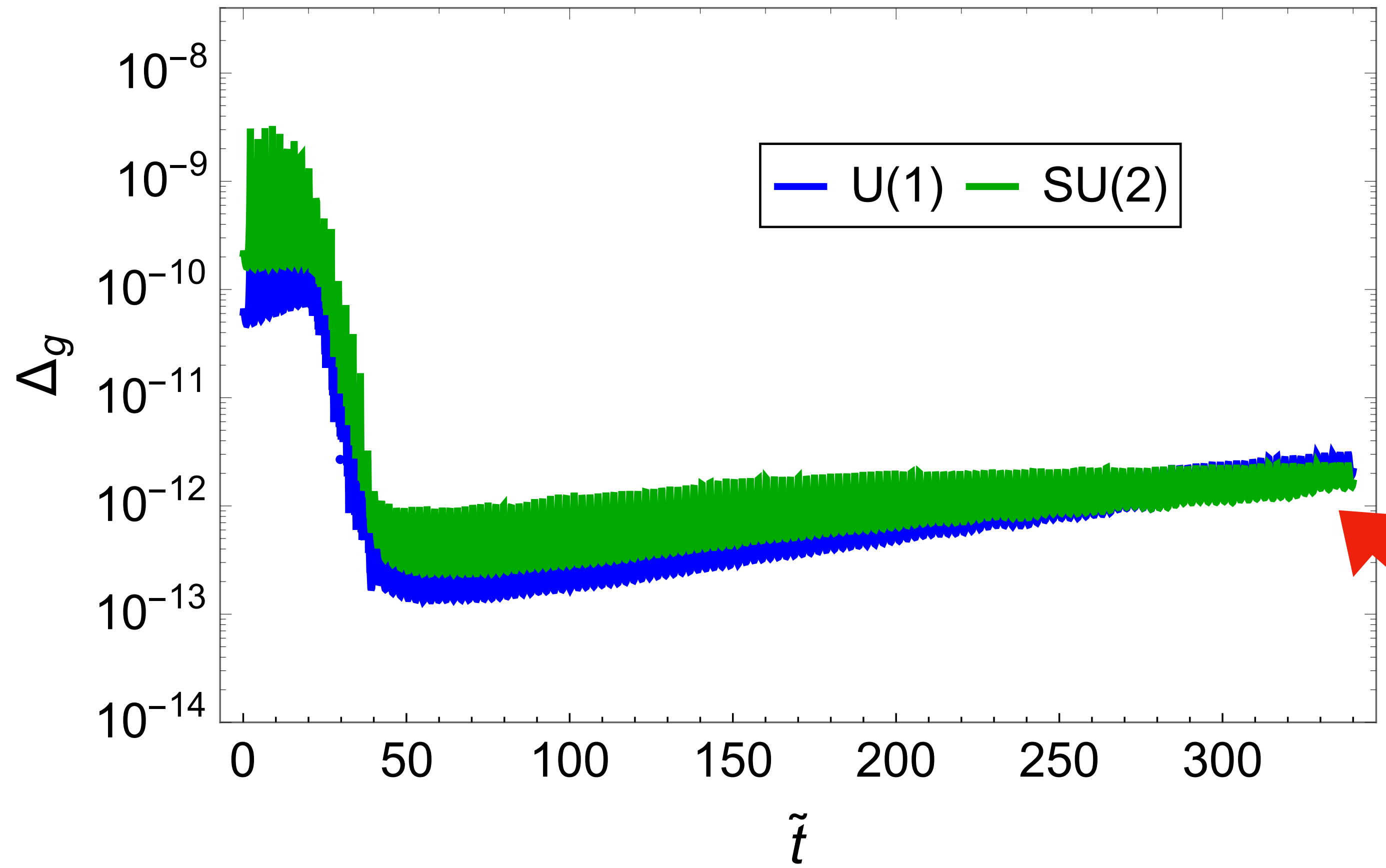
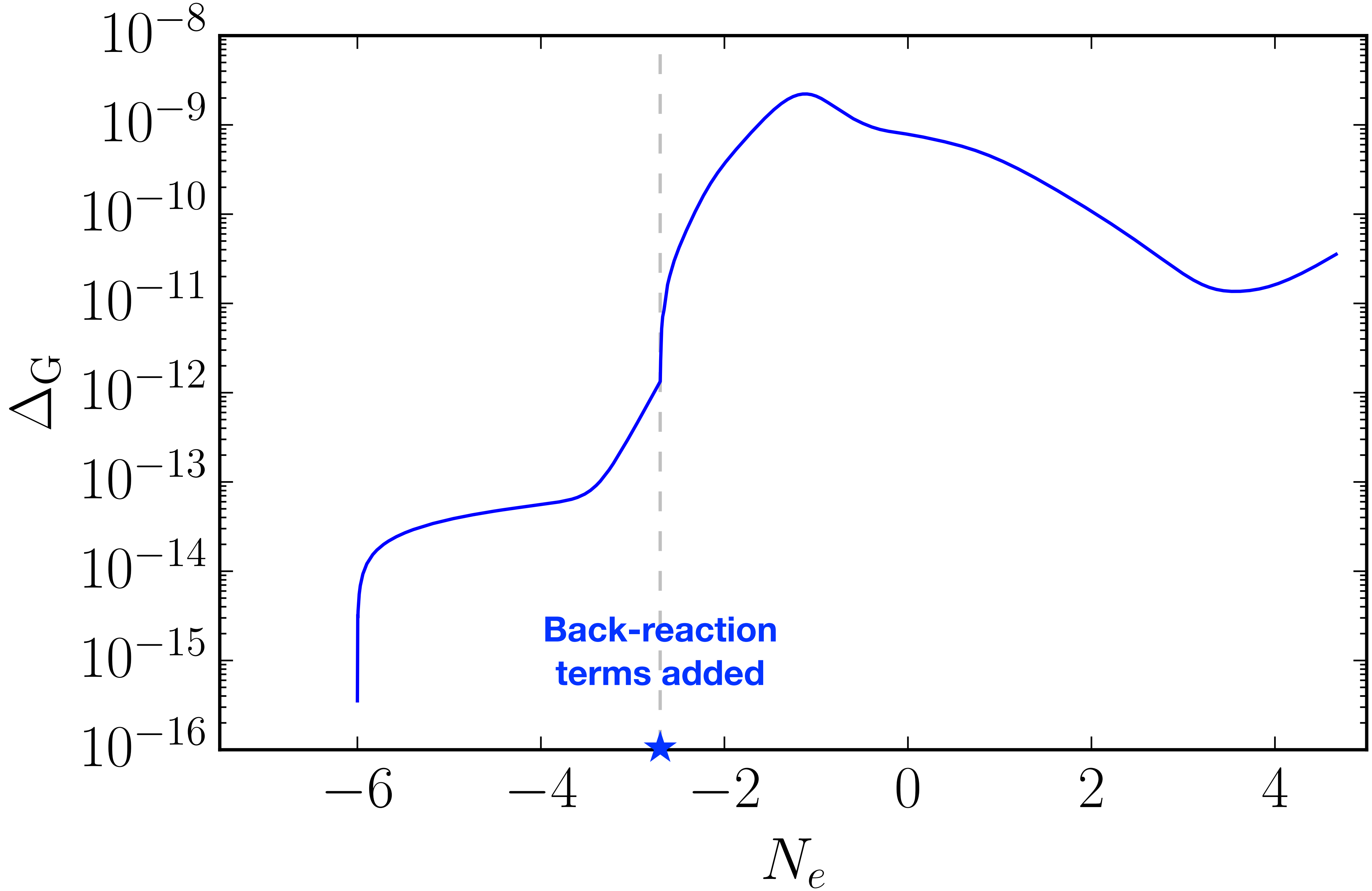


Gauss constraint preserved up to machine precision

Axion-inflation Constraints

Gauss Constraint

$$\vec{\nabla} \cdot \vec{E} = -\frac{1}{a\Lambda} \vec{\nabla}\phi \cdot \vec{B}$$



Hubble Constraint

$$\pi_a^2 = \frac{a^2}{3m_{pl}^2}(K_\phi + G_\phi + V + K_A + G_A) \quad ; \quad \pi_a \equiv \dot{a}$$