INFN

Inflationary production of dark photons and other dark sectors

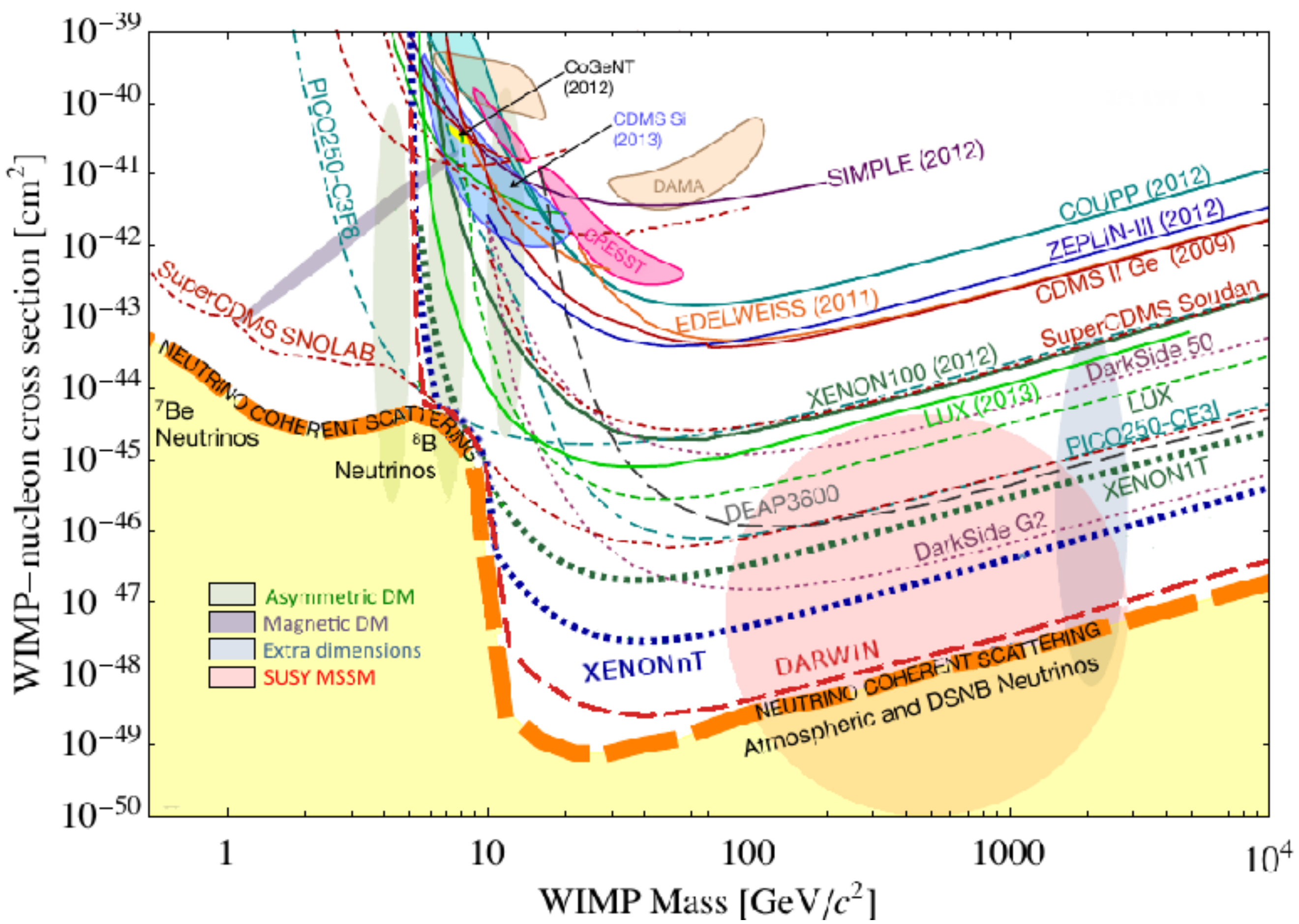
Based on 2204.14274, 2210.03108

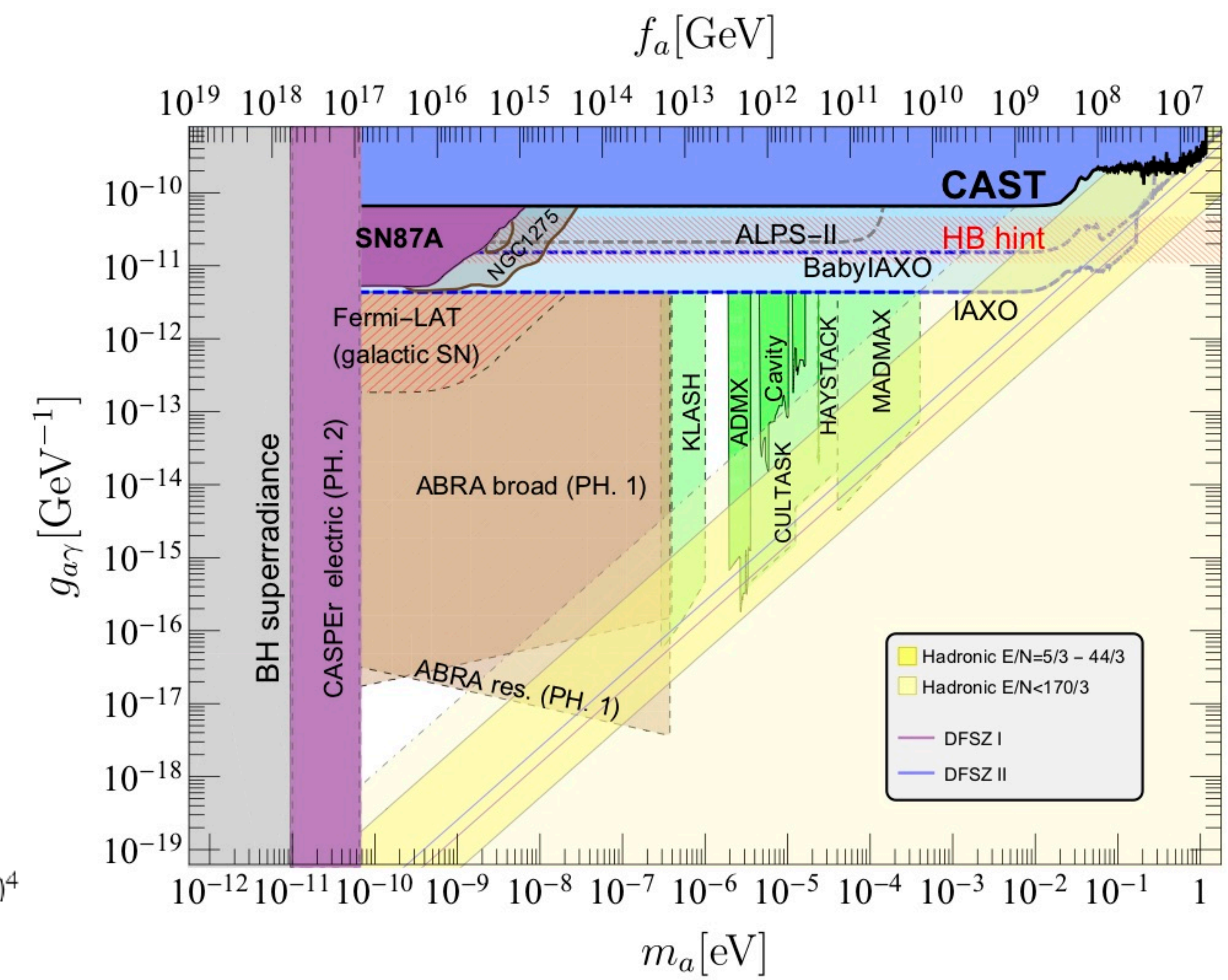
with Andrea Tesi

Madrid - 19 October 2022

All evidences for the existence of DM are due to gravity.

It would great if it had interactions with SM:





Is this wishful thinking?

What if DM interacts only gravitationally?

We may never know...

How is dark matter produced?

There are infinite model independent ways for example:

- Inflaton decay

$$\rho_D = \frac{g_D}{g_{SM}} \rho_{SM}$$

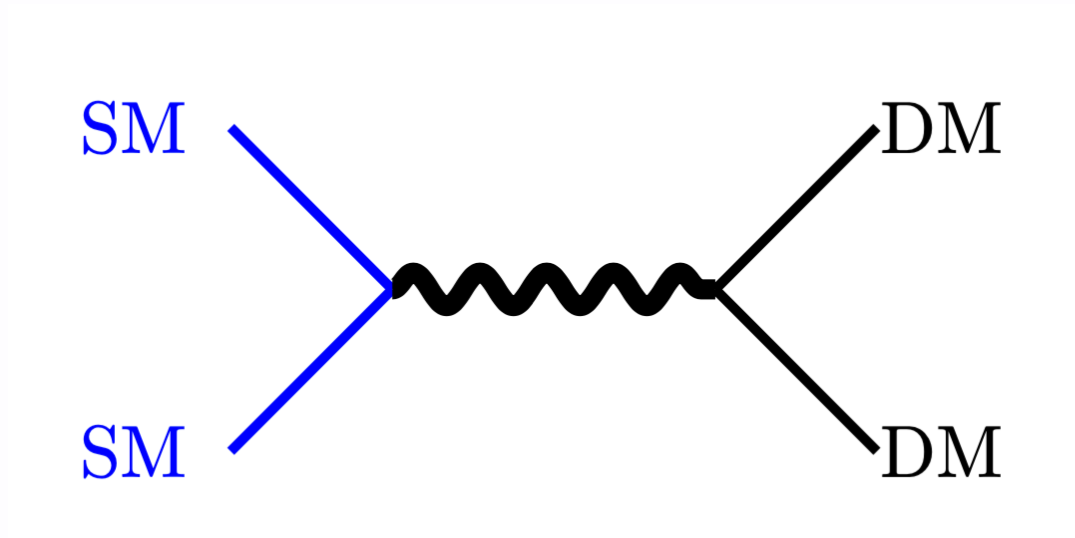
- Some bridge

$$T_D = T$$

Could dark matter be produced by gravity itself?

Gravitational freeze-in

[Garny, Sandora, Sloth '15]



$$\mathcal{A} = \frac{1}{M_p^2 s}\left(T^{\rm SM}_{\mu\nu}T^{\rm DM}_{\alpha\beta}\eta^{\mu\alpha}\eta^{\nu\beta} - \frac{1}{2}T^{\rm SM}T^{\rm DM}\right)$$

$$\frac{dY_D}{dT} = \frac{\langle\sigma v\rangle s(T)}{HT}(Y_D^2 - Y_{\rm eq}^2) \qquad Y_D(0) = \int_0^{T_R}\frac{dT}{T}\frac{\langle\sigma v\rangle s}{H}Y_{\rm eq}^2$$

$$\langle\sigma v\rangle = 4\langle\sigma_0 v\rangle + 45\langle\sigma_{1/2} v\rangle + 12\langle\sigma_1 v\rangle$$

Abundance:

$$\frac{n_{\rm D}}{n_{\rm eq}} \approx 0.0014\frac{c_D}{g_D}\left(\frac{T_R}{M_p}\right)^3 \qquad \rho_D \approx 5\cdot 10^{-4}\, c_D\left(\frac{T_R}{M_p}\right)^3 T^4$$

Thermalization increases the abundance.

[MR, Tesi, Tillim '20]

Quantum production

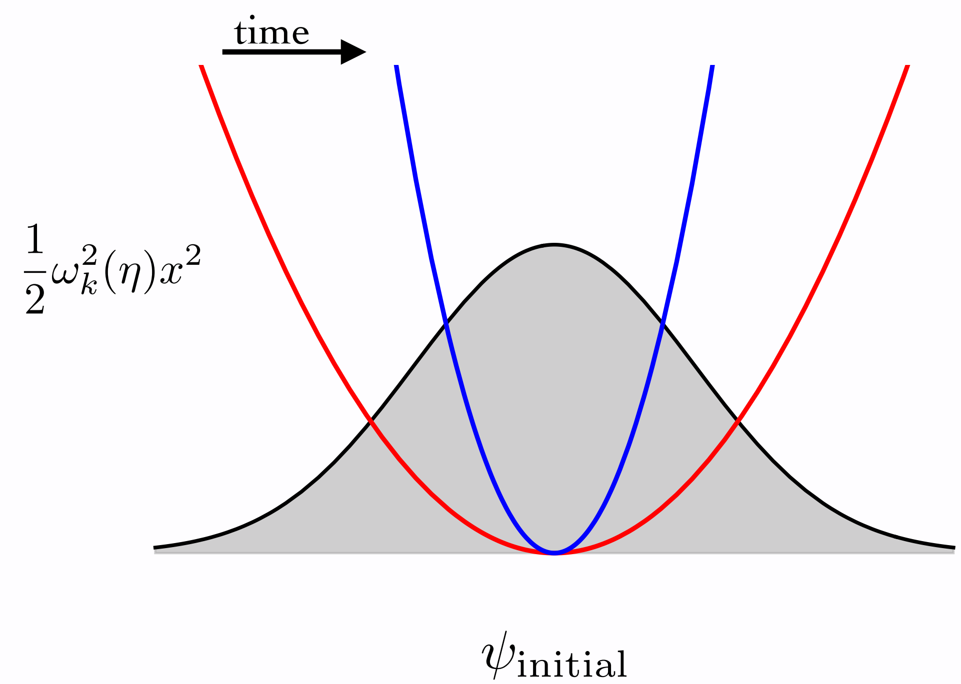
[Ford '87,
Kolb, Riotto, Giudice '90s
...]

In a time dependent background particles are produced due to the non-adiabatic evolution of the vacuum.

$$L = \frac{1}{2}(\partial\phi)^2 - \frac{M^2}{2}\phi^2 + \frac{\xi}{2}\phi^2 R \qquad a\phi = v$$

$$v_k''(\eta) + \omega_k^2(\eta) v_k(\eta) = 0\,, \mathrm{BD-vacuum}$$

$$\omega_k^2(\eta) = |\vec{k}|^2 + M^2 a^2(\eta) + \frac{a''(\eta)}{a(\eta)}(1-6\xi)$$



Solving wave-equation we determine Bogoliubov coefficients:

$$a^4 \frac{d\rho}{d\log k} = \frac{k^3}{2\pi^2}\left[\frac{|\partial_\eta v|^2}{2} + \frac{\omega_k^2 |v|^2}{2} - \frac{\omega_k}{2}\right] = \frac{k^3}{2\pi^2}\omega_k |\beta_k|^2$$

- Minimal coupling:

During inflation each mode is produced with an amplitude $H_I/(2\pi)$ that is constant till horizon re-entry. This gives an abundance:

$$\frac{1}{s}\frac{d\rho_X}{d\log k}\bigg|_{\xi=0} \approx \frac{1}{g_*(M)^{1/4}}\frac{H_I^2}{(2\pi)^2}\frac{\sqrt{M}}{M_{Pl}^{3/2}}\begin{cases}\dfrac{k_*}{k} & k \gg k_* \\ 1 & k \ll k_*\end{cases}$$

$$k_* = a_{eq}\sqrt{M H_{eq}}$$

Light non-thermal DM:

$$\frac{\Omega_a^{\xi=0}h^2}{0.12} = \frac{\rho/s}{0.44\,\mathrm{eV}} \approx \sqrt{\frac{M_a}{6\times 10^{-9}\,\mathrm{eV}}}\left(\frac{H_I}{10^{14}\,\mathrm{GeV}}\right)^2$$

The flat IR energy spectrum is grossly excluded by isocurvature perturbations.

- Conformal coupling:

Particle production vanishes in the massless limit for $\xi = 1/6$ corresponding to conformal coupling to curvature. The action becomes Weyl invariant

$$g_{\mu\nu}(x) \to \Omega(x) g_{\mu\nu}(x) \,, \qquad \phi(x) \to \Omega^{-1}(x)\phi(x)$$

More in general particle production vanishes for Weyl invariant theories:

$$T^{\mu}_{\mu} = 0$$

This is generic: fermions, gauge fields, strongly coupled CFTs. Only exception minimally coupled scalar that naturally appears as Nambu-Goldstone boson due to shift symmetry.
Relativistic $\simeq$ conformally coupled.

Inflationary production is often discussed for free theories.
In what follows I will highlight how interactions might change the results:

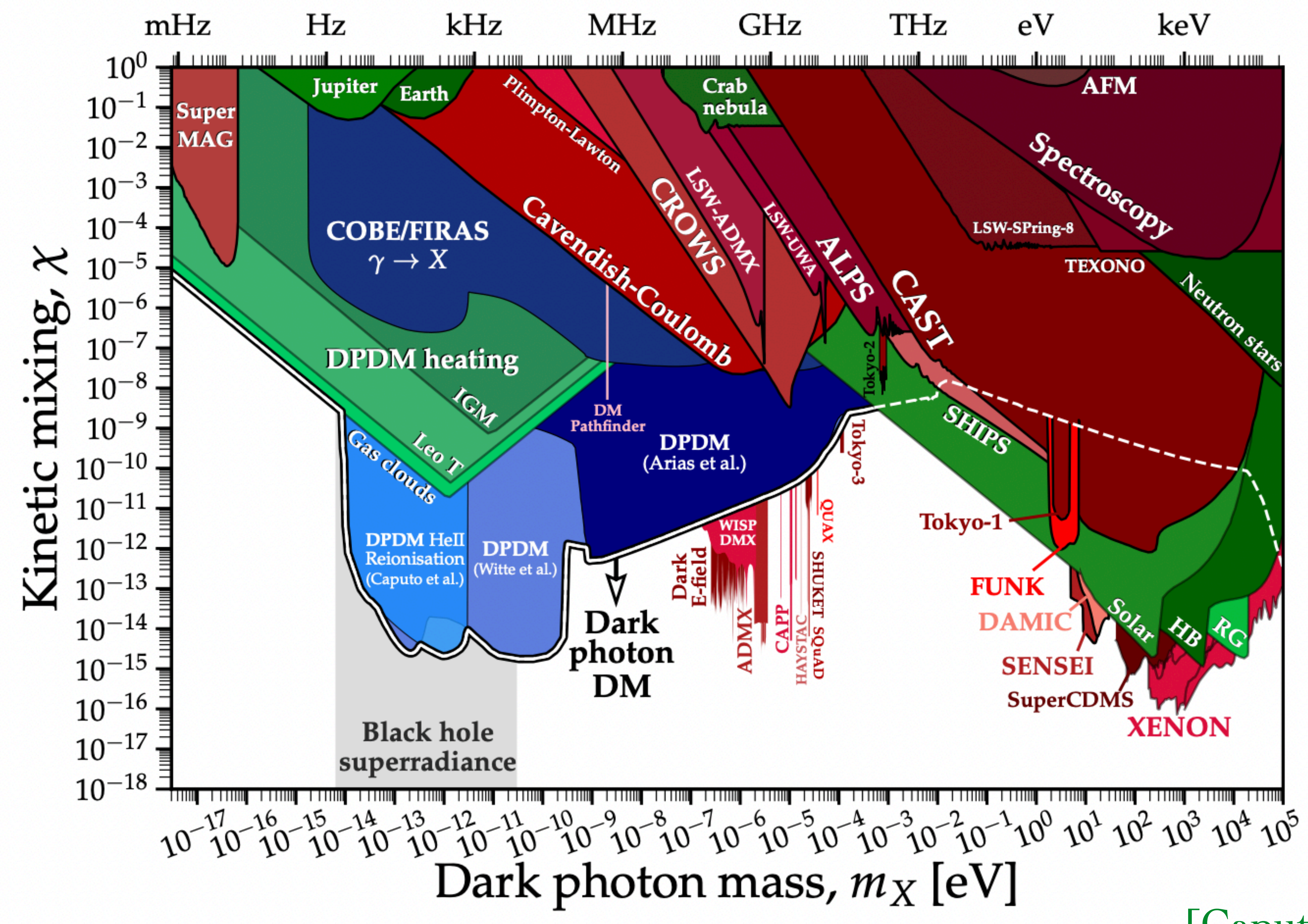
- Dark photon Dark Matter

- Phase Transitions

Vector DM

- 2204.14274 with A. Tesi

Dark photons

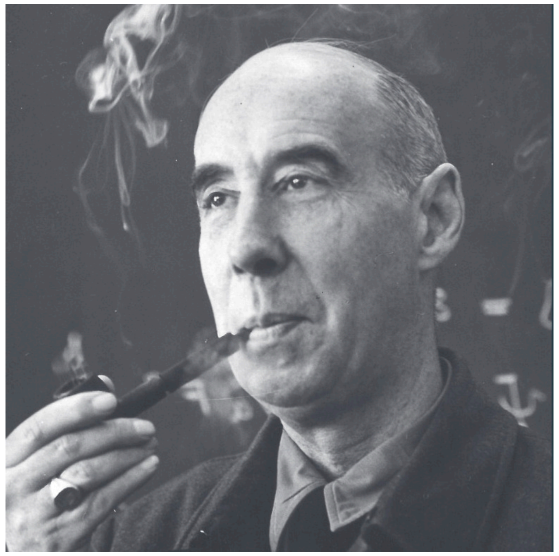


[Caputo et al '21]

Great interest in (Stueckelberg) dark photons!

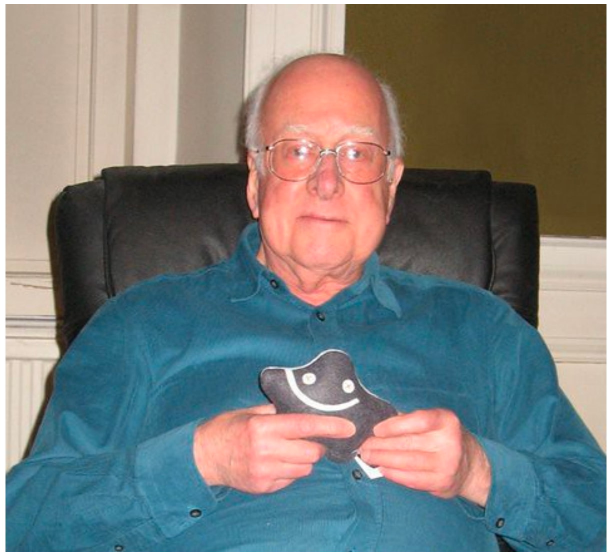
2 theories

Stueckelberg



$3 = 2 + 1$

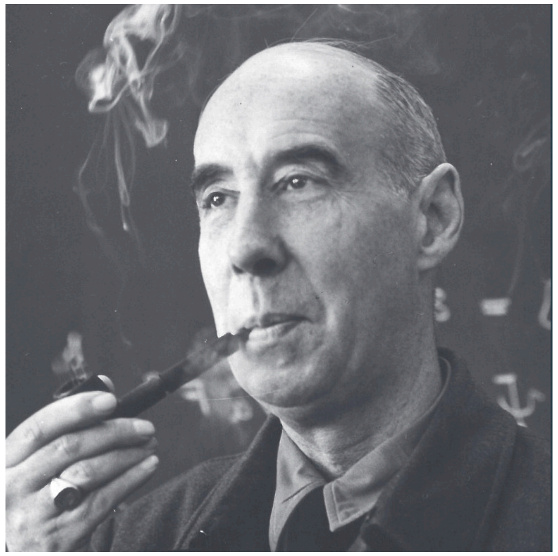
Higgs



$2 + 2 = 3 + 1 = 4$

- Stueckelberg:

Massive spin-1 field described by the Proca lagrangian:



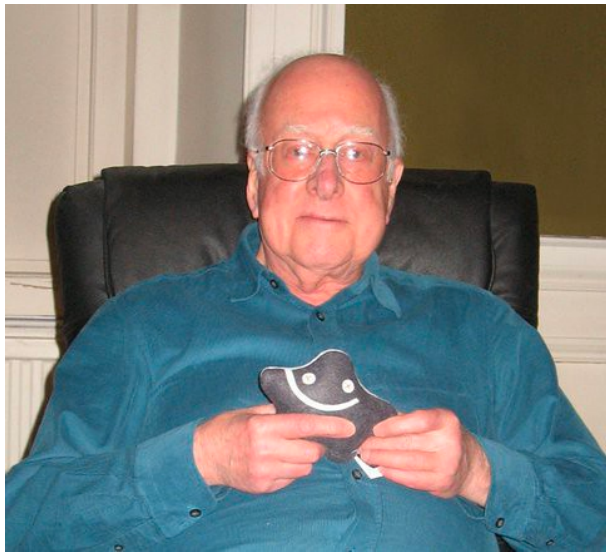
$$L = -\frac{1}{4}F_{\mu\nu}^2 + \frac{f^2}{2}(\partial_\mu\theta - gA_\mu)^2$$

3 = 2 + 1

free theory

- Higgs:

Massive spin-1 particle arise from the Higgs mechanism:

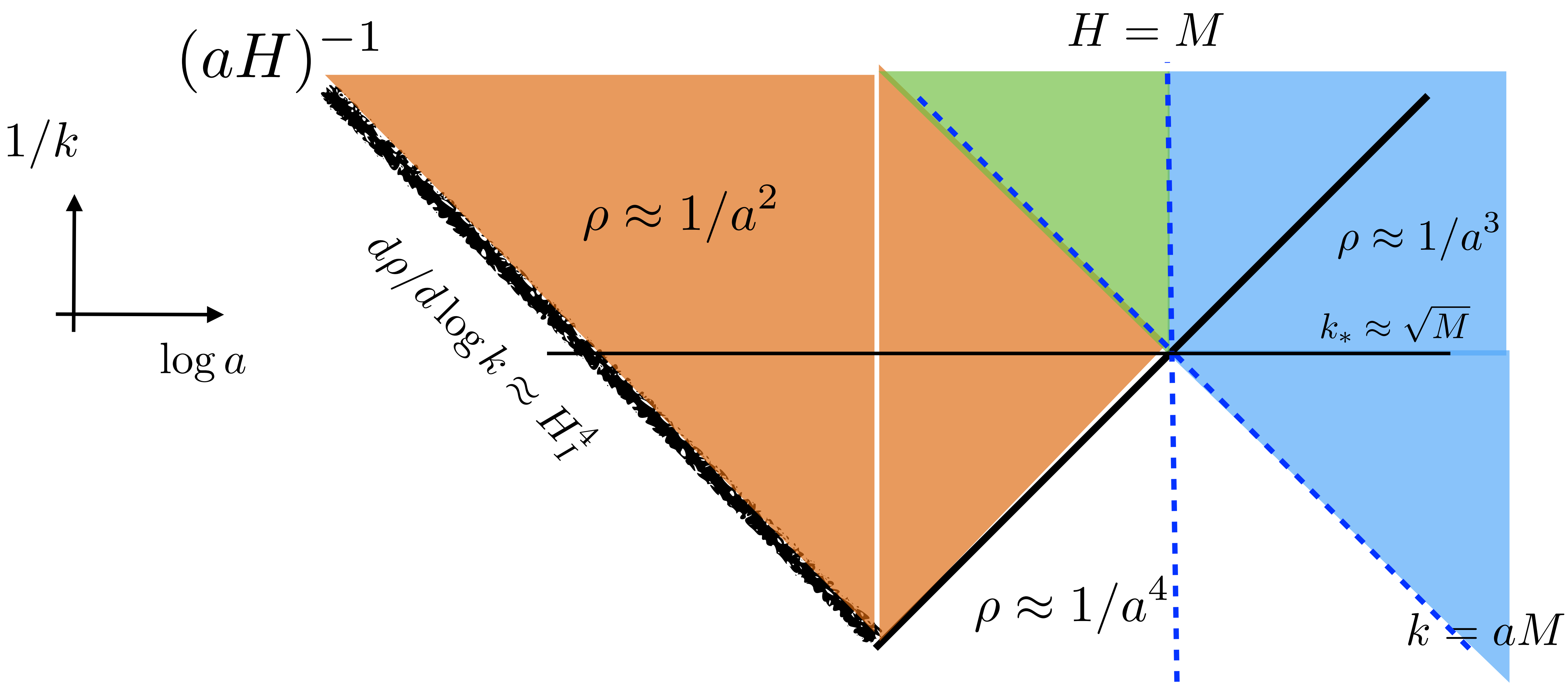


$$L = -\frac{1}{4}F_{\mu\nu}^2 + \frac{1}{2}(\partial\chi)^2 + \frac{(f+\chi)^2}{2}(\partial_\mu\theta - gA_\mu)^2 - \frac{\lambda}{4}\chi^2(2f+\chi)^2$$

4 = 3 + 1

At high energies a Stueckelberg vector is equivalent to a minimally coupled scalar + massless gauge field. Scalar is produced during inflation but evolution is different:

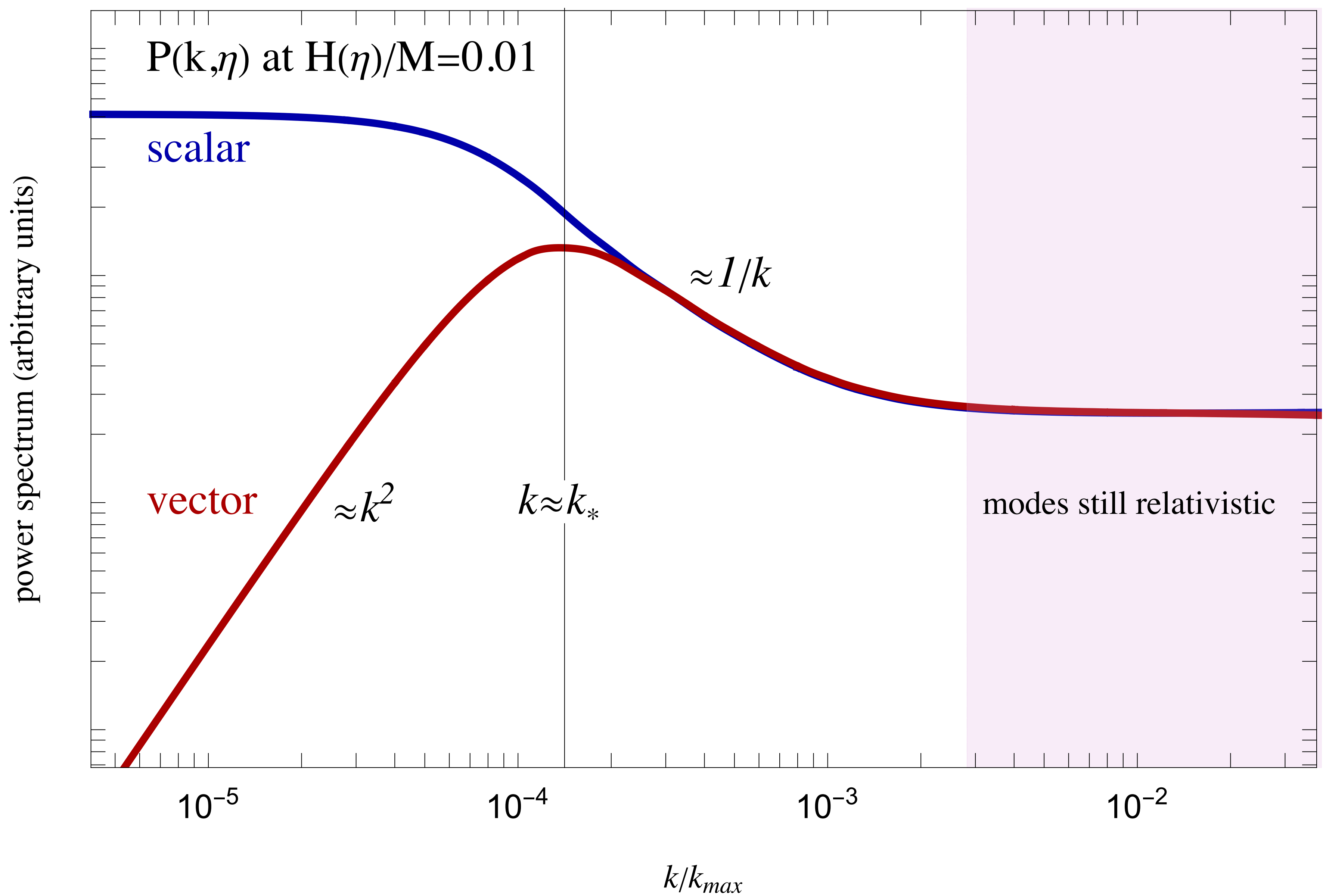
$$\ddot{A}_L + H\left(1+\frac{2k^2}{k^2+a^2M^2}\right)\dot{A}_L + \frac{k^2}{a^2}A_L + M^2 A_L = 0$$



[Graham, Mardon, Rajendran '16]

scalar vs vector

$$\frac{1}{s}\frac{d\rho_A}{d\log k}\bigg|_L \approx \frac{1}{g_*(M)^{1/4}}\frac{H_I^2}{(2\pi)^2}\frac{\sqrt{M}}{M_{pl}^{3/2}}\begin{cases} k_*/k & k \gg k_* \\ 1 & k \approx k_* \\ k^2/k_*^2 & k \ll k_* \end{cases}$$



Dangerous isocurvature perturbations eliminated!

Higgs Dark Photon

$$\mathscr{L}_D = -\frac{1}{4}F_{\mu\nu}^2 + |D_\mu \Phi|^2 - \lambda\left(|\Phi|^2 - \frac{f^2}{2}\right)^2 + \xi|\Phi|^2 R$$

In flat space U(1) symmetry is spontaneously broken:

$$M_A = gf\,, \quad M_\phi = \sqrt{2\lambda} f$$

Dynamics depends on the scale of inflation:

$$V = \lambda\left(|\Phi|^2 - \frac{f^2}{2}\right)^2 + 12H_I^2\xi|\Phi|^2$$

- Higgs phase:

$$H_I < f \qquad \& \qquad 24\xi H_I^2 < \lambda f^2 = \frac{M_\phi^2}{2}$$

Symmetry is broken during an after inflation, d.o.f. are massive vector and a Higgs. Dynamics is model dependent:

- $M_A < H_I < M_\phi$

Stueckelberg dark photon is recovered:

$$\frac{\Omega_A^{\rm GMR} h^2}{0.12} \approx \sqrt{\frac{M_A}{6\times 10^{-6}\,{\rm eV}}} \left(\frac{H_I}{10^{14}\,{\rm GeV}}\right)^2$$

Light dark photon requires ridiculous couplings:

$$g < 6\cdot 10^{-29}\left(\frac{10^{14}\,{\rm GeV}}{H_I}\right)^5 = 3\cdot 10^{-11}\left(\frac{M_A}{\rm GeV}\right)^{5/4}$$

Such small couplings might be incompatible with the weak gravity conjecture. In the simplest version

$$\Lambda < g M_p$$

Arkani-Hamed et al '06

Naively imposing the weak gravity conjecture $H_I < g M_p$.
To reproduce the abundance:

$$H_I < 10^{10}\,\mathrm{GeV}\,, \qquad M_A > 50\,\mathrm{GeV}$$

Even for Stueckelberg theories not clear if a large gap between M_A and H_I can be generated.

Reece '19

- $M_{A,\phi} < H_I$

The scalar participates to the dynamics.
For minimal coupling it is copiously produced during inflation and then decays to dark photons if kinematically allowed:

$$\frac{\rho_A}{s}\Big|_{\rm decays} \approx 5\times 10^{-3}\frac{H_I^2}{M_{\rm Pl}^{3/2}}\frac{M_A}{\sqrt{M_\phi}}\left(\frac{100}{g_*(M_\phi)}\right)^{1/4}\log\frac{k_*}{H_0}$$

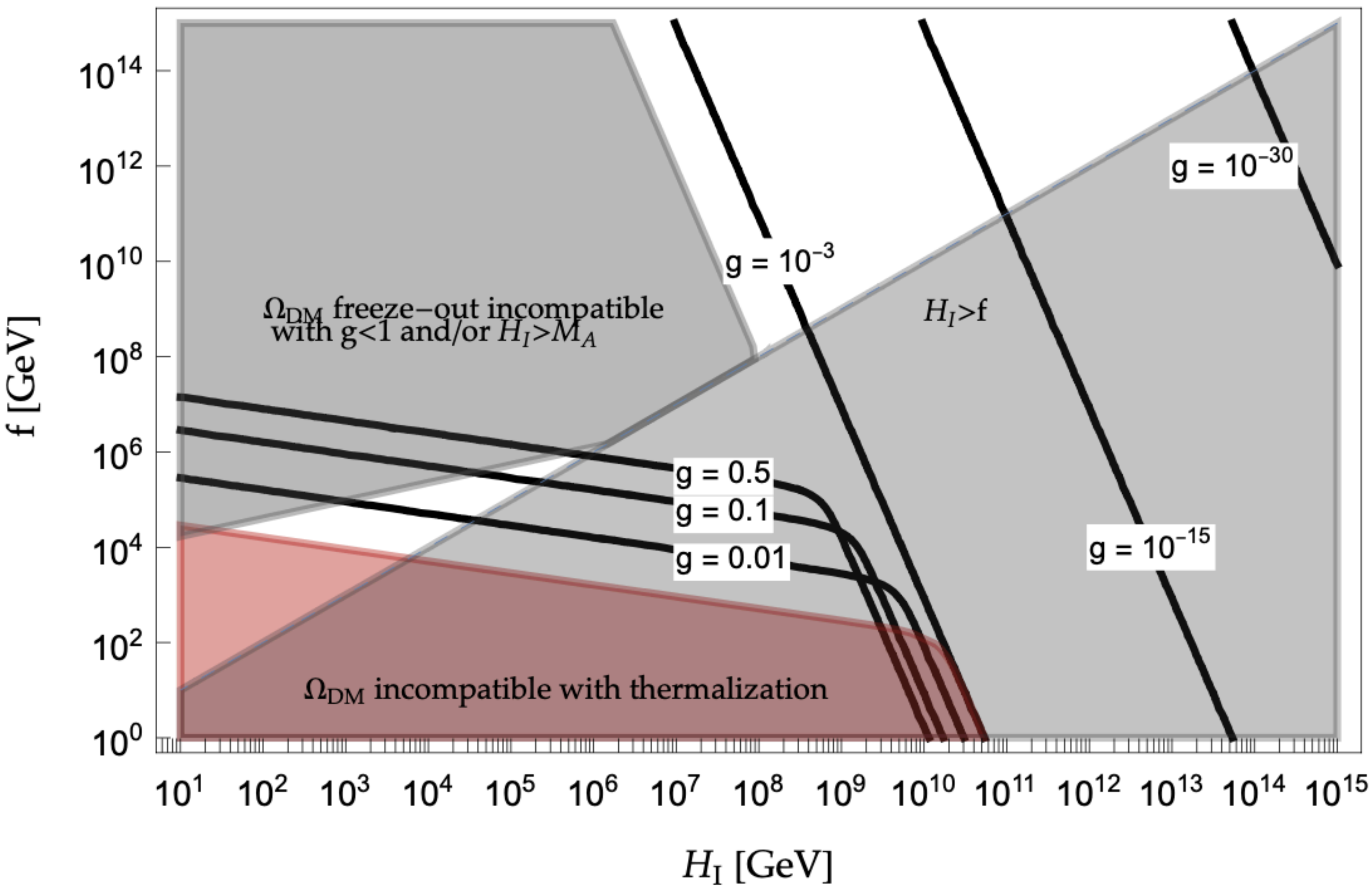
Isocurvature constraints are back to life:

$$\Delta_{\rm iso} \approx \langle(\frac{\delta\rho_{\rm iso}}{\rho})^2\rangle \lesssim 10^{-11} \quad k \approx k_{\rm cmb}$$

$$\Delta_{\rm iso} \approx \frac{M_A}{M_\phi} \quad \text{if } \phi \to \text{AA} \qquad\qquad \Delta_{\rm iso} \approx \frac{M_\phi}{M_A} \quad \text{if stable}$$

Isocurvature can also be suppressed $M_\phi \sim H_I$.

f > H_I



10^{14}
10^{12}
10^{10}
10^{8}
10^{6}
10^{4}
10^{2}
10^{0}
f [GeV]
Ω_{DM} freeze−out incompatible with g<1 and/or $H_I > M_A$
g = 10^{-3}
g = 10^{-30}
H_I>f
g = 0.5
g = 0.1
g = 0.01
g = 10^{-15}
Ω_{DM} incompatible with thermalization
10^{1} 10^{2} 10^{3} 10^{4} 10^{5} 10^{6} 10^{7} 10^{8} 10^{9} 10^{10} 10^{11} 10^{12} 10^{13} 10^{14} 10^{15}
H_I [GeV]

- Coulomb phase:

$$H_I \gg f$$

Symmetry is restored during inflation:

- $\xi = \frac{1}{6}$ $\qquad V = \lambda \left(|\Phi|^2 - \frac{f^2}{2} \right)^2 + 2H_I^2 |\Phi|^2$

Inflationary fluctuations are negligible however symmetry is restored due to the coupling to curvature. During reheating or radiation the symmetry gets broken and a string network forms through the Kibble mechanism. Dark photons are emitted similarly to axions:

$$\left.\frac{\rho_A}{s}\right|_{\text{string}} \approx \kappa \sqrt{M_A} \frac{f^2}{M_{\text{Pl}}^{3/2}} \left(\frac{100}{g_*} \right)^{1/4}$$

[Long, Wang '19]

- $\xi = 0$

Inflationary fluctuations

$$\delta\Phi \approx \frac{H_I}{2\pi}$$

The field is coherent over distances:

$$d \sim \frac{1}{H_I} \exp[\sqrt{\frac{8\pi^2}{9\lambda}}]$$

For small λ symmetry is broken during inflation so the Higgs and dark photons are produced.

$$\frac{\rho_A}{s}\Big|_{\mathrm{inf,decays}} \approx 0.005 \frac{M_A}{\lambda^{1/4}} \frac{H_I^{3/2}}{(M_p)^{3/2}} \log \frac{k_*}{H_0}$$

Avoiding isocurvature requires $\lambda \sim 1$.

It's complicated!

	$M_\phi \gg H_I \gg M_A$	$H_I \gg M_\phi \gg M_A$	$H_I \gg M_A \gg M_\phi$
$f > H_I$	GMR	GMR + ϕ-decay + (iso) $\xi = 0$	GMR + thermal-FO + (iso) $\xi = 0$
$f < H_I, \xi = \frac{1}{6}$	–	string network	string network
$f < H_I, \xi = 0$	–	string net.+ ϕ-decay +(iso)	string net. + thermal-PT +(iso)

Stueckelberg DM only recovered in tiny regions of parameters.

Strong constraint from isocurvature perturbations unless the sector is Weyl invariant.

DM from a Phase Transition

- 2210.03108 with A. Tesi

$$H_I \gg M$$

I will consider Weyl invariant dark sectors that undergoes a phase transition.

- Conformally coupled scalar:

$$L = \frac{(\partial_\mu \phi)^2}{2} - \frac{1}{2}\mu^2\phi^2 + \frac{1}{12}\phi^2 R - \frac{\lambda}{4}\phi^4$$

- $\mu^2 > 0$

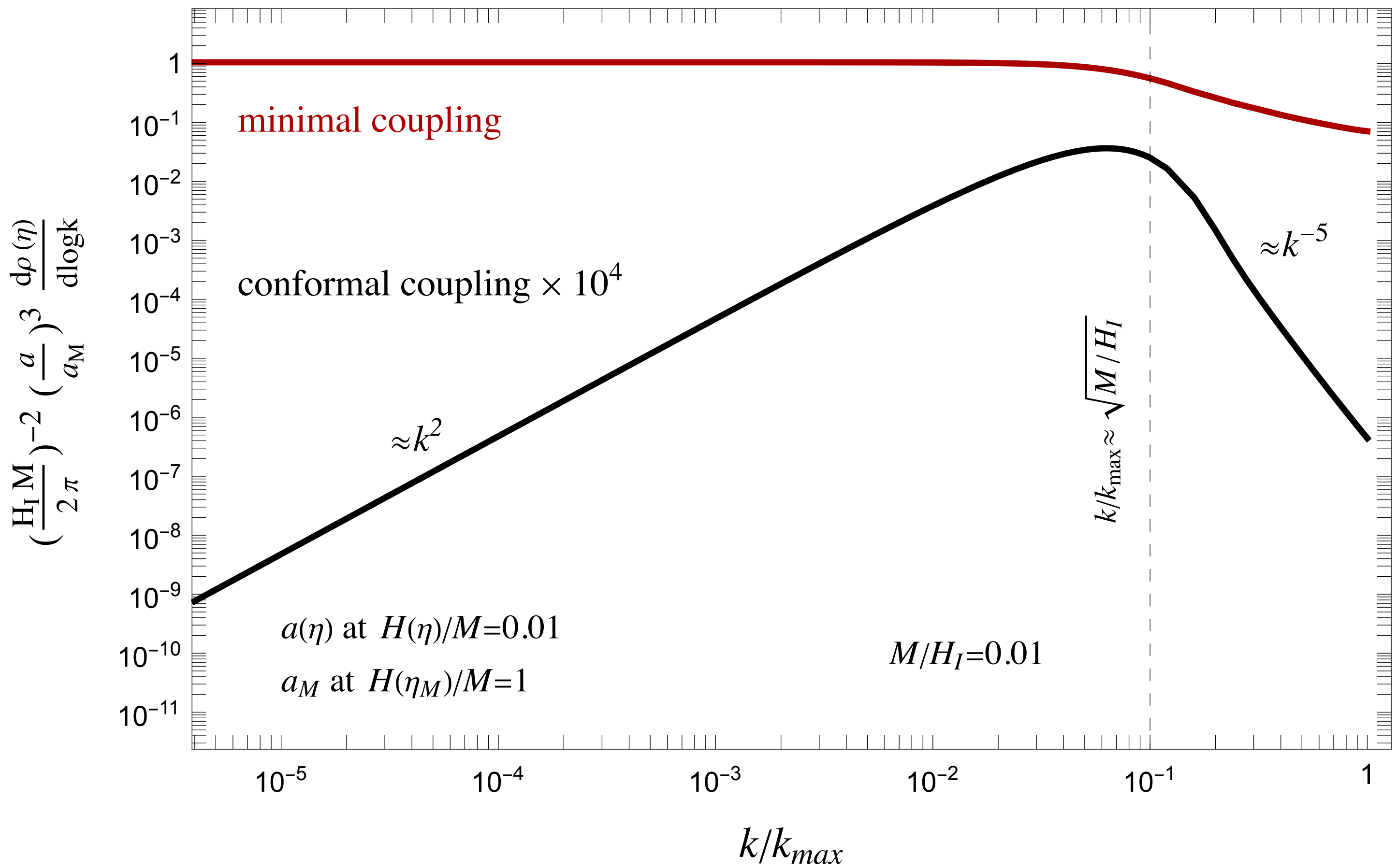
$$v'' + k^2 v + M^2 a^2 v = 0$$

Particle production peaked at:

$$H = M = \frac{k}{a}$$

$$k_{\rm peak} = \begin{cases} a_R\sqrt{H_R M} & T_R > \sqrt{M M_p} \\ a_R\sqrt{H_R M}\left(\frac{H_R}{M}\right)^{1/3} & T_R < \sqrt{M M_p} \end{cases}$$

@ radiation:



$$\left.\frac{\rho}{s}\right|_{\text{quantum}} = 0.0002\, M \left(\frac{M}{M_p}\right)^{\frac{3}{2}}$$

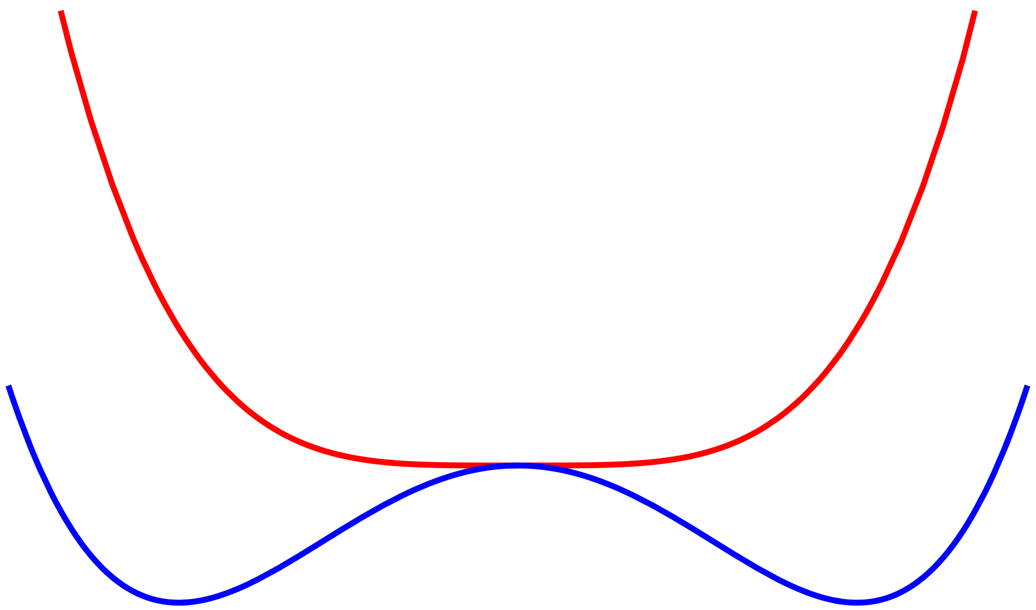
DM abundance reproduced for $M_{\text{DM}} = 5.7 \times 10^8\, \text{GeV}$

@ reheating:

$$\left.\frac{\rho}{s}\right|_{\text{quantum}} \approx 10^{-4}\, T_R \frac{M^2}{M_p^2}$$

- $\mu^2 < 0$

In flat space the minimum is at $\phi = -\mu^2/\lambda$. During inflation the conformal coupling produces a positive mass $2H_I^2$ so that the symmetry is restored. A phase transition takes place in radiation or reheating releasing the latent heat:



$$\Delta V = \frac{M^4}{16\lambda} \qquad @ \qquad H \sim M$$

The abundance is:

radiation:

$$\left.\frac{\rho_\phi}{s}\right|_{\rm PT} = \frac{\Delta V}{s(T_*)} \approx 0.1 \frac{M}{\lambda} \frac{M^{3/2}}{M_p^{3/2}}$$

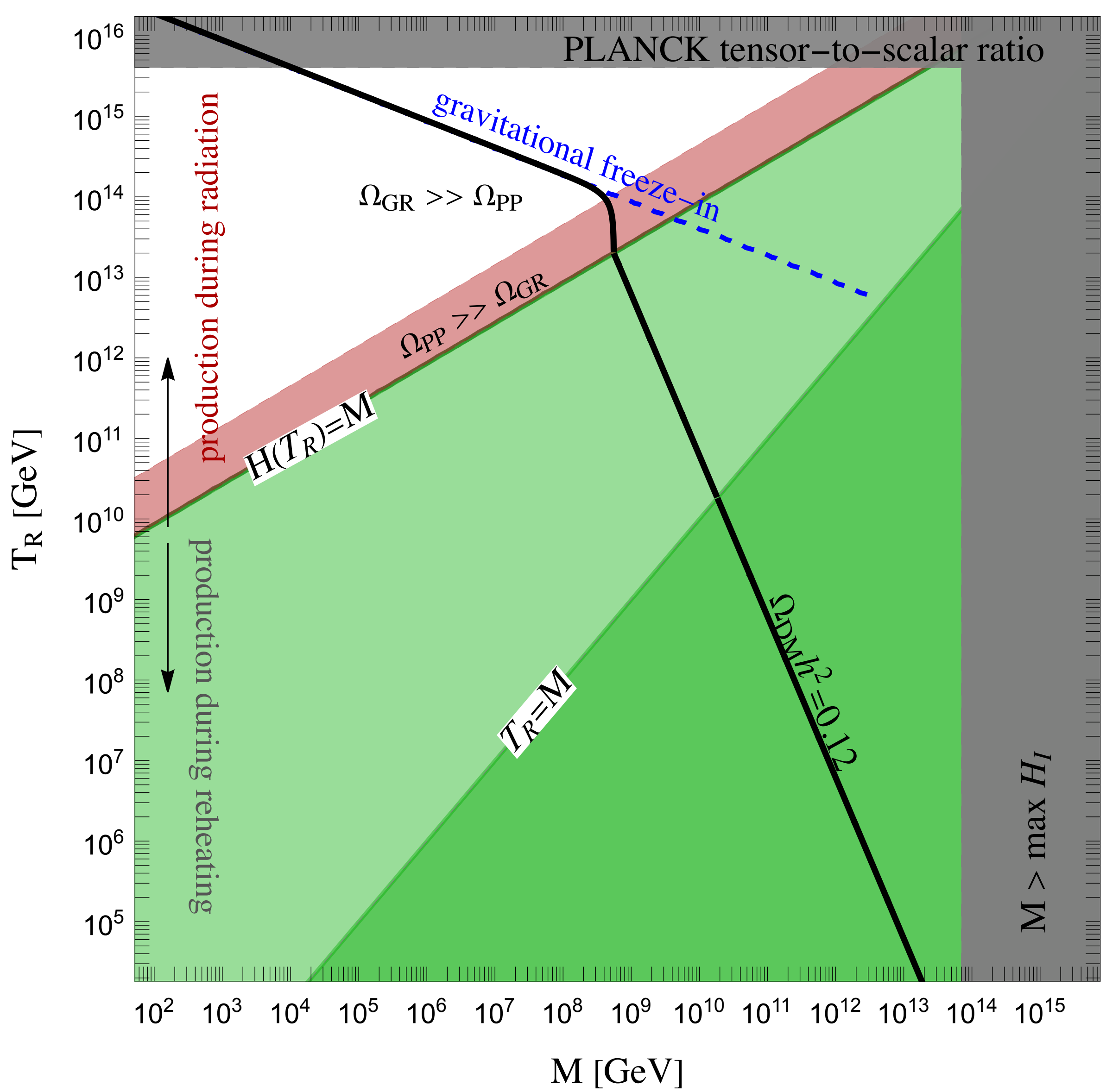
reheating:

$$\left.\frac{\rho_\phi}{s}\right|_{\rm PT} = \frac{\Delta V}{s(T_R)} \left(\frac{a_*}{a_R}\right)^3 \approx 0.5 \frac{T_R}{\lambda} \frac{M^2}{M_p^2}$$

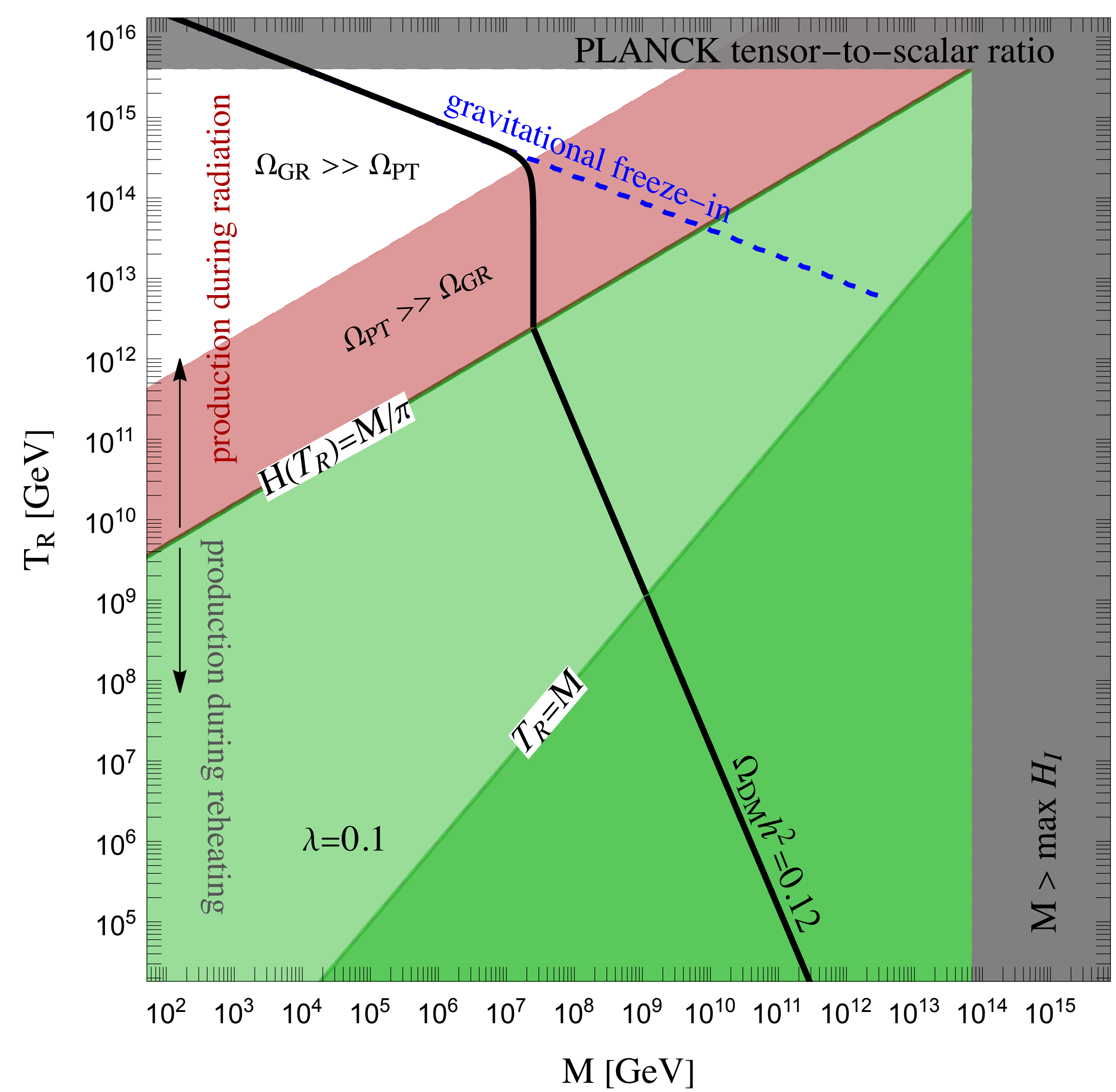
Gravitational freeze-in:

$$\left.\frac{\rho}{s}\right|_{\rm GR} = 8\times 10^{-6} M \left(\frac{T_R}{M_p}\right)^3$$

quantum vs. freeze-in



PT vs. freeze-in



Glueball DM

[see 1710.06447]

A very minimal scenario for DM is a decoupled ``pure glue" gauge theory. Simplest example is $\mathrm{SU}(3)$:

$$\frac{M_{\mathrm{DG}}}{\Lambda} \approx 5.5 \qquad \frac{L_h}{\Lambda^4} \approx 1.4$$

Lightest glueball is a CP even scalar that is automatically stable:

$$\tau \sim \frac{M_p^4}{M_{\mathrm{DG}}^5} \sim 10^{19}\,\mathrm{s}\left(\frac{10^6\,\mathrm{GeV}}{M_{\mathrm{DG}}}\right)^5$$

- thermal scenario

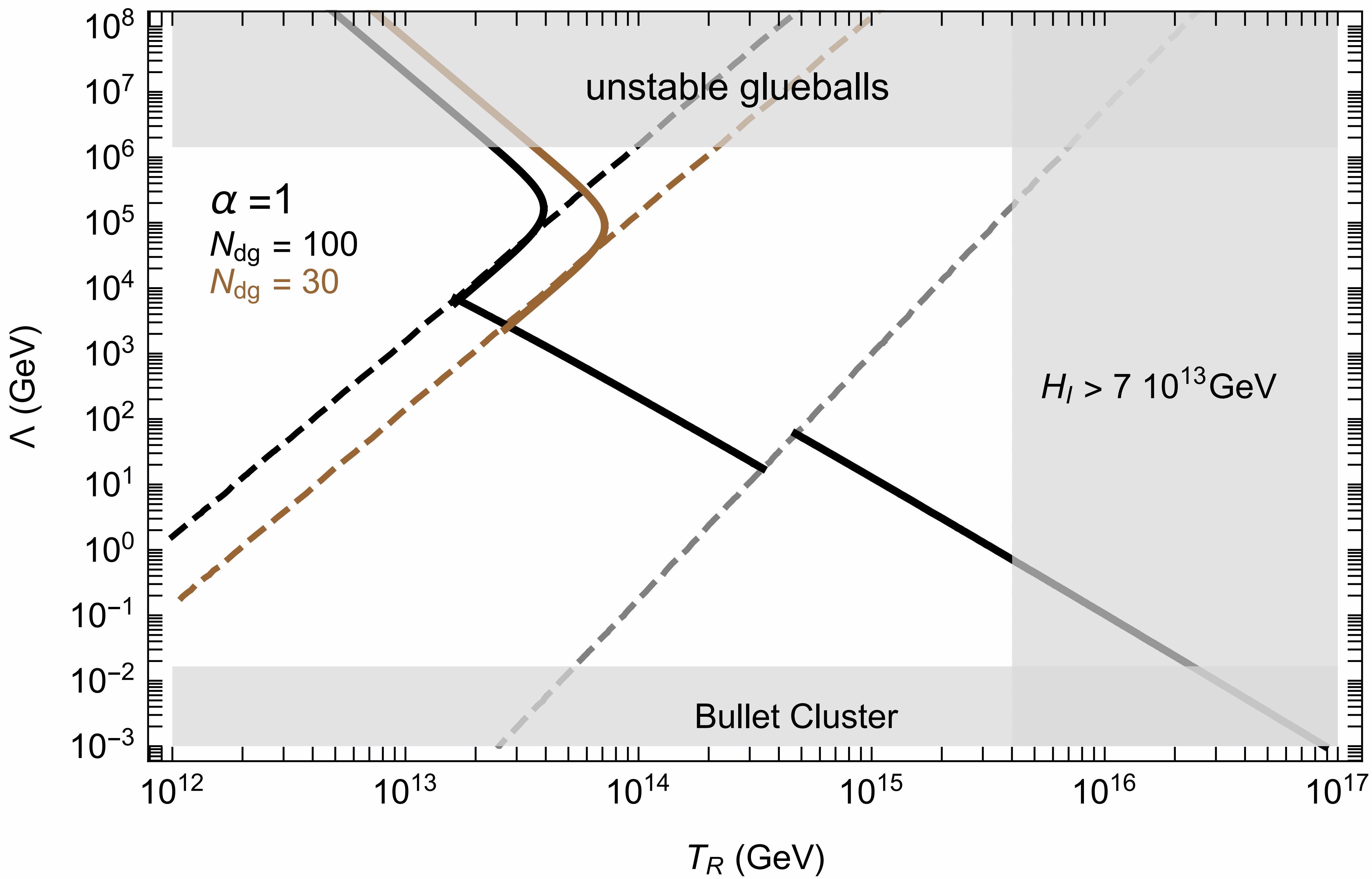
If the sector thermalizes in the relativistic regime

$$\frac{\rho}{s} \approx \frac{g_D}{g^*}\Lambda\,\xi^3 \qquad \xi = \frac{T_D}{T}$$

Gravitational freeze-in:

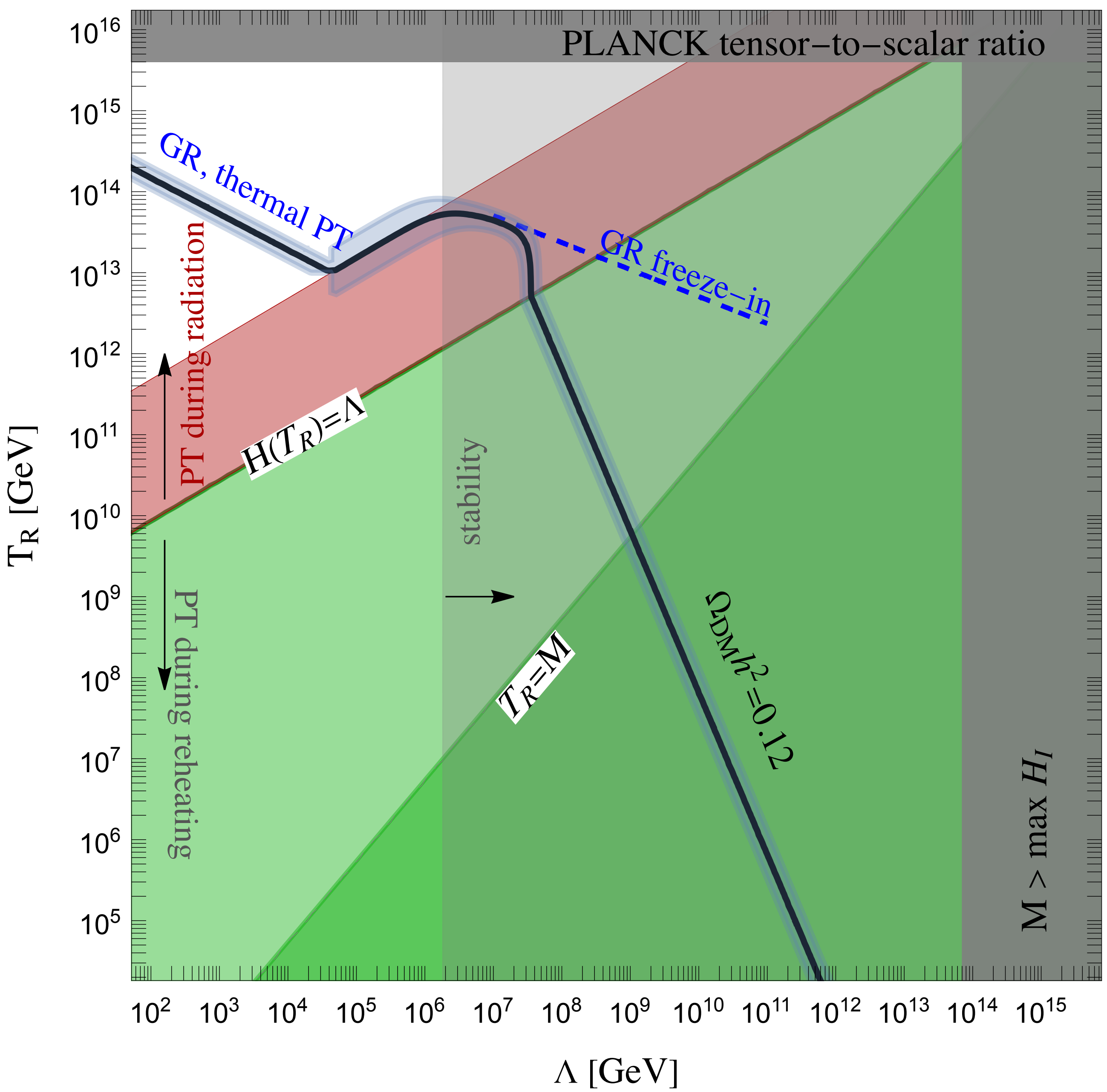
[MR, Tesi, Tillim '20]

$$\xi = 0.4 \left(\frac{T_R}{M_p} \right)^{3/4} \qquad T_R > 10^{13} \mathrm{GeV}$$



- No thermalization:

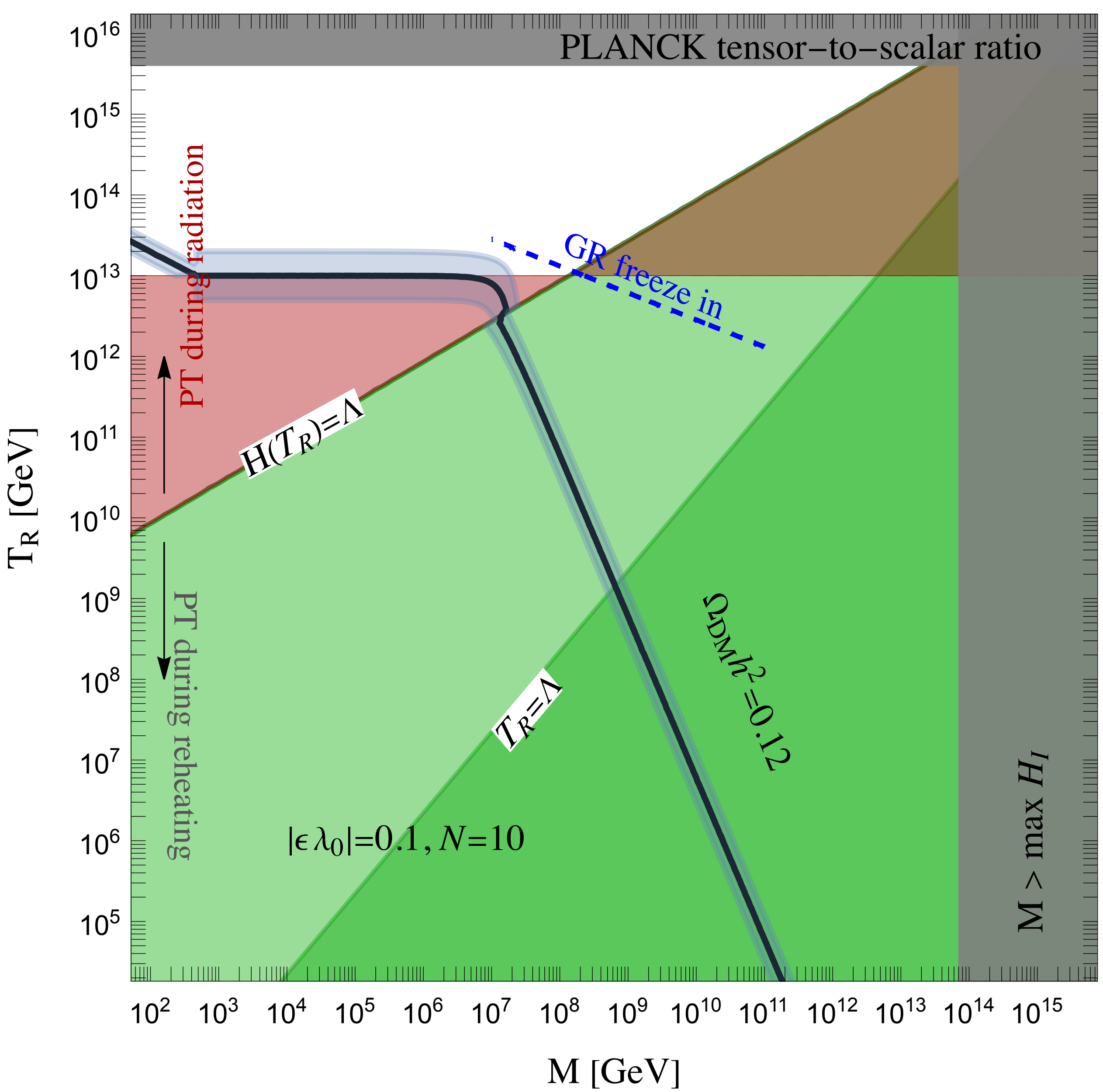
Gluons are deconfined during inflation for $H_I > \Lambda$.
Inflation prepares the system in an empty false vacuum state.
As for the scalar an energy Λ^4 will be released @ $H \sim \Lambda$:



$$\frac{\rho_{\rm DG}}{s} \sim 0.1\,\Lambda\,{\rm Min}\left[\left(\frac{\Lambda}{M_p}\right)^{3/2},\frac{\Lambda T_R}{M_p^2}\right]$$

- Dilaton Dark Matter:

Very similar conclusions in strongly coupled scenarios described by their holographic Randall-Sundrum dual (with light dilaton).



$$\Delta V \sim \frac{N^2}{64\pi^2}\epsilon\Lambda^4$$

$$M^2 \sim \epsilon\Lambda^2$$

$$\left.\frac{\rho_{\rm dilaton}}{s}\right|_{\rm PT} = \frac{N^2}{64\pi^2}\frac{1}{|\epsilon\lambda_0|}\min\left[0.5\frac{M^{5/2}}{M_p^{3/2}}, \frac{T_R M^2}{M_p^2}\right]$$

SUMMARY

- Interactions can significantly modify inflationary production of dark matter.

- Massive vector DM can be realized through the Stueckelberg lagrangian or the Higgs mechanism. Phenomenology is vastly different. Only Weyl invariant theories are safe from isocurvature constraints.

- In the second part I suggested the possibility to populate the dark sector through a phase transition. The mechanism is more simple for Weyl invariant theories where the control parameter is Hubble. Non thermal phase transitions should be further studied.

- β-functions:

For $H_I > \Lambda$ the gluons are deconfined during inflation. Since the action is Weyl invariant inflationary production is small. In curved space 1PI effective action,

$$L = -\frac{1}{4g^2}\left[1 + \frac{g^2 b_0}{16\pi^2}\log(\frac{-\Box}{M^2})\right] G^2_{\mu\nu} + \left(\frac{b_0}{32\pi^2}\right)\log a(t)\, G^2_{\mu\nu} + \cdots$$

$$A''_T + k^2 A_T - 2\Delta\,\frac{a'}{a}A'_T = 0\,, \quad \Delta = -\frac{b_0 g^2}{16\pi^2}$$

One can compute the abundance of gluons by solving the wave-equation,

$$\rho_{\text{gluons}} \sim (N^2-1)\frac{H_I^4}{\pi^2}\Delta^2\frac{a_e^4}{a^4} \ll H^4$$