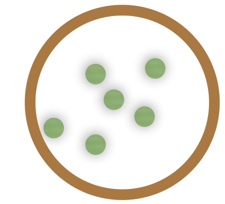
Wave dark matter (axion), Love numbers & quasi-normal modes

Lam Hui
Columbia University

Wave dark matter (axion or axion-like)

- wave regime: $\underbrace{\text{particle separation}}_{(\rho/m)^{-1/3}} < \underbrace{\lambda_{\text{de Broglie}}}_{1/mv} \longrightarrow m < 30\,\text{eV}$



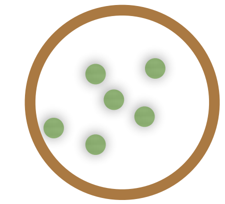
e.g. $\lambda_{\text{de Broglie}} \sim 100\,\text{pc}$ for $m = 10^{-22}\,\text{eV}$ $\qquad$ $\lambda_{\text{de Broglie}} \sim 100\,\text{m}$ for $m = 10^{-6}\,\text{eV}$

1610.08297 with Ostriker, Tremaine, Witten
2004.01188 with Joyce, Landry, Li
2101.11735 review

Wave dark matter (axion or axion-like)

- wave regime: particle separation $< \lambda_{\rm de\,Broglie}$ $\longrightarrow$ $m < 30\,{\rm eV}$

  $(\rho/m)^{-1/3}$ $\qquad$ $1/mv$



e.g. $\lambda_{\rm de\,Broglie} \sim 100\,{\rm pc}$ for $m = 10^{-22}\,{\rm eV}$ $\qquad$ $\lambda_{\rm de\,Broglie} \sim 100\,{\rm m}$ for $m = 10^{-6}\,{\rm eV}$

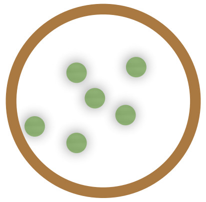
- simplest mis-alignment mechanism: (not QCD axion)

$$\Omega_{\rm matter} \sim 0.1 \left(\frac{F}{10^{17}\,{\rm GeV}}\right)^2 \left(\frac{m}{10^{-22}\,{\rm eV}}\right)^{1/2}$$

1610.08297 with Ostriker, Tremaine, Witten
2004.01188 with Joyce, Landry, Li
2101.11735 review

Wave dark matter (axion or axion-like)

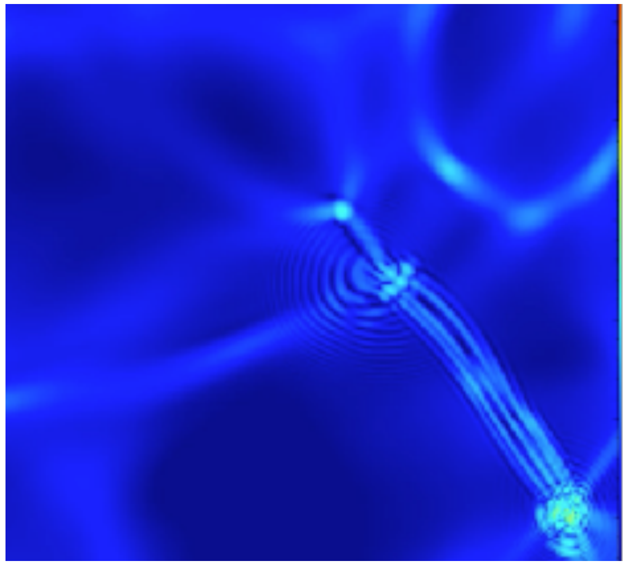
- wave regime: $\underset{(\rho/m)^{-1/3}}{\text{particle separation}} < \underset{1/mv}{\lambda_{\text{de Broglie}}} \longrightarrow m < 30\,\text{eV}$

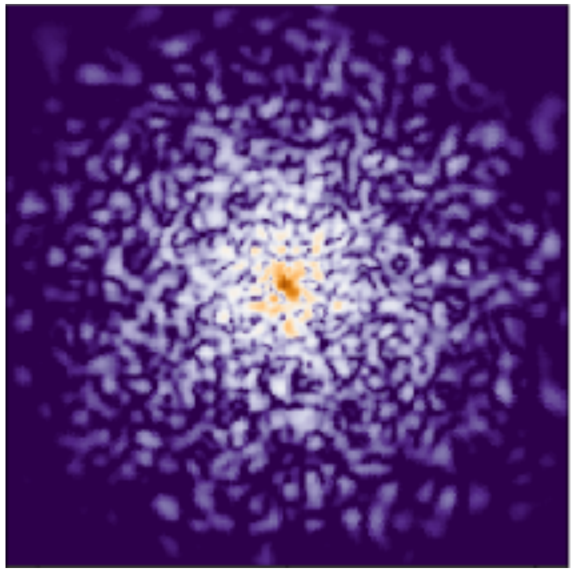


e.g. $\lambda_{\text{de Broglie}} \sim 100\,\text{pc for } m = 10^{-22}\,\text{eV}$ $\quad\lambda_{\text{de Broglie}} \sim 100\,\text{m for } m = 10^{-6}\,\text{eV}$

- simplest mis-alignment mechanism: (not QCD axion) $\quad\Omega_{\text{matter}} \sim 0.1 \left(\frac{F}{10^{17}\,\text{GeV}}\right)^2 \left(\frac{m}{10^{-22}\,\text{eV}}\right)^{1/2}$

- wave interference:



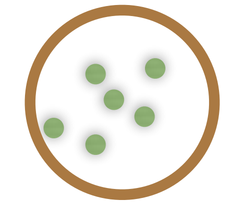


1610.08297 with Ostriker, Tremaine, Witten
2004.01188 with Joyce, Landry, Li
2101.11735 review

Wave dark matter (axion or axion-like)

- wave regime: particle separation $<\lambda_{\text{de Broglie}}$ $\longrightarrow$ $m < 30\,\text{eV}$

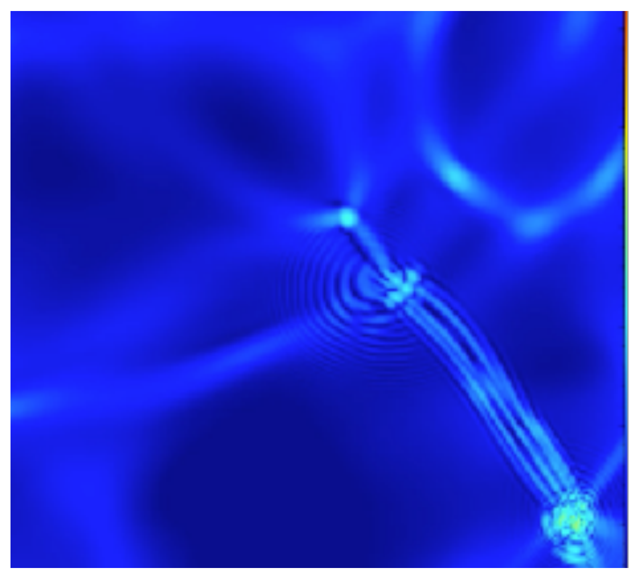
  $(\rho/m)^{-1/3}$ $\qquad$ $1/mv$

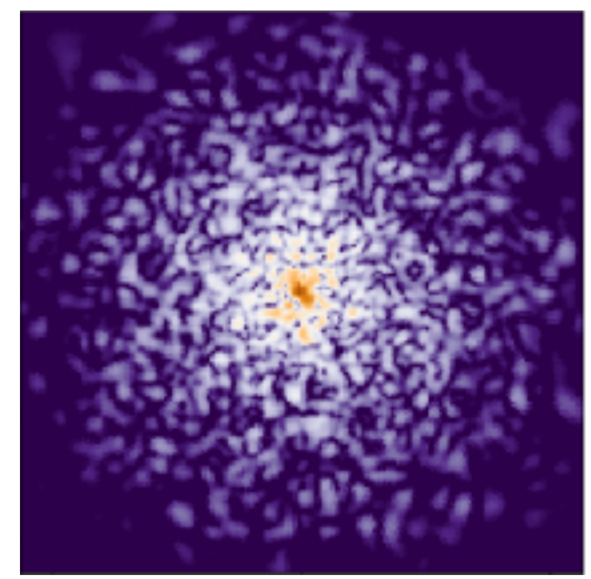


  e.g. $\lambda_{\text{de Broglie}} \sim 100\,\text{pc}$ for $m = 10^{-22}\,\text{eV}$ $\qquad$ $\lambda_{\text{de Broglie}} \sim 100\,\text{m}$ for $m = 10^{-6}\,\text{eV}$

- simplest mis-alignment mechanism: $\Omega_{\text{matter}} \sim 0.1 \left(\frac{F}{10^{17}\,\text{GeV}}\right)^2 \left(\frac{m}{10^{-22}\,\text{eV}}\right)^{1/2}$
  (not QCD axion)

- wave interference:

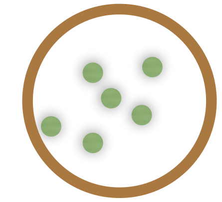




- non-relativistic regime: $\phi = e^{-imt}\psi + \text{ c.c.}$ $\qquad$ $\rho = m|\psi|^2$

1610.08297 with Ostriker, Tremaine, Witten
2004.01188 with Joyce, Landry, Li
2101.11735 review

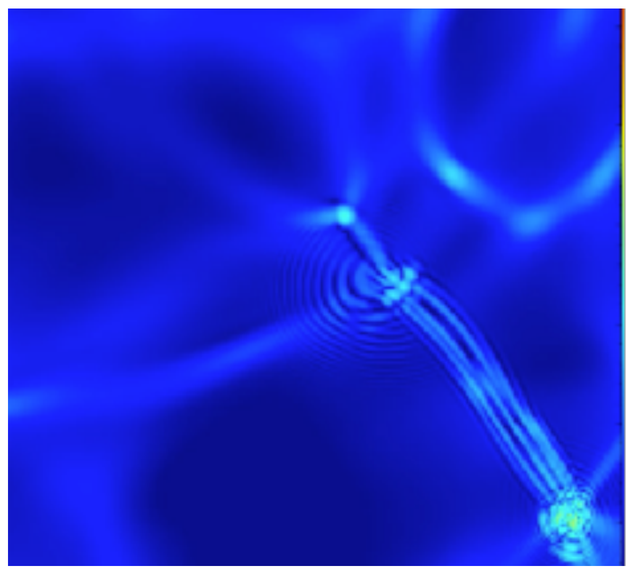
Wave dark matter (axion or axion-like)

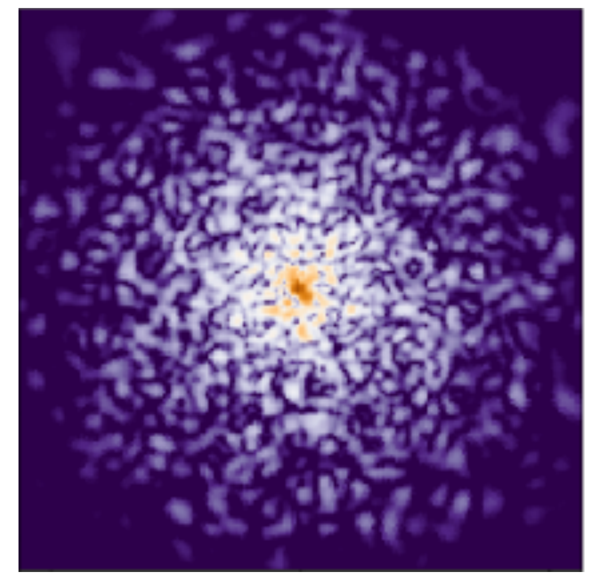
- wave regime: $\underset{(\rho/m)^{-1/3}}{\text{particle separation}} < \underset{1/mv}{\lambda_{\rm de\,Broglie}} \longrightarrow m < 30\,{\rm eV}$ 

  e.g. $\lambda_{\rm de\,Broglie} \sim 100\,{\rm pc}$ for $m = 10^{-22}\,{\rm eV}$ $\qquad\lambda_{\rm de\,Broglie} \sim 100\,{\rm m}$ for $m = 10^{-6}\,{\rm eV}$

- simplest mis-alignment mechanism: $\Omega_{\rm matter} \sim 0.1 \left(\frac{F}{10^{17}\,{\rm GeV}}\right)^2 \left(\frac{m}{10^{-22}\,{\rm eV}}\right)^{1/2}$
  (not QCD axion)

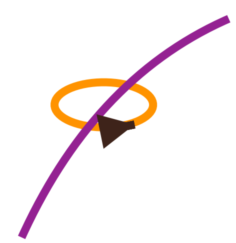
- wave interference:





- non-relativistic regime: $\phi = e^{-imt}\psi + \text{ c.c.}$ $\qquad \rho = m|\psi|^2$

- complete destructive interference - vortex:
  (not axion string)

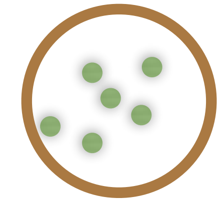


1610.08297 with Ostriker, Tremaine, Witten
2004.01188 with Joyce, Landry, Li
2101.11735 review

Wave dark matter (axion or axion-like)

- wave regime: particle separation $<\lambda_{\text{de Broglie}}$ $\longrightarrow$ $m < 30\,\text{eV}$

  $(\rho/m)^{-1/3}$ $\quad\quad$ $1/mv$

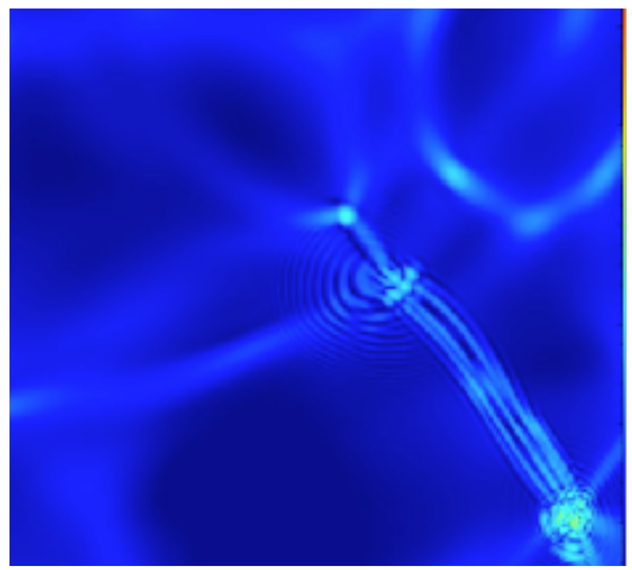


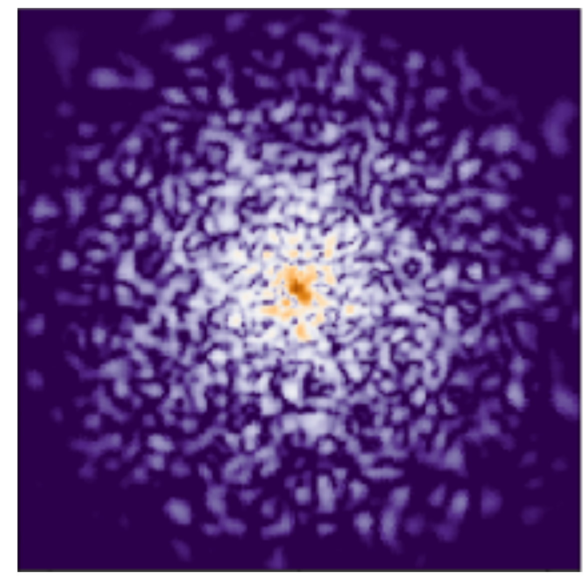
  e.g. $\lambda_{\text{de Broglie}} \sim 100\,\text{pc}$ for $m = 10^{-22}\,\text{eV}$ $\quad\quad$ $\lambda_{\text{de Broglie}} \sim 100\,\text{m}$ for $m = 10^{-6}\,\text{eV}$

- simplest mis-alignment mechanism: (not QCD axion)

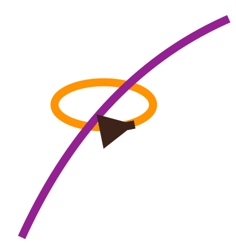
$$\Omega_{\text{matter}} \sim 0.1 \left(\frac{F}{10^{17}\,\text{GeV}}\right)^2 \left(\frac{m}{10^{-22}\,\text{eV}}\right)^{1/2}$$

- wave interference:





- non-relativistic regime: $\phi = e^{-imt}\psi + \text{ c.c.}$ $\quad\quad$ $\rho = m|\psi|^2$
- complete destructive interference - vortex: (not axion string)



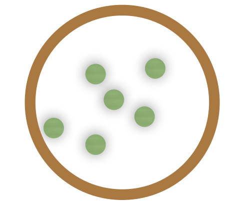
- on average, 1 vortex ring per de Broglie volume

1610.08297 with Ostriker, Tremaine, Witten
2004.01188 with Joyce, Landry, Li
2101.11735 review

Wave dark matter (axion or axion-like)

- wave regime: particle separation $< \lambda_{\rm de\,Broglie}$ $\longrightarrow$ $m < 30\,{\rm eV}$

  $(\rho/m)^{-1/3}$ $\qquad$ $1/mv$

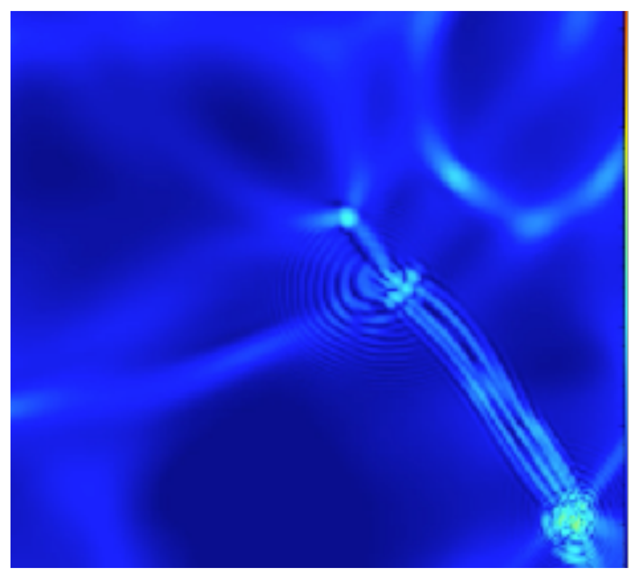


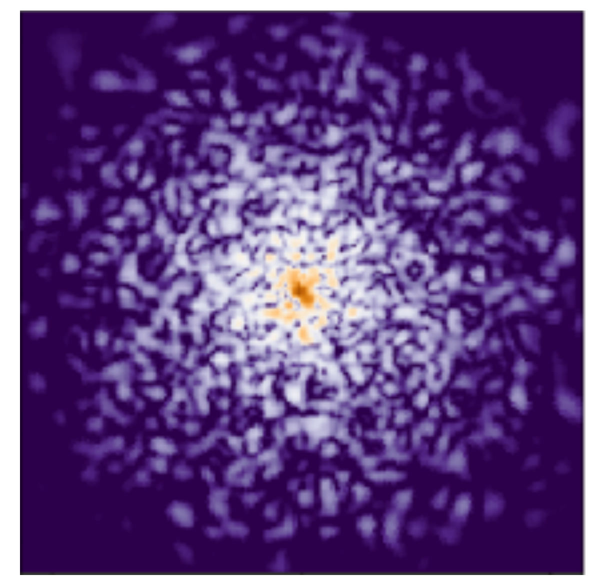
  e.g. $\lambda_{\rm de\,Broglie} \sim 100\,{\rm pc}$ for $m = 10^{-22}\,{\rm eV}$ $\qquad$ $\lambda_{\rm de\,Broglie} \sim 100\,{\rm m}$ for $m = 10^{-6}\,{\rm eV}$

- simplest mis-alignment mechanism: $\quad \Omega_{\rm matter} \sim 0.1 \left(\frac{F}{10^{17}\,{\rm GeV}}\right)^2 \left(\frac{m}{10^{-22}\,{\rm eV}}\right)^{1/2}$

  (not QCD axion)

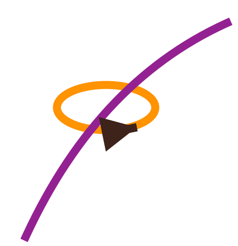
- wave interference:





- non-relativistic regime: $\quad \phi = e^{-imt}\psi + \text{ c.c.} \qquad \rho = m|\psi|^2$
- complete destructive interference - vortex:

  (not axion string)

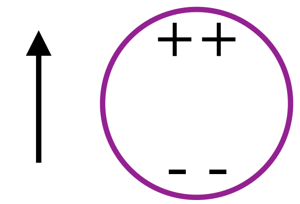


- on average, 1 vortex ring per de Broglie volume
- implications for astrophysical observations and experimental detection

1610.08297 with Ostriker, Tremaine, Witten
2004.01188 with Joyce, Landry, Li
2101.11735 review

Black hole love numbers (static tidal response)

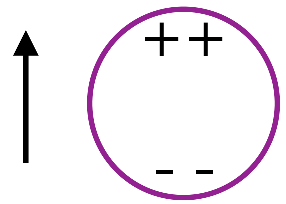
- EM example: External const. electric field induces dipole moment, creating dipolar response field.



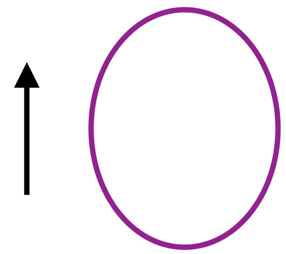
2010.00593, 2105.01069, 2203.08832 with Joyce, Penco, Santoni, Solomon (see also Charalambous, Dubovsky, Ivanov)

Black hole love numbers (static tidal response)

- EM example: External const. electric field induces dipole moment, creating dipolar response field.



- Gravitational example: External tidal field $\phi \sim r^\ell$ distorts object, creating multipolar response field $\phi \sim 1/r^{\ell+1}$.

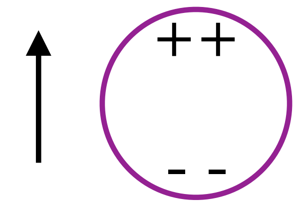


$$\phi \sim r^\ell + ... + \frac{\lambda}{r^{\ell+1}} + ... \quad , \quad \lambda \sim \text{Love number} \ \sim \ \text{size}^{2\ell+1}$$

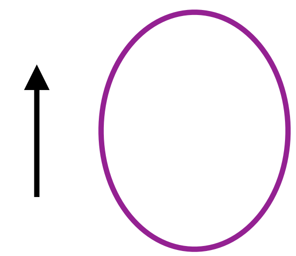
2010.00593, 2105.01069, 2203.08832 with Joyce, Penco, Santoni, Solomon (see also Charalambous, Dubovsky, Ivanov)

Black hole love numbers (static tidal response)

- EM example: External const. electric field induces dipole moment, creating dipolar response field.



- Gravitational example: External tidal field $\phi \sim r^\ell$ distorts object, creating multipolar response field $\phi \sim 1/r^{\ell+1}$.



$$\phi \sim r^\ell + ... + \frac{\lambda}{r^{\ell+1}} + ... \quad , \quad \lambda \sim \text{Love number} \sim \text{size}^{2\ell+1}$$

- $g = g_{\mathrm{BH}} + \phi \longrightarrow [\partial^2 + V]\phi = 0$

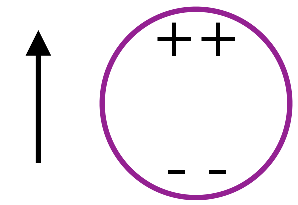
impose $\phi \sim r^\ell$ as $r \to \infty$ and regularity at horizon $\longrightarrow$ $\underline{\lambda = 0\ !}$

Fang, Lovelace, Damour, Lecian, Nagar, Bennington, Poisson, Landry, Kol, Smolkin, Chia ...

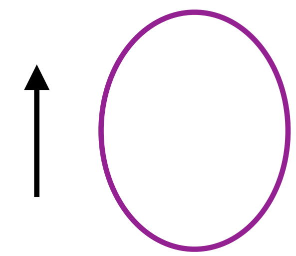
2010.00593, 2105.01069, 2203.08832 with Joyce, Penco, Santoni, Solomon (see also Charalambous, Dubovsky, Ivanov)

Black hole love numbers (static tidal response)

- EM example: External const. electric field induces dipole moment, creating dipolar response field.



- Gravitational example: External tidal field $\phi \sim r^\ell$ distorts object, creating multipolar response field $\phi \sim 1/r^{\ell+1}$.



$$\phi \sim r^\ell + \ldots + \frac{\lambda}{r^{\ell+1}} + \ldots \quad , \quad \lambda \sim \text{Love number} \sim \text{size}^{2\ell+1}$$

- $g = g_{\rm BH} + \phi \longrightarrow [\partial^2 + V]\phi = 0$

impose $\phi \sim r^\ell$ as $r \to \infty$ and regularity at horizon $\longrightarrow$ $\lambda = 0$!

Fang, Lovelace, Damour, Lecian, Nagar, Bennington, Poisson, Landry, Kol, Smolkin, Chia ...

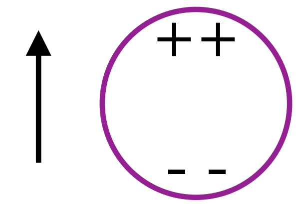
- $S_{\rm worldline} \sim \int d\tau\, \lambda\, (\partial^\ell \phi)^2 + \ldots \longleftarrow$ why does λ vanish ?

More precisely, different possible contractions Weyl tensor give E and B tidal fields.

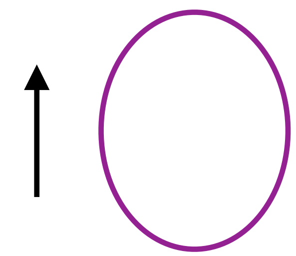
2010.00593, 2105.01069, 2203.08832 with Joyce, Penco, Santoni, Solomon (see also Charalambous, Dubovsky, Ivanov)

Black hole love numbers (static tidal response)

- EM example: External const. electric field induces dipole moment, creating dipolar response field.



- Gravitational example: External tidal field $\phi \sim r^\ell$ distorts object, creating multipolar response field $\phi \sim 1/r^{\ell+1}$.



$$\phi \sim r^\ell + \ldots + \frac{\lambda}{r^{\ell+1}} + \ldots \quad , \quad \lambda \sim \text{Love number} \sim \text{size}^{2\ell+1}$$

- $g = g_{\rm BH} + \phi \longrightarrow [\partial^2 + V]\phi = 0$

  impose $\phi \sim r^\ell$ as $r \to \infty$ and regularity at horizon $\longrightarrow$ $\underline{\lambda = 0 \; !}$

  Fang, Lovelace, Damour, Lecian, Nagar, Bennington, Poisson, Landry, Kol, Smolkin, Chia ...

- $S_{\rm worldline} \sim \int d\tau\, \lambda\, (\partial^\ell \phi)^2 + \ldots \longleftarrow$ why does λ vanish ?

More precisely, different possible contractions Weyl tensor give E and B tidal fields.

- Teukolsky equation:

$$\partial_r \left(\Delta \partial_r \phi_\ell^{(s)}\right) + s(2r - r_s)\partial_r \phi_\ell^{(s)} + \left(\frac{a^2 m^2 + is(2r - r_s)am}{\Delta} - (\ell - s)(\ell + s + 1)\right)\phi_\ell^{(s)} = 0$$

$\Delta \equiv r^2 - r r_s + a^2$ static: $\partial_t = 0$ expand in spin-weighted spherical harmonics

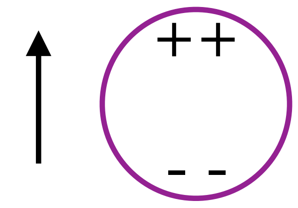
$a = \text{BH spin} \quad , \quad r_s = 2GM \quad , \quad s = 0, 1, 2 \quad , \quad \ell, m = \text{ang. mom. quantum no.}$

$\phi_\ell^{(s)} =$ Newman Penrose scalar $\sim$ Weyl tensor projected onto null tetrad
electric and magnetic types contained in real and imaginary parts thereof

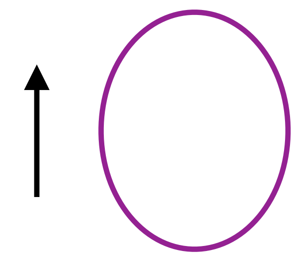
2010.00593, 2105.01069, 2203.08832 with Joyce, Penco, Santoni, Solomon (see also Charalambous, Dubovsky, Ivanov)

Black hole love numbers (static tidal response)

- EM example: External const. electric field induces dipole moment, creating dipolar response field.



- Gravitational example: External tidal field $\phi \sim r^\ell$ distorts object, creating multipolar response field $\phi \sim 1/r^{\ell+1}$.



$$\phi \sim r^\ell + \ldots + \frac{\lambda}{r^{\ell+1}} + \ldots \quad , \quad \lambda \sim \text{Love number} \sim \text{size}^{2\ell+1}$$

- $g = g_{\rm BH} + \phi \longrightarrow [\partial^2 + V]\phi = 0$

  impose $\phi \sim r^\ell$ as $r \to \infty$ and regularity at horizon $\longrightarrow$ $\lambda = 0$!

  Fang, Lovelace, Damour, Lecian, Nagar, Bennington, Poisson, Landry, Kol, Smolkin, Chia ...

- $S_{\rm worldline} \sim \int d\tau \, \lambda \, (\partial^\ell \phi)^2 + \ldots \longleftarrow$ why does λ vanish ?

More precisely, different possible contractions Weyl tensor give E and B tidal fields.

- Teukolsky equation:

$$\partial_r \left(\Delta \partial_r \phi_\ell^{(s)} \right) + s(2r - r_s)\partial_r \phi_\ell^{(s)} + \left(\frac{a^2 m^2 + is(2r - r_s)am}{\Delta} - (\ell - s)(\ell + s + 1) \right) \phi_\ell^{(s)} = 0$$

$\Delta \equiv r^2 - r r_s + a^2$ static: $\partial_t = 0$ expand in spin-weighted spherical harmonics

$a =$ BH spin , $r_s = 2GM$, $s = 0, 1, 2$, $\ell, m =$ ang. mom. quantum no.

$\phi_\ell^{(s)} =$ Newman Penrose scalar $\sim$ Weyl tensor projected onto null tetrad

electric and magnetic types contained in real and imaginary parts thereof

- A spin ladder - by taking derivatives and multiplying by polynomials - allows us to construct s=2 solution from s=0 solution, generalizing the Starobinsky-Teukolsky identity.

2010.00593, 2105.01069, 2203.08832 with Joyce, Penco, Santoni, Solomon (see also Charalambous, Dubovsky, Ivanov)

Black hole love numbers (ladder symmetries)

- Let us thus focus on s=0, and for pedagogy a=0.

$$\partial_r(\Delta\partial_r\phi_\ell) - \ell(\ell+1)\phi_\ell = 0 \qquad \text{i.e.} \quad \Box\,\phi = 0 \quad \text{in Schwarz. backgd.} \qquad \Delta \equiv r^2 - r r_s$$

Black hole love numbers (ladder symmetries)

- Let us thus focus on s=0, and for pedagogy a=0.

$$\partial_r(\Delta\partial_r\phi_\ell) - \ell(\ell+1)\phi_\ell = 0 \qquad \text{i.e.} \quad \Box\,\phi = 0 \quad \text{in Schwarz. backgd.} \qquad \Delta \equiv r^2 - r r_s$$

- Asymptotics:

$$\phi_\ell \sim \# \; + \# \ln\left[(r - r_s)/r_s\right] \qquad\qquad \phi_\ell \sim \# \, r^\ell \; + \frac{\#}{r^{\ell+1}}$$

$$r \sim r_s \qquad\qquad r \to \infty$$

Black hole love numbers (ladder symmetries)

- Let us thus focus on s=0, and for pedagogy a=0.

$$\partial_r(\Delta\partial_r\phi_\ell) - \ell(\ell+1)\phi_\ell = 0 \qquad \text{i.e.} \quad \Box\,\phi = 0 \quad \text{in Schwarz. backgd.} \qquad \Delta \equiv r^2 - r r_s$$

- Asymptotics: $\phi_\ell \sim \# \; + \# \ln[(r-r_s)/r_s]$ $\longleftrightarrow$ $\phi_\ell \sim \# \, r^\ell \; + \dfrac{\#}{r^{\ell+1}}$

  $r \sim r_s$ $\qquad$ $r \to \infty$

- Vanishing of Love number: $\phi_\ell \sim \#$ $\longleftrightarrow$ $\phi_\ell \sim \# \, r^\ell$

  Solution regular at horizon is purely growing at large r.

What's the symmetry behind this behavior? Think about the analogy with scattering.

Black hole love numbers (ladder symmetries)

- Let us thus focus on s=0, and for pedagogy a=0.

$$\partial_r(\Delta\partial_r\phi_\ell) - \ell(\ell+1)\phi_\ell = 0 \qquad \text{i.e.} \quad \Box\,\phi = 0 \quad \text{in Schwarz. backgd.} \qquad \Delta \equiv r^2 - r r_s$$

- Asymptotics: $\phi_\ell \sim \# \; + \# \ln[(r-r_s)/r_s]$ $\longleftrightarrow$ $\phi_\ell \sim \# \, r^\ell \; + \dfrac{\#}{r^{\ell+1}}$

  $r \sim r_s$ $\qquad\qquad$ $r \to \infty$

- Vanishing of Love number: $\phi_\ell \sim \#$ $\longleftrightarrow$ $\phi_\ell \sim \# \, r^\ell$

  Solution regular at horizon is purely growing at large r.

What's the symmetry behind this behavior? Think about the analogy with scattering.

- For $\ell = 0$: $\quad \partial_r(\Delta\partial_r\,\phi_0) = 0$ tells us $P_0 \equiv \Delta\partial_r\phi_0$ is conserved (r indep).

  Can show symmetry is : $\delta\phi_0 = \Delta\partial_r\phi_0$ for $S_0 = 2\pi\displaystyle\int \frac{dr}{\Delta}\phi_0\Delta\partial_r(\Delta\partial_r)\phi_0$

  Love number: $P_0[\#] = 0$ $\longleftrightarrow$ $P_0[\# \, r^0] = 0$

Nice, but does it generalize to higher harmonics?

Black hole love numbers (ladder symmetries)

- Let us thus focus on s=0, and for pedagogy a=0.

$$\partial_r(\Delta\partial_r\phi_\ell) - \ell(\ell+1)\phi_\ell = 0 \qquad \text{i.e.} \quad \Box\,\phi = 0 \quad \text{in Schwarz. backgd.} \qquad \Delta \equiv r^2 - r r_s$$

- Asymptotics:

$$\phi_\ell \sim \# \;+\; \# \ln[(r-r_s)/r_s] \qquad\qquad \phi_\ell \sim \# \, r^\ell \;+\; \frac{\#}{r^{\ell+1}}$$

$$r \sim r_s \longleftrightarrow r \to \infty$$

- Vanishing of Love number: $\phi_\ell \sim \#$ $\longleftrightarrow$ $\phi_\ell \sim \# \, r^\ell$

Solution regular at horizon is purely growing at large r.

What's the symmetry behind this behavior? Think about the analogy with scattering.

- For $\ell = 0$: $\partial_r(\Delta\partial_r\,\phi_0) = 0$ tells us $P_0 \equiv \Delta\partial_r\phi_0$ is conserved (r indep).

Can show symmetry is : $\delta\phi_0 = \Delta\partial_r\phi_0$ for $S_0 = 2\pi\int\frac{dr}{\Delta}\phi_0\Delta\partial_r(\Delta\partial_r)\phi_0$

Love number: $P_0[\#] = 0$ $\longleftrightarrow$ $P_0[\# \, r^0] = 0$

Nice, but does it generalize to higher harmonics?

- Use ladder structure: $D^+\phi_\ell = \phi_{\ell+1}$ $\qquad D^-\phi_\ell = \phi_{\ell-1}$ $\qquad D^\pm \equiv \mp\Delta\partial_r + \ldots$

$P_\ell = \Delta\partial_r([D^-]^\ell\phi_\ell)$ is conserved. Symmetry is $\delta\phi_\ell = [D^+]^\ell\Delta\partial_r([D^-]^\ell\phi_\ell)\,.$ (reminiscent of higher spin symm.)

Black hole love numbers (ladder symmetries)

- Let us thus focus on s=0, and for pedagogy a=0.

$$\partial_r(\Delta\partial_r\phi_\ell) - \ell(\ell+1)\phi_\ell = 0 \qquad \text{i.e.} \quad \Box\,\phi = 0 \quad \text{in Schwarz. backgd.} \qquad \Delta \equiv r^2 - r r_s$$

- Asymptotics:

$$\phi_\ell \sim \# \; + \# \ln\left[(r - r_s)/r_s\right] \quad (r \sim r_s) \qquad \longleftrightarrow \qquad \phi_\ell \sim \# \, r^\ell \; + \frac{\#}{r^{\ell+1}} \quad (r \to \infty)$$

- Vanishing of Love number: $\phi_\ell \sim \#$ $\longleftrightarrow$ $\phi_\ell \sim \# \, r^\ell$

Solution regular at horizon is purely growing at large r.

What's the symmetry behind this behavior? Think about the analogy with scattering.

- For $\ell = 0$: $\partial_r(\Delta\partial_r\,\phi_0) = 0$ tells us $P_0 \equiv \Delta\partial_r\phi_0$ is conserved (r indep).

Can show symmetry is : $\delta\phi_0 = \Delta\partial_r\phi_0$ for $S_0 = 2\pi\int \frac{dr}{\Delta}\phi_0\Delta\partial_r(\Delta\partial_r)\phi_0$

Love number: $P_0[\,\#] = 0$ $\longleftrightarrow$ $P_0[\,\# \, r^0] = 0$

Nice, but does it generalize to higher harmonics?

- Use ladder structure: $D^+\phi_\ell = \phi_{\ell+1}$ $\qquad D^-\phi_\ell = \phi_{\ell-1}$ $\qquad D^\pm \equiv \mp\Delta\partial_r + \ldots$

$P_\ell = \Delta\partial_r([D^-]^\ell\phi_\ell)$ is conserved. $\qquad$ Symmetry is $\delta\phi_\ell = [D^+]^\ell\Delta\partial_r([D^-]^\ell\phi_\ell)\,.$ $\qquad$ (reminiscent of higher spin symm.)

- Ladder structure has a geometric origin: static scalar in Schwarz. (suitably rescaled) effectively lives in 3D hyperbolic space (EAdS). Ladder structure follows from translation (or CKV in orig. space).

Black hole love numbers (ladder symmetries)

- Let us thus focus on s=0, and for pedagogy a=0.

$$\partial_r(\Delta\partial_r\phi_\ell) - \ell(\ell+1)\phi_\ell = 0 \qquad \text{i.e.} \quad \Box\phi = 0 \quad \text{in Schwarz. backgd.} \qquad \Delta \equiv r^2 - r r_s$$

- Asymptotics: $\phi_\ell \sim \# \ + \# \ln\left[(r - r_s)/r_s\right]$ $\longleftrightarrow$ $\phi_\ell \sim \# r^\ell + \frac{\#}{r^{\ell+1}}$

  $r \sim r_s$ $\qquad$ $r \to \infty$

- Vanishing of Love number: $\phi_\ell \sim \#$ $\longleftrightarrow$ $\phi_\ell \sim \# r^\ell$

  Solution regular at horizon is purely growing at large r.

What's the symmetry behind this behavior? Think about the analogy with scattering.

- For $\ell = 0$: $\quad \partial_r(\Delta\partial_r\,\phi_0) = 0 \quad$ tells us $\quad P_0 \equiv \Delta\partial_r\phi_0$ is conserved (r indep).

  Can show symmetry is : $\quad \delta\phi_0 = \Delta\partial_r\phi_0 \quad$ for $\quad S_0 = 2\pi\int\frac{dr}{\Delta}\phi_0\Delta\partial_r(\Delta\partial_r)\phi_0$

  Love number: $P_0[\#] = 0$ $\longleftrightarrow$ $P_0[\# r^0] = 0$

Nice, but does it generalize to higher harmonics?

- Use ladder structure: $\quad D^+\phi_\ell = \phi_{\ell+1} \qquad D^-\phi_\ell = \phi_{\ell-1} \qquad D^\pm \equiv \mp\Delta\partial_r + \ldots$

  $P_\ell = \Delta\partial_r([D^-]^\ell\phi_\ell)$ is conserved. $\qquad$ Symmetry is $\delta\phi_\ell = [D^+]^\ell\Delta\partial_r([D^-]^\ell\phi_\ell)$. $\qquad$ (reminiscent of higher spin symm.)

- Ladder structure has a geometric origin: static scalar in Schwarz. (suitably rescaled) effectively lives in 3D hyperbolic space (EAdS). Ladder structure follows from translation (or CKV in orig. space).
- Ladder structure also implies a supersymmetry (levels come in pairs) and the potential in $[\partial^2 + V]\phi = 0$ takes the Poschl-Teller form i.e. reflectionless potential!

Black hole love numbers (ladder symmetries)

- Let us thus focus on s=0, and for pedagogy a=0.

$$\partial_r(\Delta\partial_r\phi_\ell) - \ell(\ell+1)\phi_\ell = 0 \qquad \text{i.e.} \quad \Box\,\phi = 0 \quad \text{in Schwarz. backgd.} \qquad \Delta \equiv r^2 - r r_s$$

- Asymptotics: $\phi_\ell \sim \# \; + \# \ln[(r - r_s)/r_s]$ at $r \sim r_s$; $\phi_\ell \sim \# \, r^\ell \; + \dfrac{\#}{r^{\ell+1}}$ at $r \to \infty$

- Vanishing of Love number: $\phi_\ell \sim \#$ (near the horizon), $\phi_\ell \sim \# \, r^\ell$ (at large r)

  Solution regular at horizon is purely growing at large r.

What's the symmetry behind this behavior? Think about the analogy with scattering.

- For $\ell = 0$: $\quad \partial_r(\Delta\partial_r\,\phi_0) = 0 \quad$ tells us $\quad P_0 \equiv \Delta\partial_r\phi_0$ is conserved (r indep).

  Can show symmetry is : $\quad \delta\phi_0 = \Delta\partial_r\phi_0 \qquad$ for $\quad S_0 = 2\pi \int \dfrac{dr}{\Delta}\phi_0\Delta\partial_r(\Delta\partial_r)\phi_0$

  Love number: $P_0[\#] = 0$ (near the horizon), $P_0[\# \, r^0] = 0$ (at large r)

Nice, but does it generalize to higher harmonics?

- Use ladder structure: $\quad D^+\phi_\ell = \phi_{\ell+1} \qquad D^-\phi_\ell = \phi_{\ell-1} \qquad D^\pm \equiv \mp\Delta\partial_r + \ldots$

  $P_\ell = \Delta\partial_r([D^-]^\ell\phi_\ell)$ is conserved. $\qquad$ Symmetry is $\delta\phi_\ell = [D^+]^\ell\Delta\partial_r([D^-]^\ell\phi_\ell)$. $\qquad$ (reminiscent of higher spin symm.)

- Ladder structure has a geometric origin: static scalar in Schwarz. (suitably rescaled) effectively lives in 3D hyperbolic space (EAdS). Ladder structure follows from translation (or CKV in orig. space).
- Ladder structure also implies a supersymmetry (levels come in pairs) and the potential in $[\partial^2 + V]\phi = 0$ takes the Poschl-Teller form i.e. reflectionless potential!
- At large r, the geometric symmetry takes the form of (3D) special conformal transformation.

Black hole love numbers (ladder symmetries)

- Let us thus focus on s=0, and for pedagogy a=0.

$$\partial_r(\Delta\partial_r\phi_\ell) - \ell(\ell+1)\phi_\ell = 0 \qquad \text{i.e.} \quad \Box\,\phi = 0 \quad \text{in Schwarz. backgd.} \qquad \Delta \equiv r^2 - r r_s$$

- Asymptotics: $\phi_\ell \sim \# \;+\; \# \ln\left[(r - r_s)/r_s\right]$ at $r \sim r_s$; $\phi_\ell \sim \# r^\ell \;+\; \dfrac{\#}{r^{\ell+1}}$ at $r \to \infty$

- Vanishing of Love number: $\phi_\ell \sim \#$ (at horizon) $\longleftrightarrow$ $\phi_\ell \sim \# r^\ell$ (at large r)

  Solution regular at horizon is purely growing at large r.

What's the symmetry behind this behavior? Think about the analogy with scattering.

- For $\ell = 0:$ $\quad \partial_r(\Delta\partial_r\,\phi_0) = 0 \quad$ tells us $\quad P_0 \equiv \Delta\partial_r\phi_0$ is conserved (r indep).

  Can show symmetry is : $\quad \delta\phi_0 = \Delta\partial_r\phi_0 \qquad$ for $\quad S_0 = 2\pi\displaystyle\int \frac{dr}{\Delta}\phi_0\Delta\partial_r(\Delta\partial_r)\phi_0$

  Love number: $P_0[\#] = 0 \longleftrightarrow P_0[\# r^0] = 0$

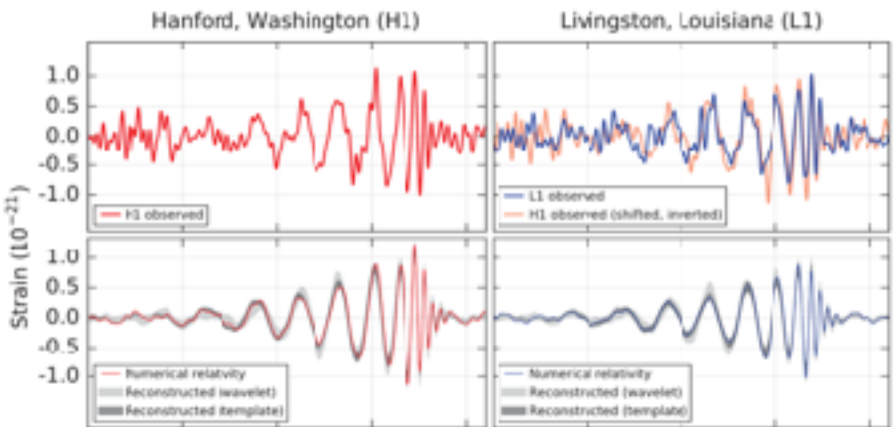
Nice, but does it generalize to higher harmonics?

- Use ladder structure: $\quad D^+\phi_\ell = \phi_{\ell+1} \qquad D^-\phi_\ell = \phi_{\ell-1} \qquad D^\pm \equiv \mp\Delta\partial_r + \ldots$

  $P_\ell = \Delta\partial_r([D^-]^\ell\phi_\ell)$ is conserved. $\qquad$ Symmetry is $\delta\phi_\ell = [D^+]^\ell\Delta\partial_r([D^-]^\ell\phi_\ell)\,.$ $\qquad$ (reminiscent of higher spin symm.)

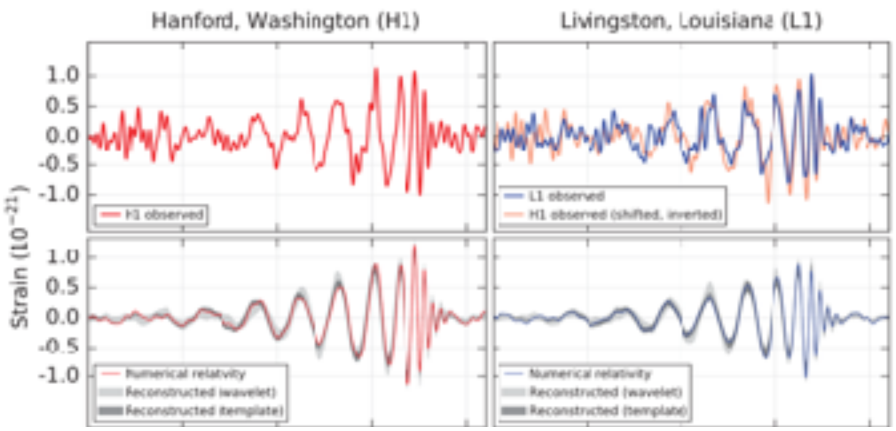
- Ladder structure has a geometric origin: static scalar in Schwarz. (suitably rescaled) effectively lives in 3D hyperbolic space (EAdS). Ladder structure follows from translation (or CKV in orig. space).
- Ladder structure also implies a supersymmetry (levels come in pairs) and the potential in $[\partial^2 + V]\phi = 0$ takes the Poschl-Teller form i.e. reflectionless potential!
- At large r, the geometric symmetry takes the form of (3D) special conformal transformation.
- No-hair property can be shown using the same conserved charges.

Black hole ring down (quadratic quasi-normal modes)

- 

Abbott et al. 2016

Black hole ring down (quadratic quasi-normal modes)

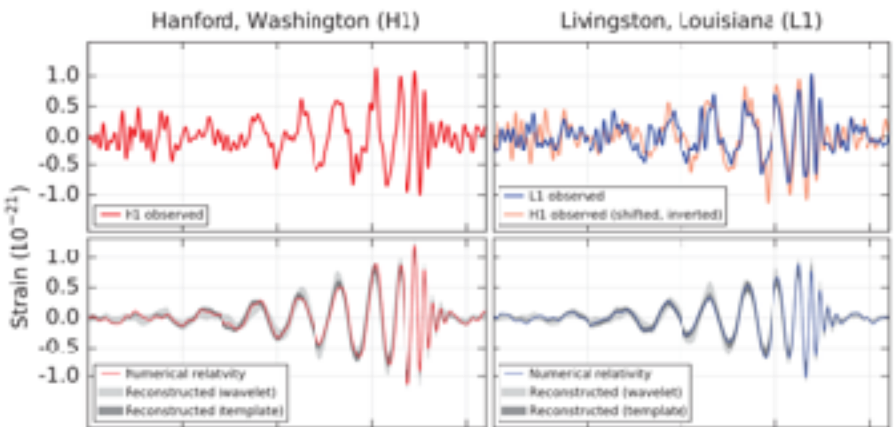
- 

  Abbott et al. 2016

- Solve $[\partial^2 + V]\phi = 0$ with outgoing b.c. at infinity and ingoing b.c. at horizon.

  This picks out special frequencies: the quasi-normal spectrum $\omega_{\ell mn}$.

Black hole ring down (quadratic quasi-normal modes)

- 

  Abbott et al. 2016

- Solve $[\partial^2 + V]\phi = 0$ with outgoing b.c. at infinity and ingoing b.c. at horizon.

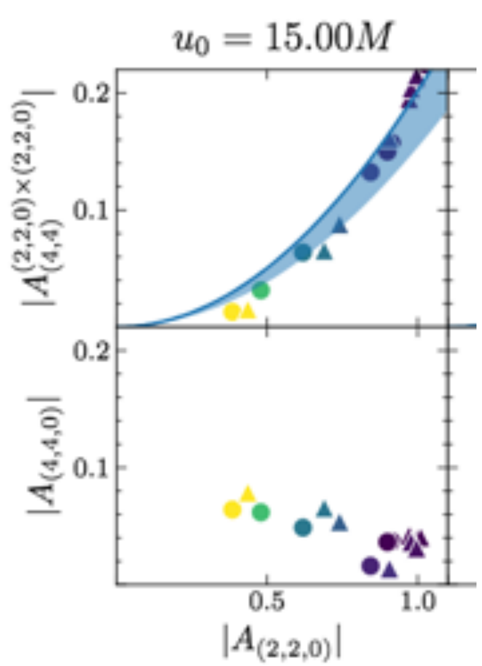
  This picks out special frequencies: the quasi-normal spectrum $\omega_{\ell m n}$.

- Go beyond linear theory: $[\partial^2 + V]\phi \sim \partial^2\phi^2$

  Expect quadratic quasi-normal modes from pairs of linear quasi-normal modes.

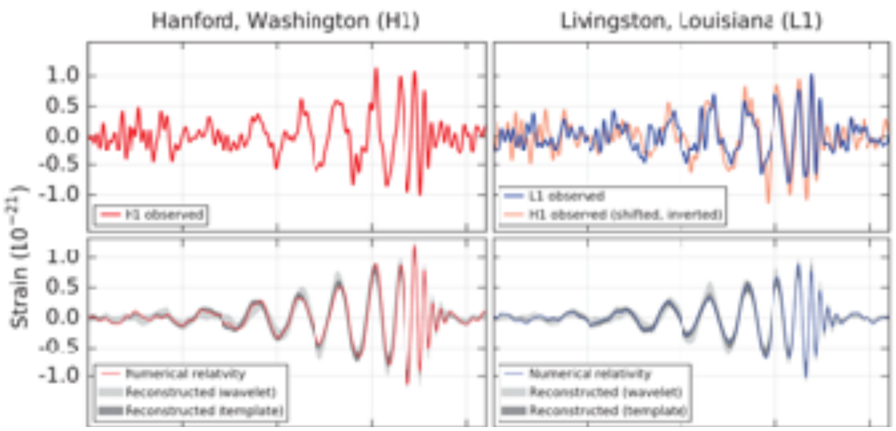
  e.g. from pairs of ω_{220} we get quadratic $\omega = 2\omega_{220}$

  Confirmed in merger simulations:



2208.07380 with Mitman et al.

Black hole ring down (quadratic quasi-normal modes)

- 

  Abbott et al. 2016

- Solve $[\partial^2 + V]\phi = 0$ with outgoing b.c. at infinity and ingoing b.c. at horizon.

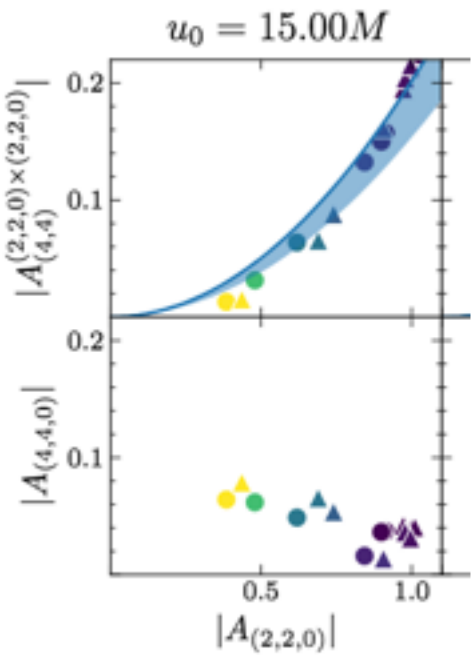
  This picks out special frequencies: the quasi-normal spectrum $\omega_{\ell mn}$.

- Go beyond linear theory: $[\partial^2 + V]\phi \sim \partial^2\phi^2$

  Expect quadratic quasi-normal modes from pairs of linear quasi-normal modes.

  e.g. from pairs of ω_{220} we get quadratic $\omega = 2\omega_{220}$

  Confirmed in merger simulations:

  2208.07380 with Mitman et al.

- Gleiser et al. 2019 suggested perturbation theory prediction for quasi-normal modes work even at merger. See Green function analysis in 2208.07379 (with Lagos).