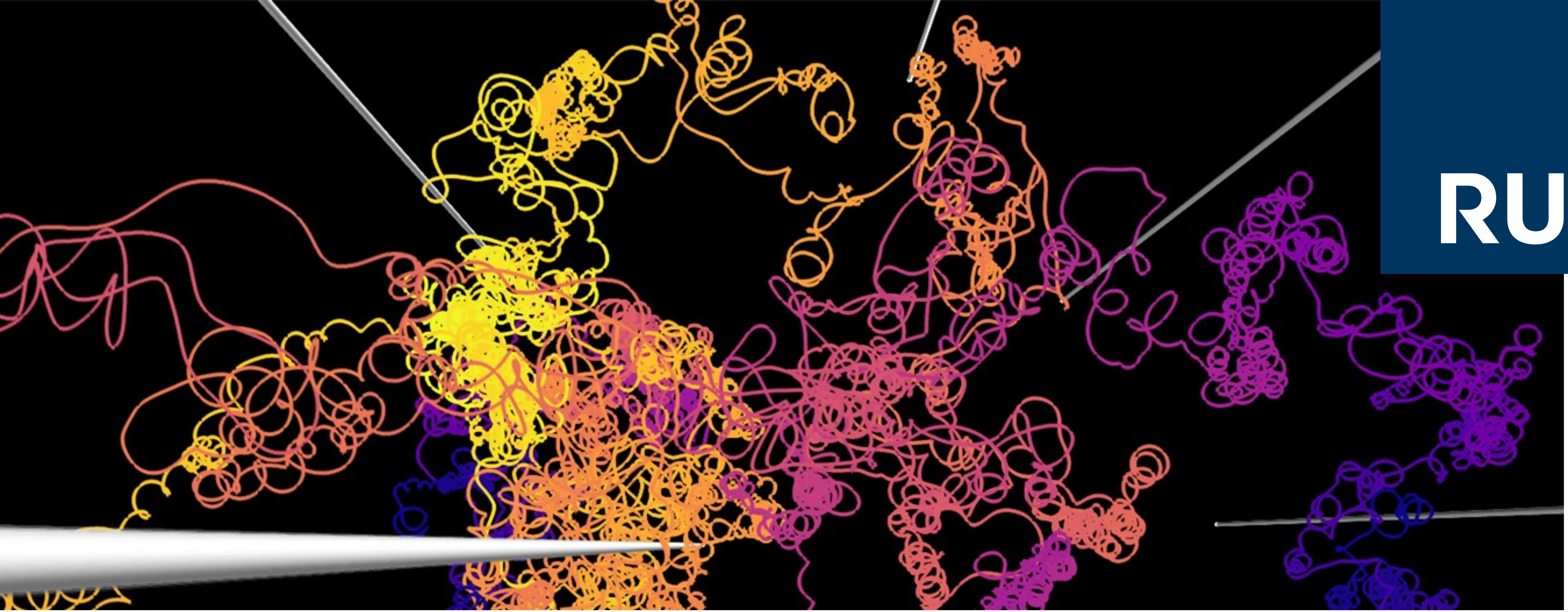

RUB

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MOMENTUM DIFFUSION: STATUS QUO AND PERSPECTIVE

CRPropa Workshop | Madrid | 14.-16.9.22 | Dr. Lukas Merten

RAPP Center

Contents

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Transport Equation

Transport Equation

$$\frac{\partial f}{\partial t} = \vec{\nabla} \cdot (\mathcal{K} \cdot \vec{\nabla} f - \vec{V} f) - \vec{W} \cdot \vec{\nabla} f + \Omega \frac{\partial f}{\partial q} + \frac{1}{q^2} \frac{\partial}{\partial q} \left(q^2 D_{qq} \frac{\partial f}{\partial q} \right) - Lf + S.$$

K – spatial diffusion tensor
D_{qq} – momentum diffusion scalar
$\vec{V}$ – conservative drift
$\vec{W}$ – non-conservative drift term
Ω – momentum loss, e.g., adiabatic cooling Ω_{ad}=q/3$\nabla \cdot \vec{u}$
q – absolute value of the momentum vector
L – catastrophic losses

Alternative formulations, e.g, in my Dissertation.

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General Fokker Planck Equation

- Time-forward equation:

$$\frac{\partial f}{\partial t} = -\sum_i \frac{\partial}{\partial x_i}(A_{F,i} f) + \frac{1}{2}\sum_{i,j} \frac{\partial^2}{\partial x_i \partial x_j}(C_{F,ij} f) - L_F f + S. \tag{13}$$

- Time-backward equation:

$$\frac{\partial f}{\partial t} = \sum_i A_{B,i} \frac{\partial f}{\partial x_i} + \frac{1}{2}\sum_{i,j} C_{B,ij} \frac{\partial^2 f}{\partial x_i \partial x_j} - L_B f + S. \tag{14}$$

- Transforming the transport equation into this form
- Reading off the coefficients for stochastic differential equation (SDE)

Time forward CR transport equation

$$\frac{\partial f}{\partial t} = \vec{\nabla}\cdot(\vec{\nabla}\cdot[{}^t\mathcal{K}f]) - \vec{\nabla}\cdot([(\vec{\nabla}\cdot{}^t\mathcal{K}) + \vec{U}]f)$$
$$+ \frac{\partial^2}{\partial q^2}(D_{qq}f) - \frac{\partial}{\partial q}\left(\left[\frac{\partial D_{qq}}{\partial q} - \hat{\Omega}\right]f\right)$$
$$- L_F f + S.$$

$$\vec{U} = \vec{V} + \vec{W},$$
$$\hat{\Omega} = \Omega + \frac{2}{q}D_{qq},$$
$$L_B = L + \vec{\nabla}\cdot\vec{V} + \frac{\partial\Omega}{\partial q},$$
$$L_F = L - \vec{\nabla}\cdot\vec{W},$$

Here, I got different results and suppose there is just a *typo/mixup* in the paper:

$$L_B = L + \nabla\cdot\vec{V}$$
$$L_F = L - \nabla\cdot\vec{W} + \frac{\partial\hat{\Omega}}{\partial q}$$

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2

Stochastic Differential Equation

Time forward SDE

$$d\vec{r} = \left(\left(\vec{\nabla} \cdot {}^t\mathcal{K}\right) + \vec{U}\right) dt + \mathcal{B} \cdot d\vec{\omega}_r ,$$

$$dq = \left(\frac{\partial D_{qq}}{\partial q} - \hat{\Omega}\right) dt + \sqrt{2D_{qq}}\, d\omega_q .$$

3

Implementation

Current implementation

Separation into:

spatial terms → DiffusionSDE

energy losses/gains → MomentumDiffusion

DiffusionSDE

$d\vec{r} = \vec{U}dt + Bd\overrightarrow{\omega_r}$, **as** $\nabla \cdot \mathrm{K} \approx 0$

MomentumDiffusion

$$dq = \left(\frac{\partial D_{qq}}{\partial q} - \frac{2}{q} D_{qq}\right) dt + \sqrt{2D_{qq}} d\omega_q$$

Further assumption in CRPropa

Connection between spatial and momentum diffusion:

$D_{xx} \cdot D_{pp} \propto 4v_A^2 p^2$

The treatment of the term L_f is usually done with weighting of the form:
$w_j = w_{j-1} e^{-L_j dt} = \exp(-\sum_{k=0}^{j}(L_k dt))$
However, **catastrophic losses** are treated in CRPropa with **dedicated modules**.

So especially this part needs to be checked carefully.→ Work in Progress

The diffusion tool itself might in principle be used already, e.g., for pure propagation (without losses) studies.

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Testing

- **We are looking for simple analytical solutions to compare with**

- **Sophie Aerdker will look especially into Fermi I/II order acceleration**
- **Different implementations for ideal / non-ideal shocks are needed.**

Will be presented when some significant progress was made.

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Summary and Outlook

Summary

The implementation principle is relatively clear

- A first *untestet* version is available (#PR 279)

Testing not too easy

- Strong influence of relative size of propagation step
- Not too many simple analytical solutions available
- Statistical tests needed

Large potential

- Will allow for much better source modelling and e.g. re-acceletation

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Outlook

Spatially varying diffusion tensor Eigen-values

- More terms must be included in DiffusionSDE (e.g. drift)

Ideal Shock description

- Can probably be done by clever transformations of SDE
- Idea was already introduced in *MING ZHANG ApJ 541 428-435 (2000)*

More flexible choice of spatial and momentum diffusion components

- Can be based on ideas for varying diffusion tensor