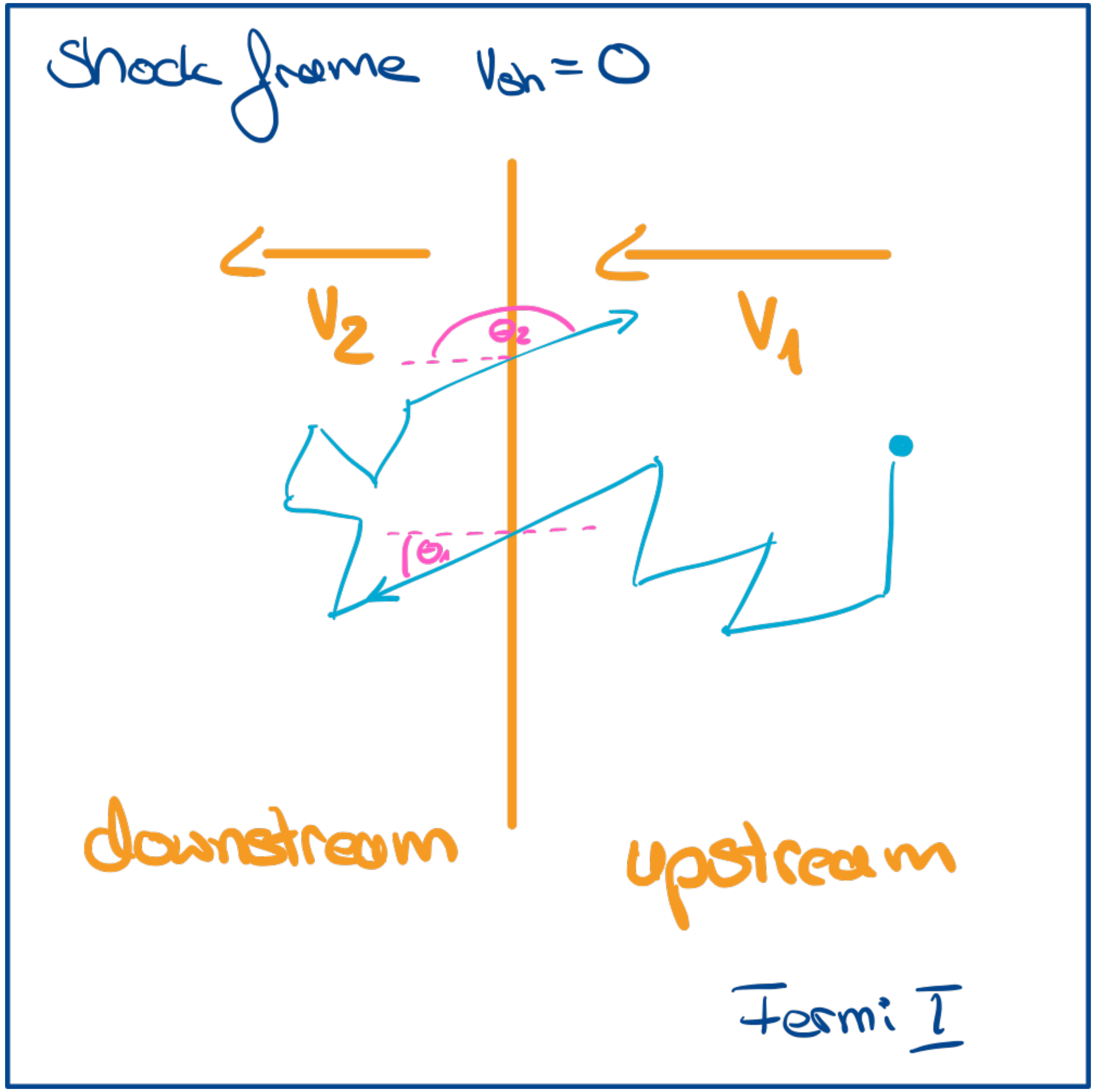
Modeling Diffusive Shock Acceleration with CRPropa

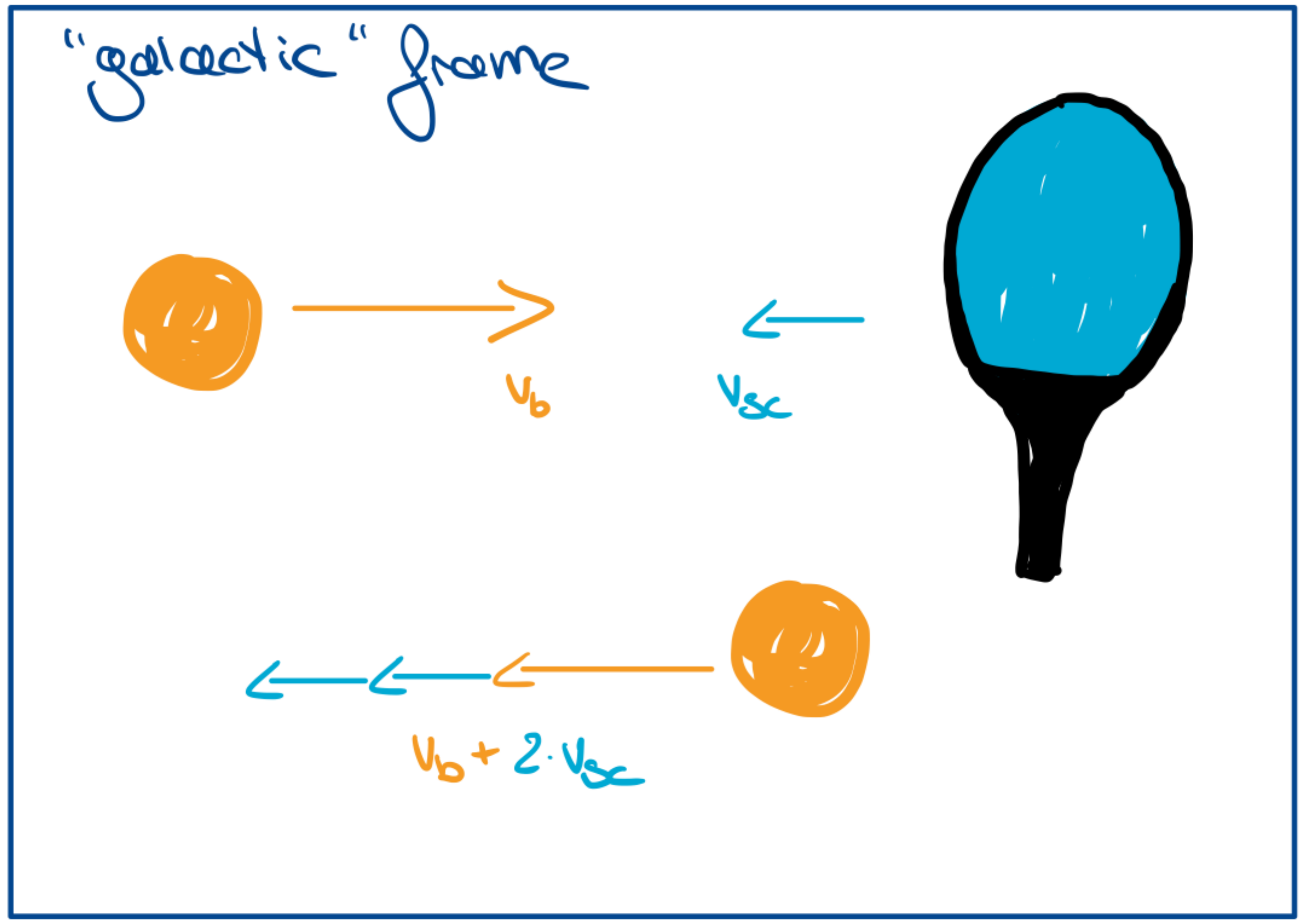


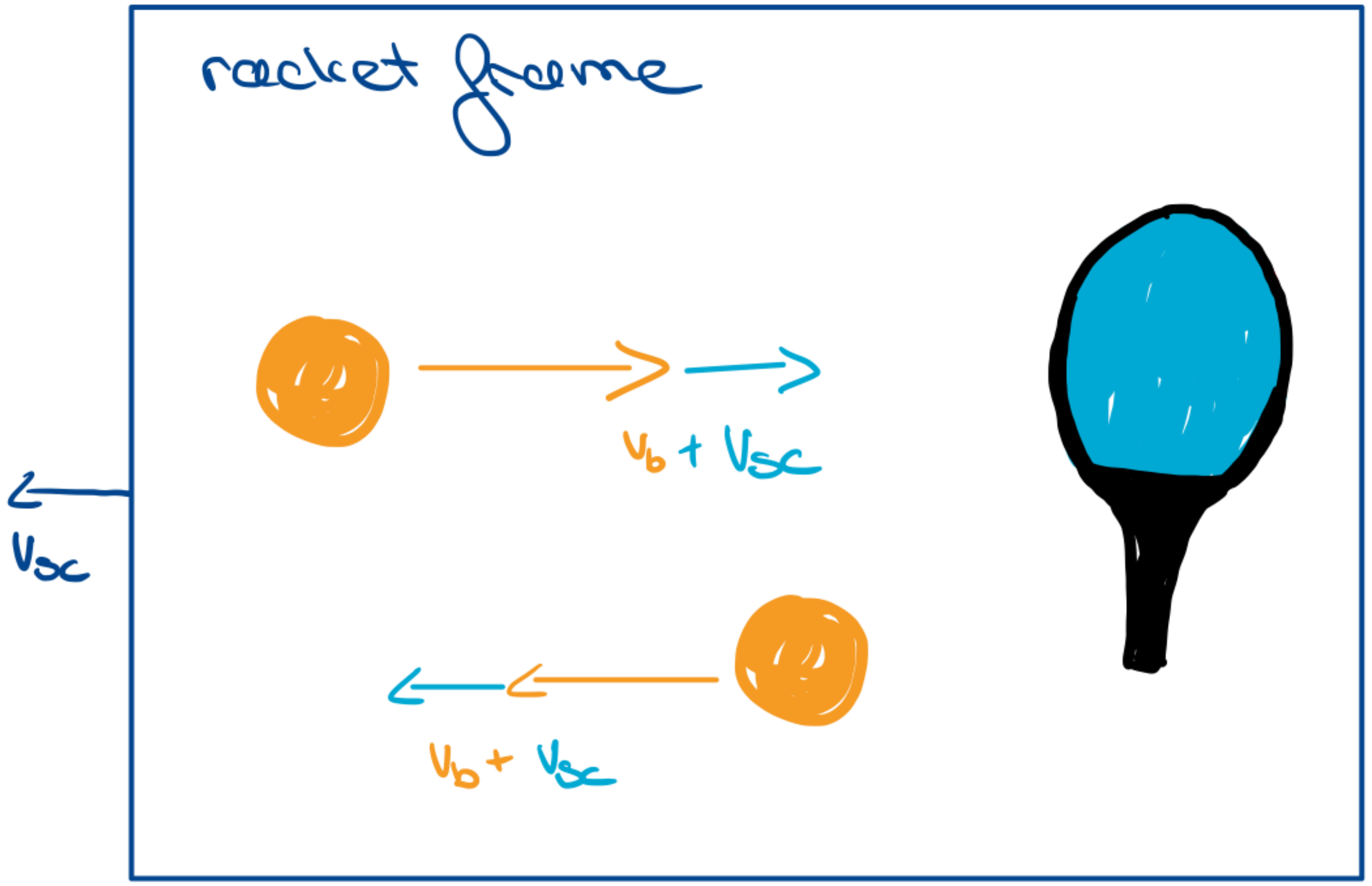
Sophie Aerdker, Ruhr-University Bochum, TPIV
CRPropa Workshop 2022, Madrid

CR Acceleration

Brief Overview

- Cosmic ray particles scatter with magnetic field turbulences
- In the reference frame of the *scatter center,* the magnetic field reflects them, without accelerating
- Assuming that the scatter centers are moving as well, particles on average are accelerated in the *galactic reference frame*



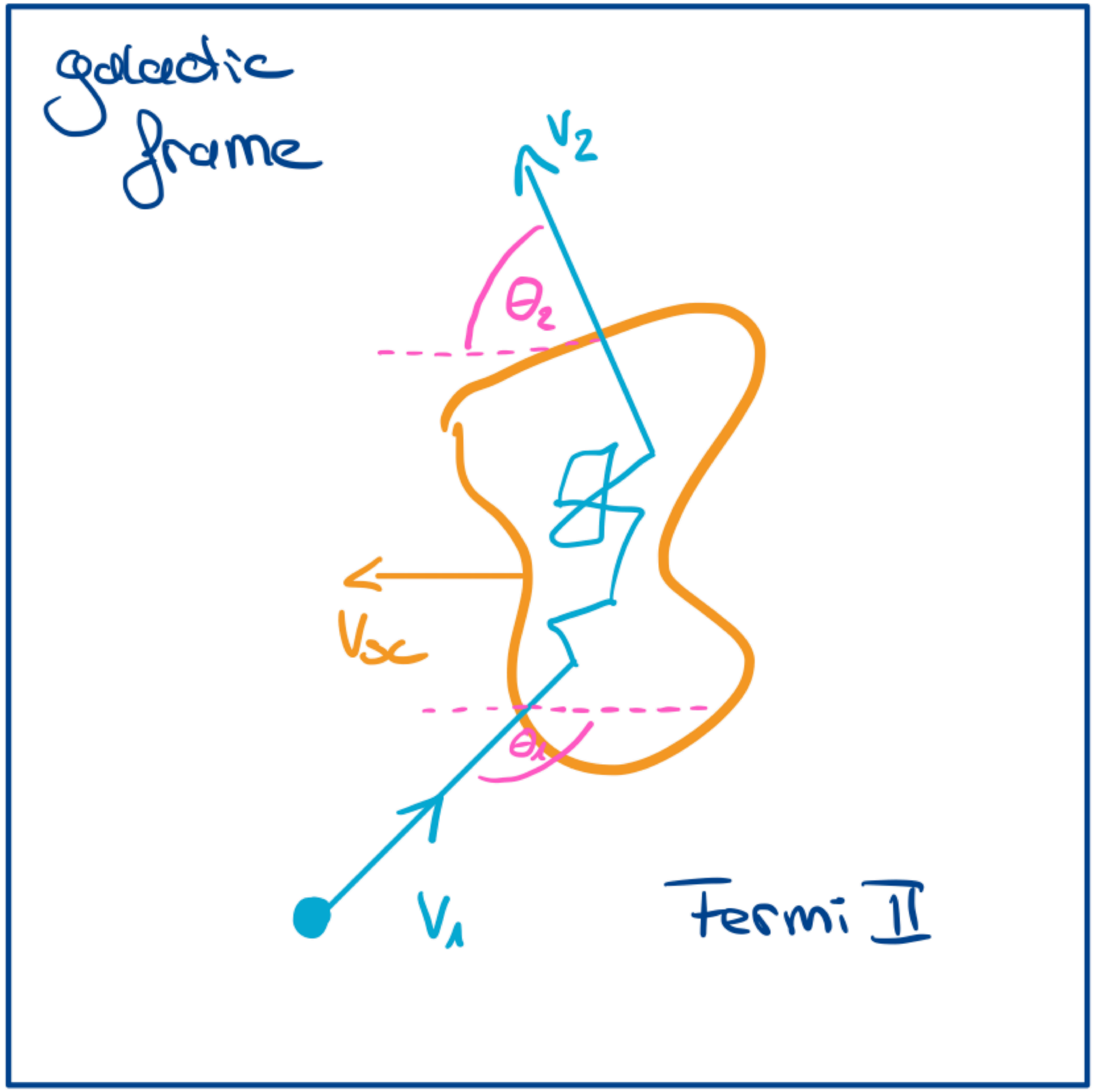


Moving Clouds Scenario

Second Order Fermi Acceleration

- Particles enter moving magnetized clouds, elastic scattering, leave in random direction
- "Head-on" and "tail-on" collisions, depending on relative velocity, head-on are more likely than tail-on
- Lorentz transformations from galactic -> cloud -> galactic frame and averaging over scattering angles yields average energy gain:

$$\left\langle \frac{\Delta E}{E_{in}} \right\rangle \simeq \frac{4}{3} \beta_{cloud}^2$$

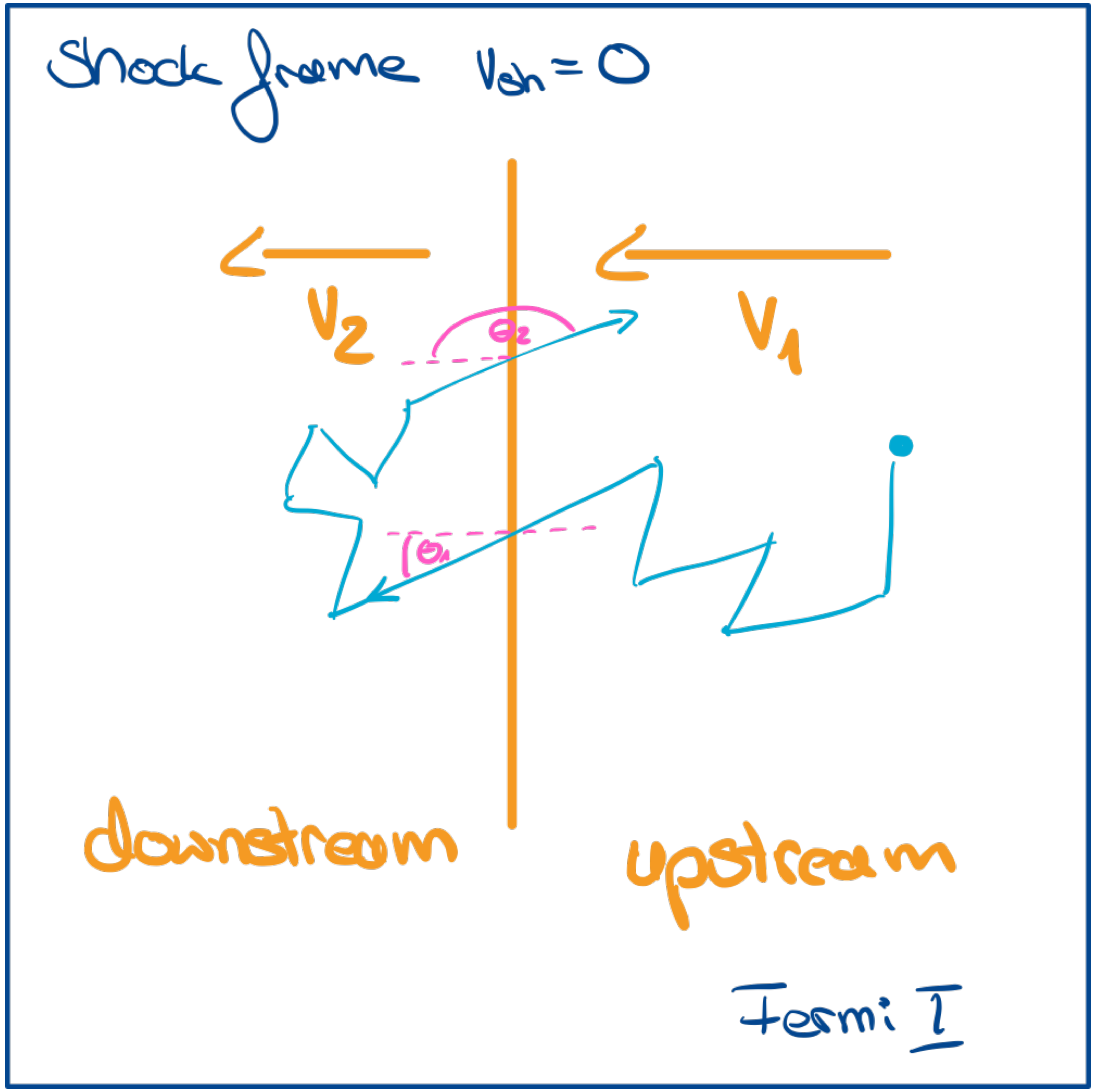


Diffusive Shock Acceleration

First Order Fermi Acceleration

- Particles cross shock front repeatedly
- "Head-on" collisions only, with $v_{\mathrm{sc}} = v_{\mathrm{up}} - v_{\mathrm{down}}$ seen from *upstream* or *downstream* frame of reference
- Lorentz transformations from upstream -> downstream -> upstream frame and averaging over scattering angles yields average energy gain:

$$\left\langle \frac{\Delta E}{E_{in}} \right\rangle = \frac{4}{3}\beta_{sh}$$



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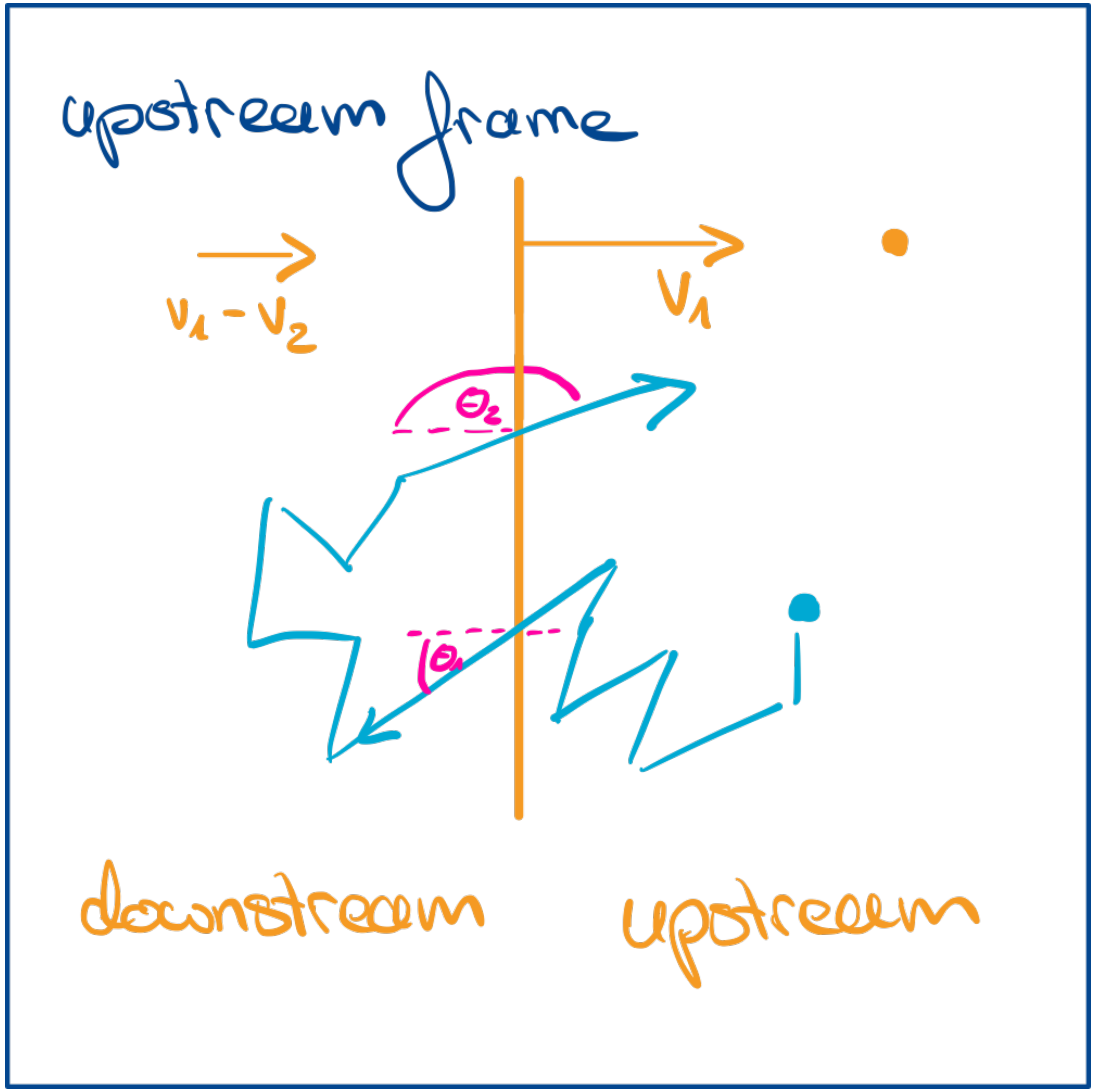

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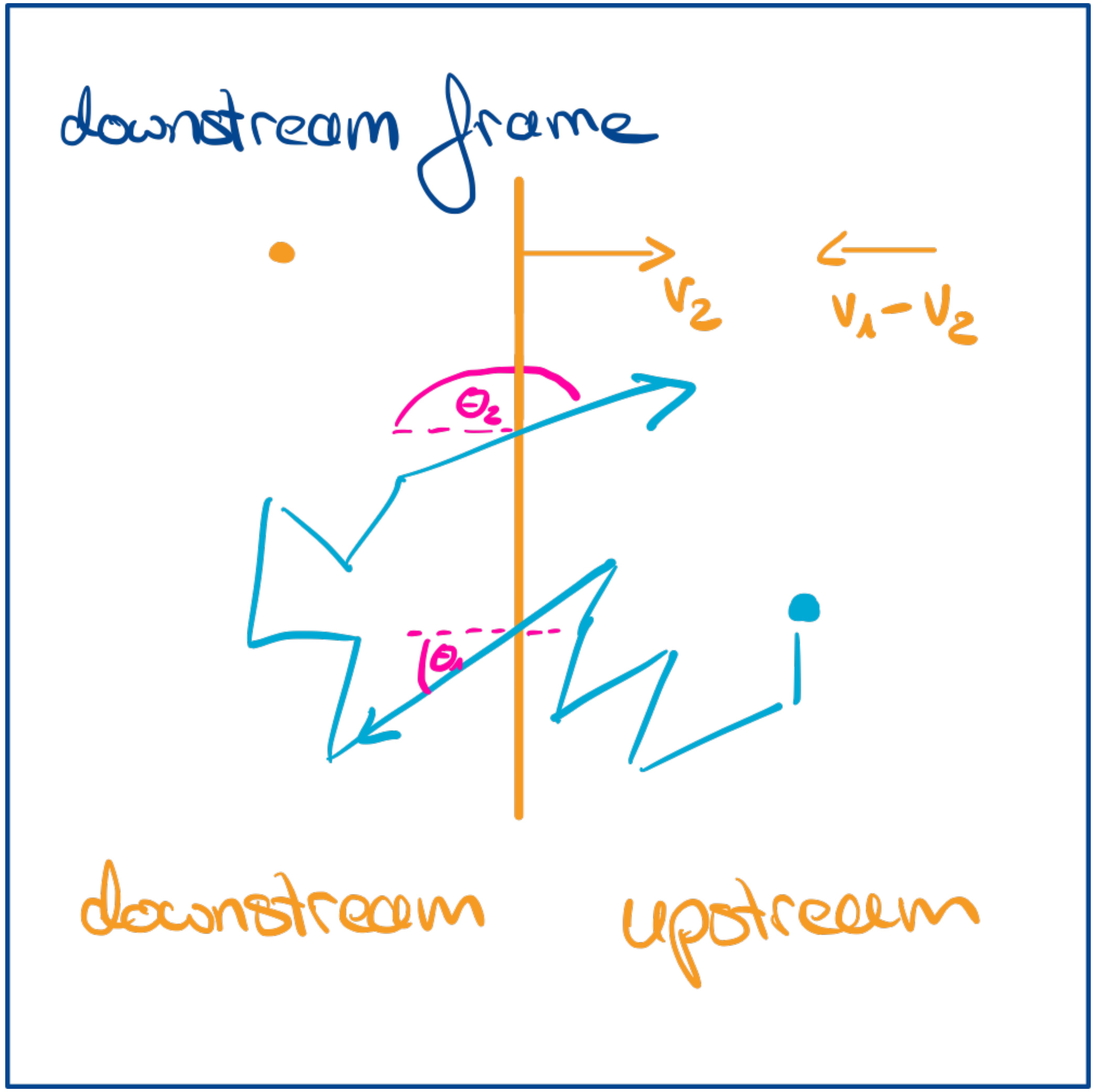

Stochastic Differential Equations

First & Second Order Fermi Acceleration

$$\mathrm{d}x(t) = \sqrt{2\kappa}\,\mathrm{d}W_x + \left(\frac{\partial \kappa}{\partial x} + V \right) \mathrm{d}t$$

$$\mathrm{d}p(t) = \sqrt{2D}\,\mathrm{d}W_p + \left(\frac{\partial D}{\partial p} - \frac{1}{3}\frac{\partial V}{\partial x} p \right) \mathrm{d}t$$

Stochastic Differential Equations

First & Second Order Fermi Acceleration

$$\mathrm{d}x(t) = \sqrt{2\kappa}\,\mathrm{d}W_x + \left(\frac{\partial \kappa}{\partial x} + V\right)\mathrm{d}t$$

Spatial Diffusion Advection/Drift

$$\mathrm{d}p(t) = \sqrt{2D}\,\mathrm{d}W_p + \left(\frac{\partial D}{\partial p} - \frac{1}{3}\frac{\partial V}{\partial x}p\right)\mathrm{d}t$$

Momentum Diffusion Adiabatic Energy Change

Spatial Diffusion Coefficient

Advection Velocity

Momentum Diffusion Coefficient

Stochastic Differential Equations

First & Second Order Fermi Acceleration

$$\mathrm{d}x(t) = \sqrt{2\kappa}\,\mathrm{d}W_x + \left(\frac{\partial \kappa}{\partial x} + V\right)\mathrm{d}t$$

Spatial Diffusion Coefficient

Advection Velocity

$$\mathrm{d}p(t) = \sqrt{2D}\,\mathrm{d}W_p + \left(\frac{\partial D}{\partial p} - \frac{1}{3}\frac{\partial V}{\partial x}p\right)\mathrm{d}t$$

Momentum Diffusion Coefficient

Stochastic Differential Equations

First & Second Order Fermi Acceleration

$$\mathrm{d}x(t) = \sqrt{2\kappa}\,\mathrm{d}W_x + \left(\frac{\partial \kappa}{\partial x} + V\right)\mathrm{d}t$$

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Spatial Diffusion

Advection

Momentum Diffusion

… implemented in CRPropa so far

Computation of cosmic-ray acceleration by Itô's stochastic differential equations

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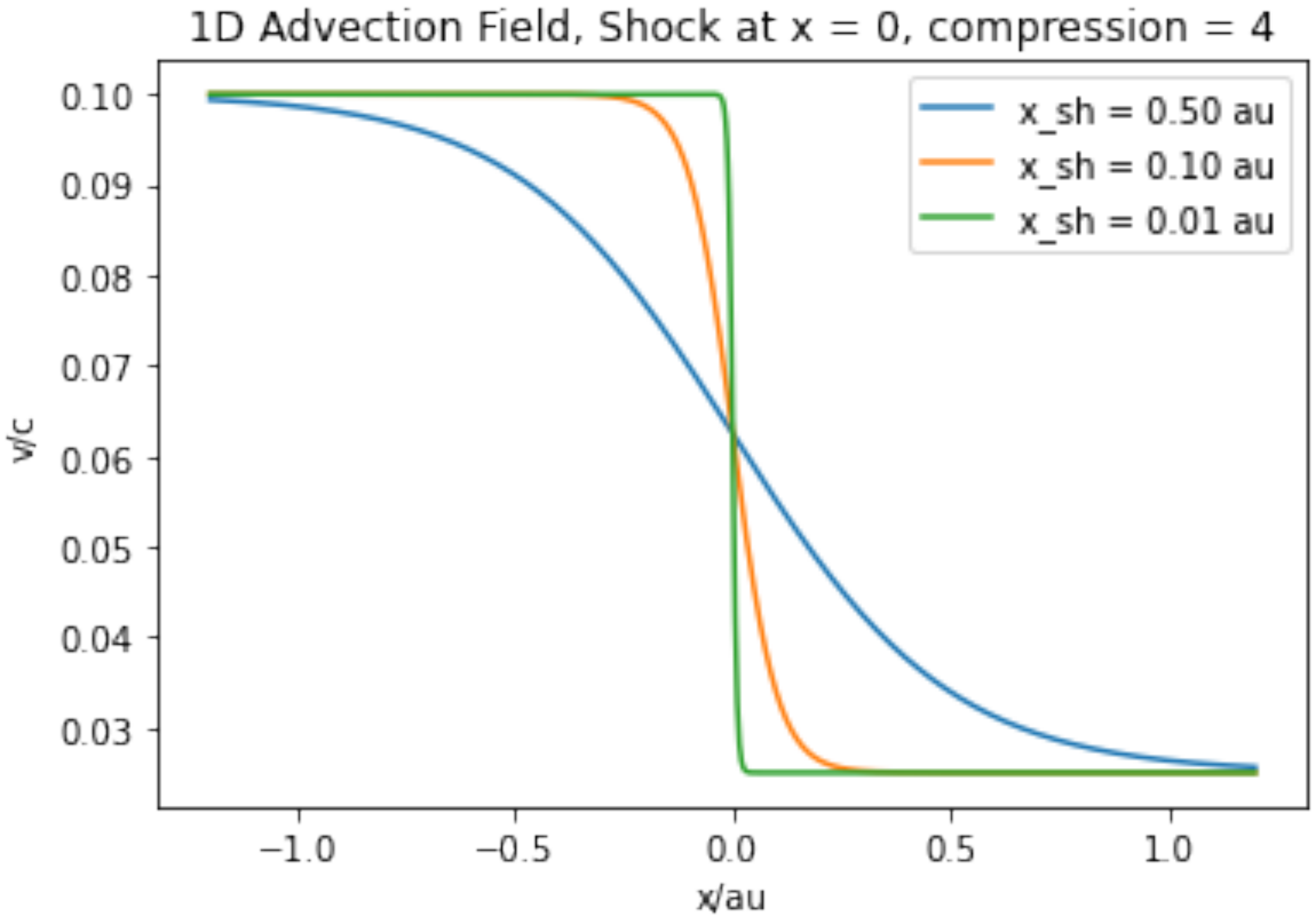
Diffusive Shock Acceleration

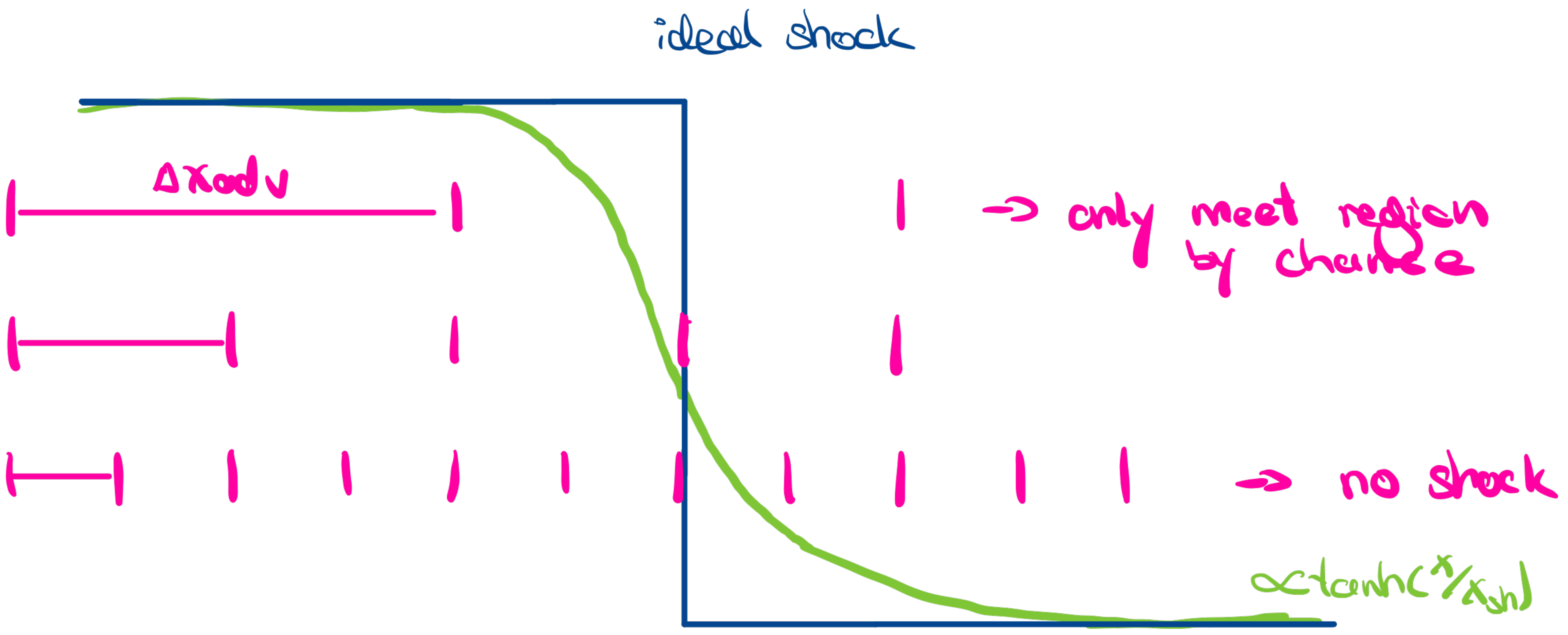
Continuous velocity profile for SDE approach

- Influence of shock wave given by change in streaming velocity with energy gain proportional to velocity gradient
- For SDE approach: no discontinuity, “smooth” transition instead:

$$\bar{\beta}(x) = a - b \tanh(x/X_{\text{sh}}) \, ,$$



Shock width, step length and spectrum



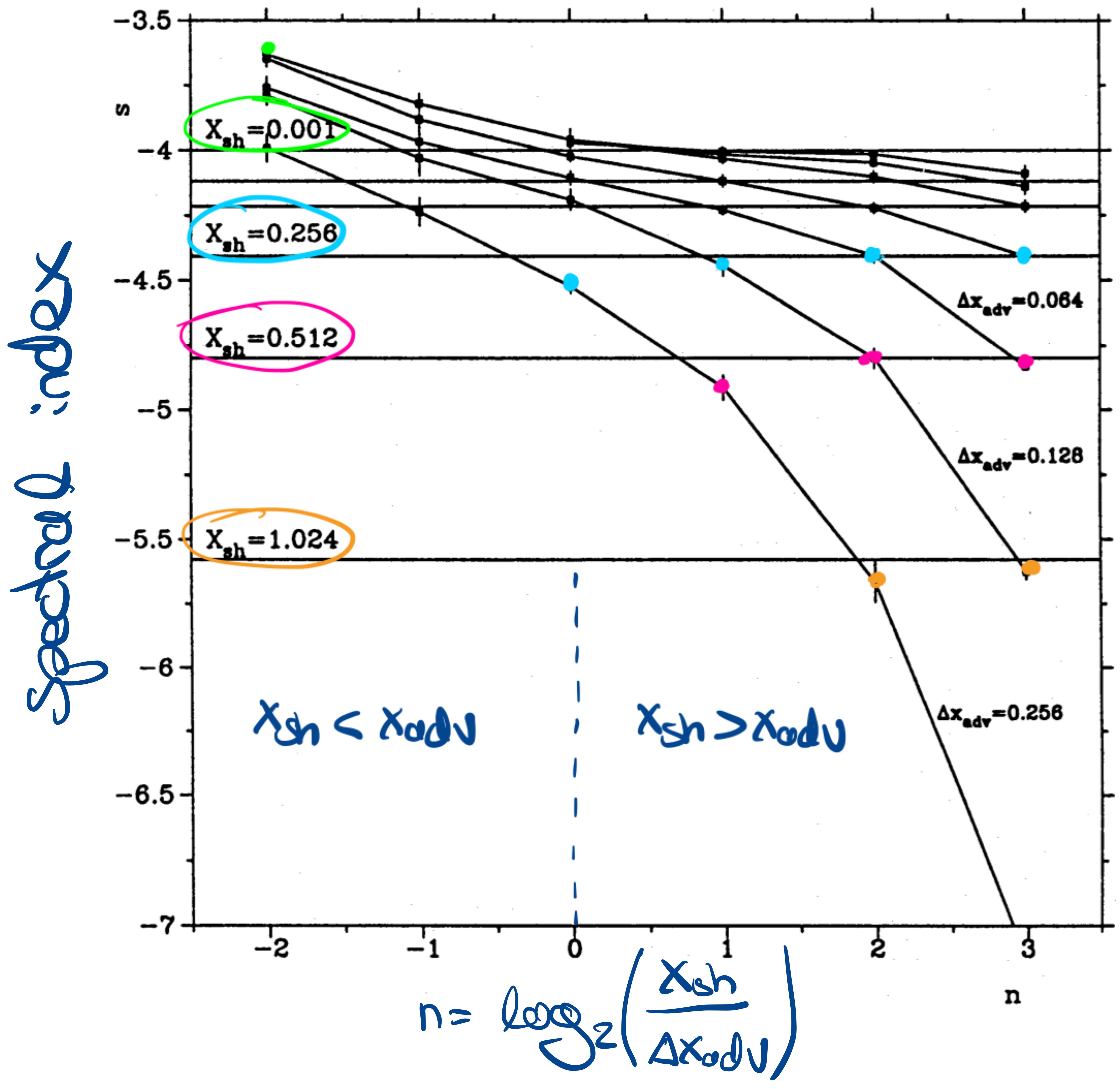
Shock width, step length and spectrum

- Advective step must be smaller than the shock width, otherwise pseudo-particles only by chance meet the area where velocity changes

$$\Delta x_{adv} = \bar{\beta} \cdot \Delta \tau$$

- But: Small advective steps (n > 0) don't "see" a discrete shock but a smooth velocity gradient

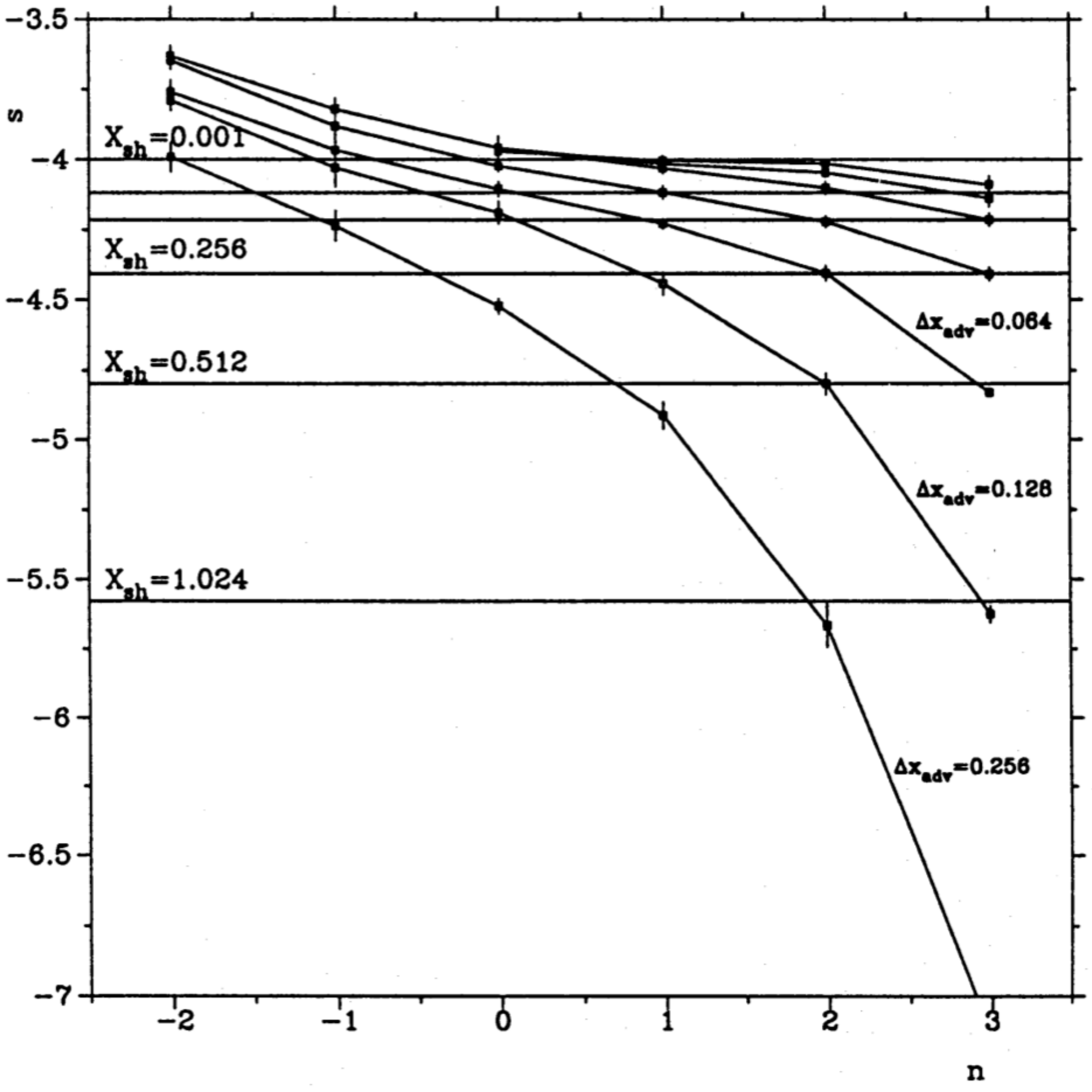
- For ideal shock solution: very small shock width and thus very high resolution in advective step need to be simulated



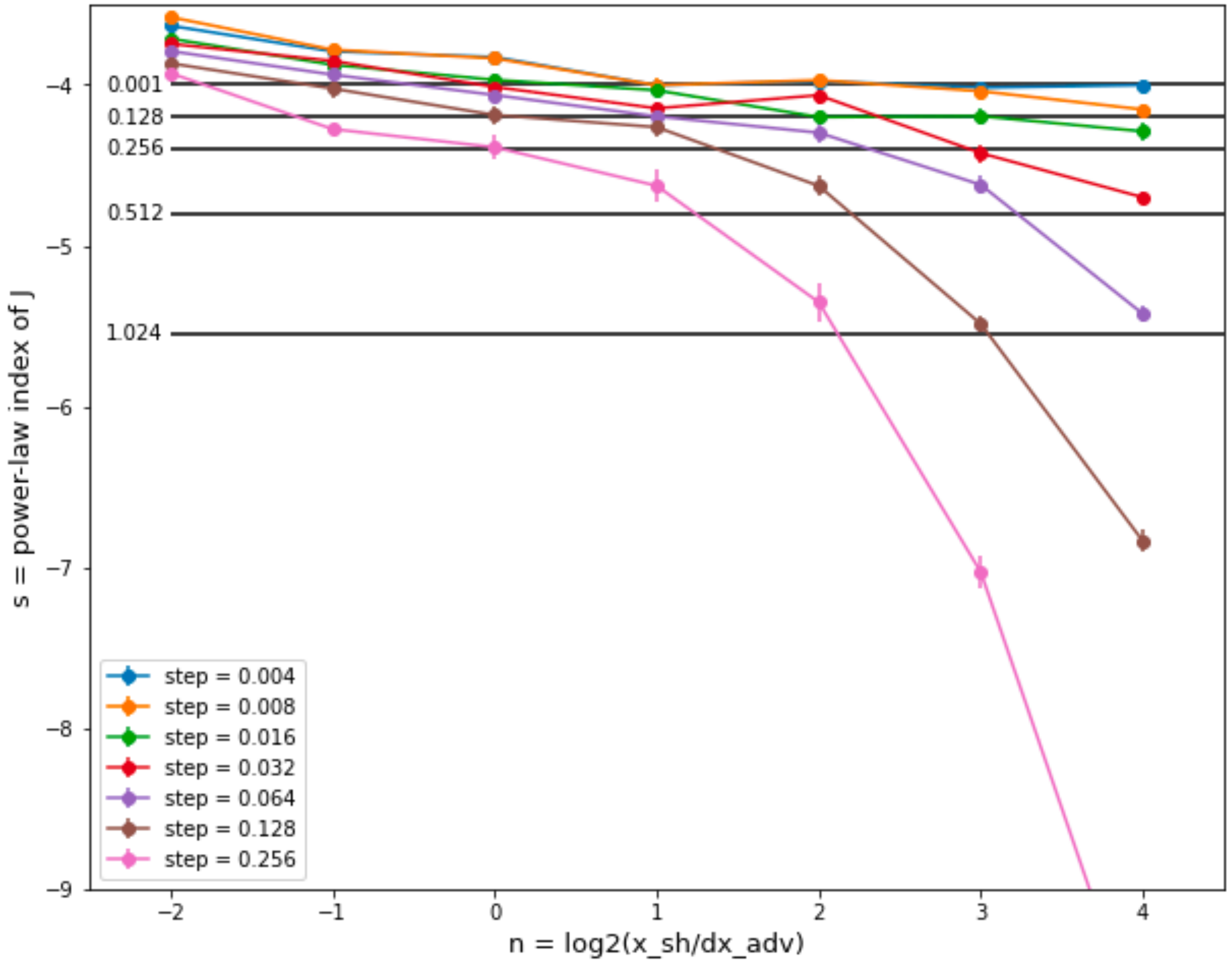
Adapted from Krülls&Achterberg, 1993

Shock width and step length

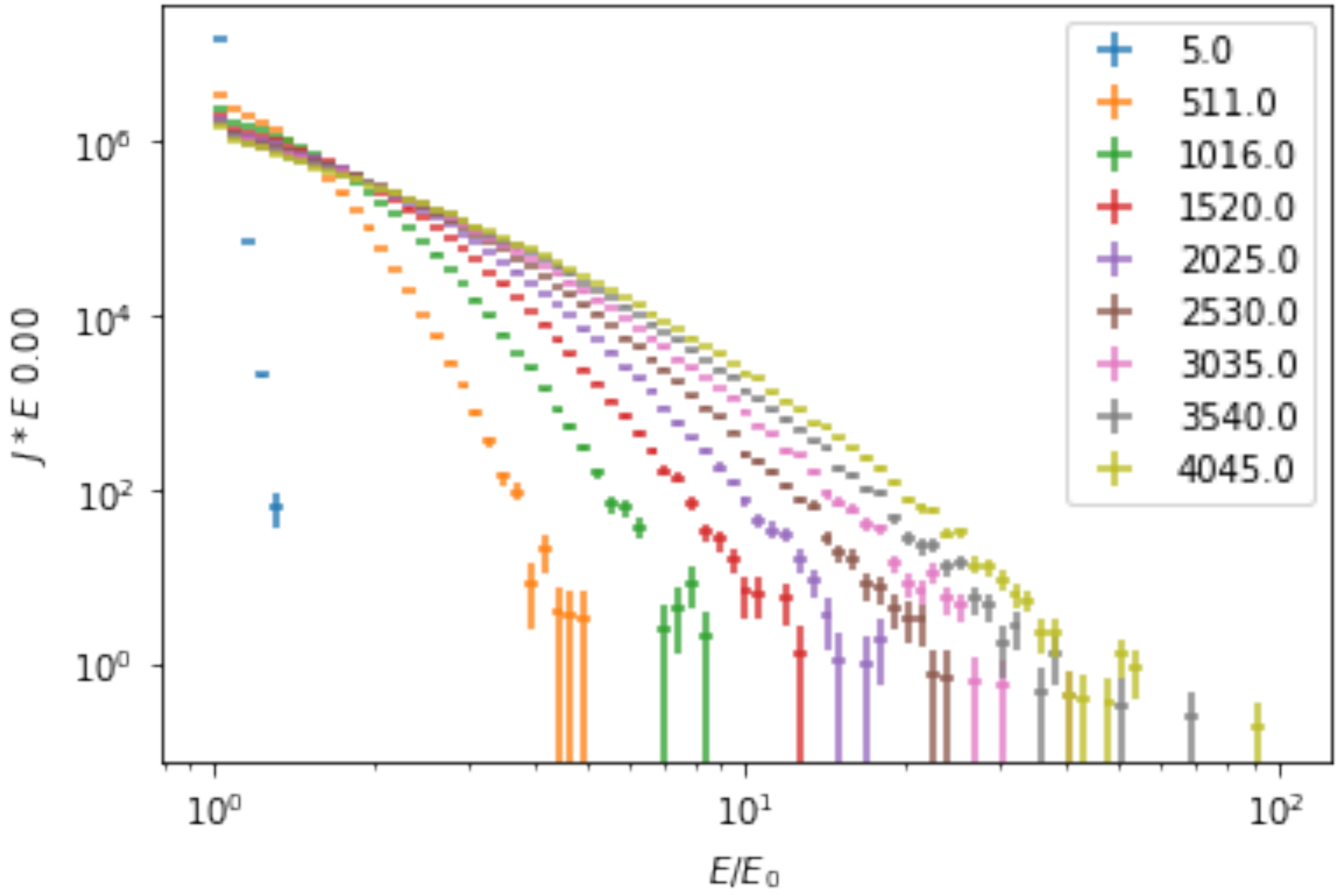
$\kappa = \text{const.} \quad \Delta x_{dif} = \sqrt{\kappa \cdot \Delta\tau} \propto \sqrt{\Delta x_{adv}}$

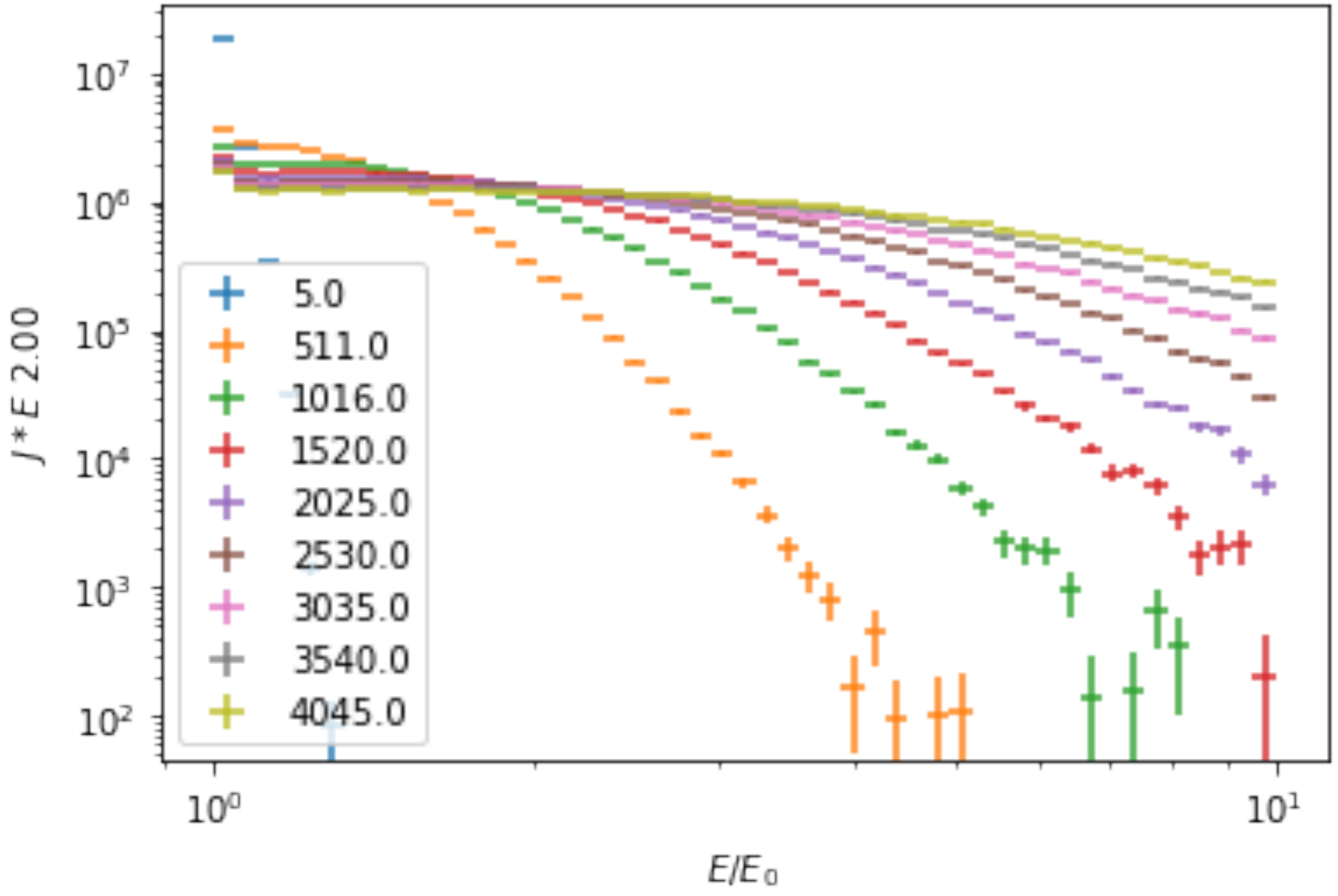


Krülls&Achterberg, 1993



Time Evolution of Resulting Spectrum





Reference Solution with varying Diff. Coeff.

- For analytic solution by Toptyghin diffusion coefficient changes over shock:

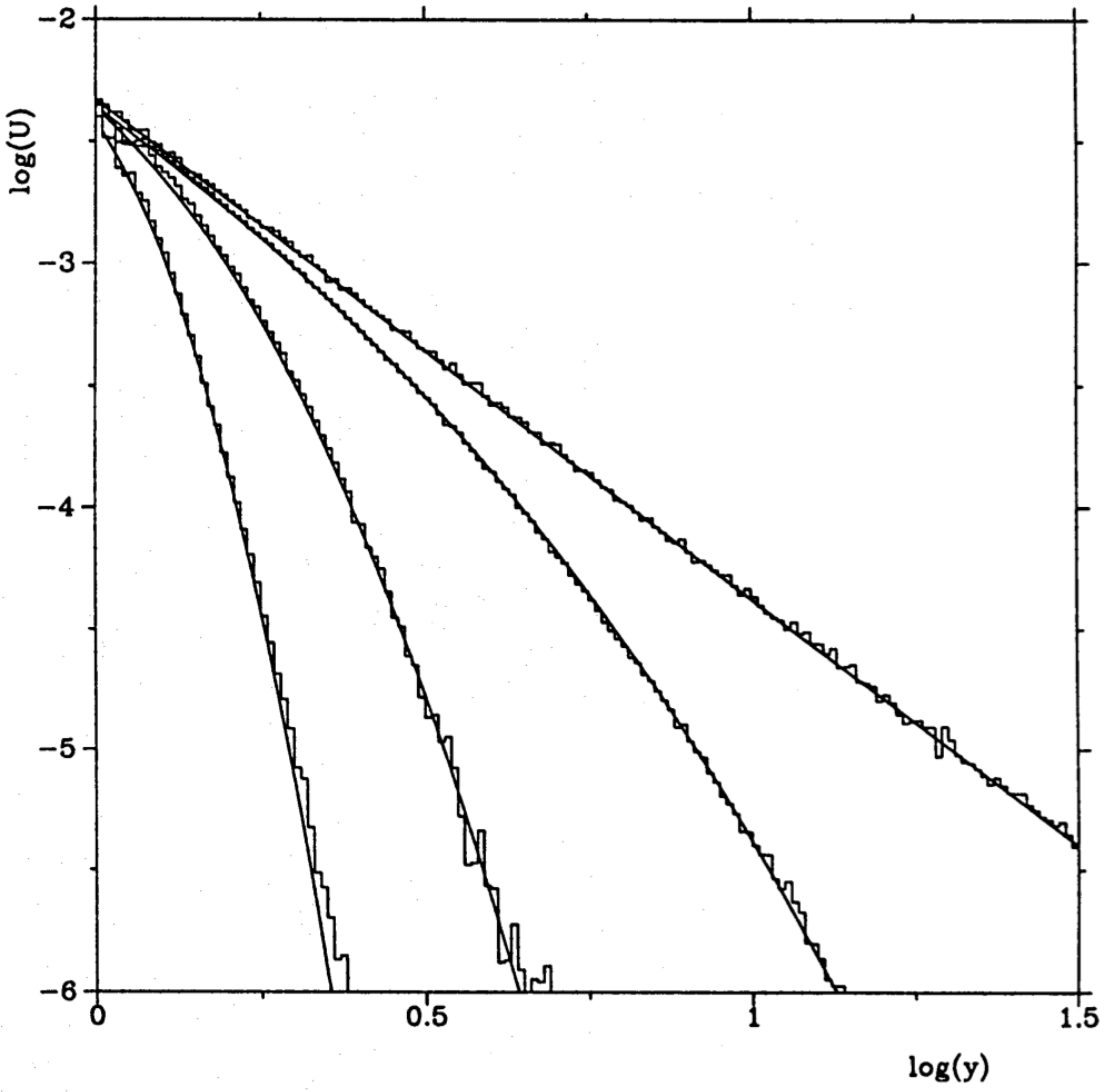
$$\bar{\beta}^2 / \bar{\kappa} = \text{const.}$$

- A spatial varying Diffusion Coefficient also adds to advective step:

$$\Delta x_{adv} = \left(\frac{\partial \bar{\kappa}}{\partial x} + \bar{\beta} \right) \Delta \tau$$

- CRPropa Module specifying Diffusion Coefficient and its derivative, analogous to AdvectionField Module
- For simulation of an ideal shock:

$$\Delta x'_{adv} < x_{sh} < \Delta x_{diff}$$



Time development of number density at the shock. U = y^2 F. T = 0.64, 2.0, 6.4 and infinity. Solid lines represent analytical results of Toptyghin, 1980. Krülls&Achterberg, 1993

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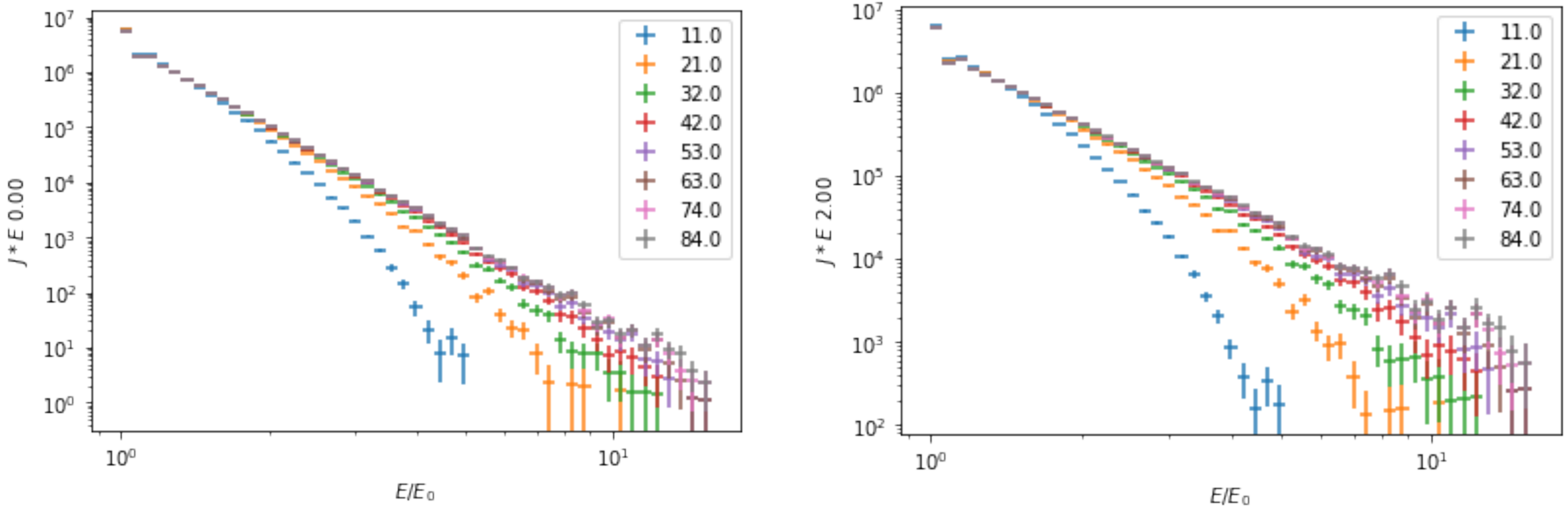

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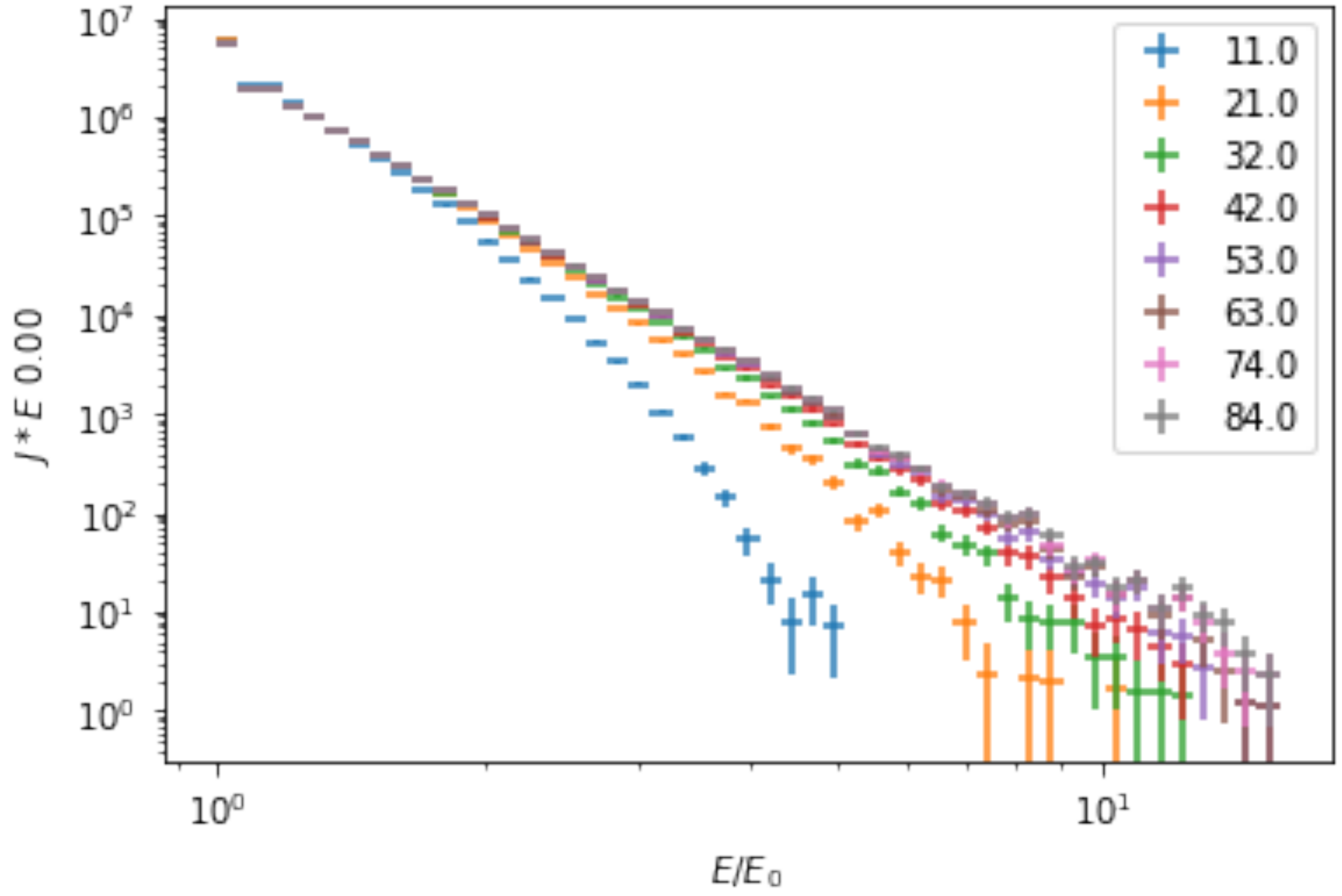

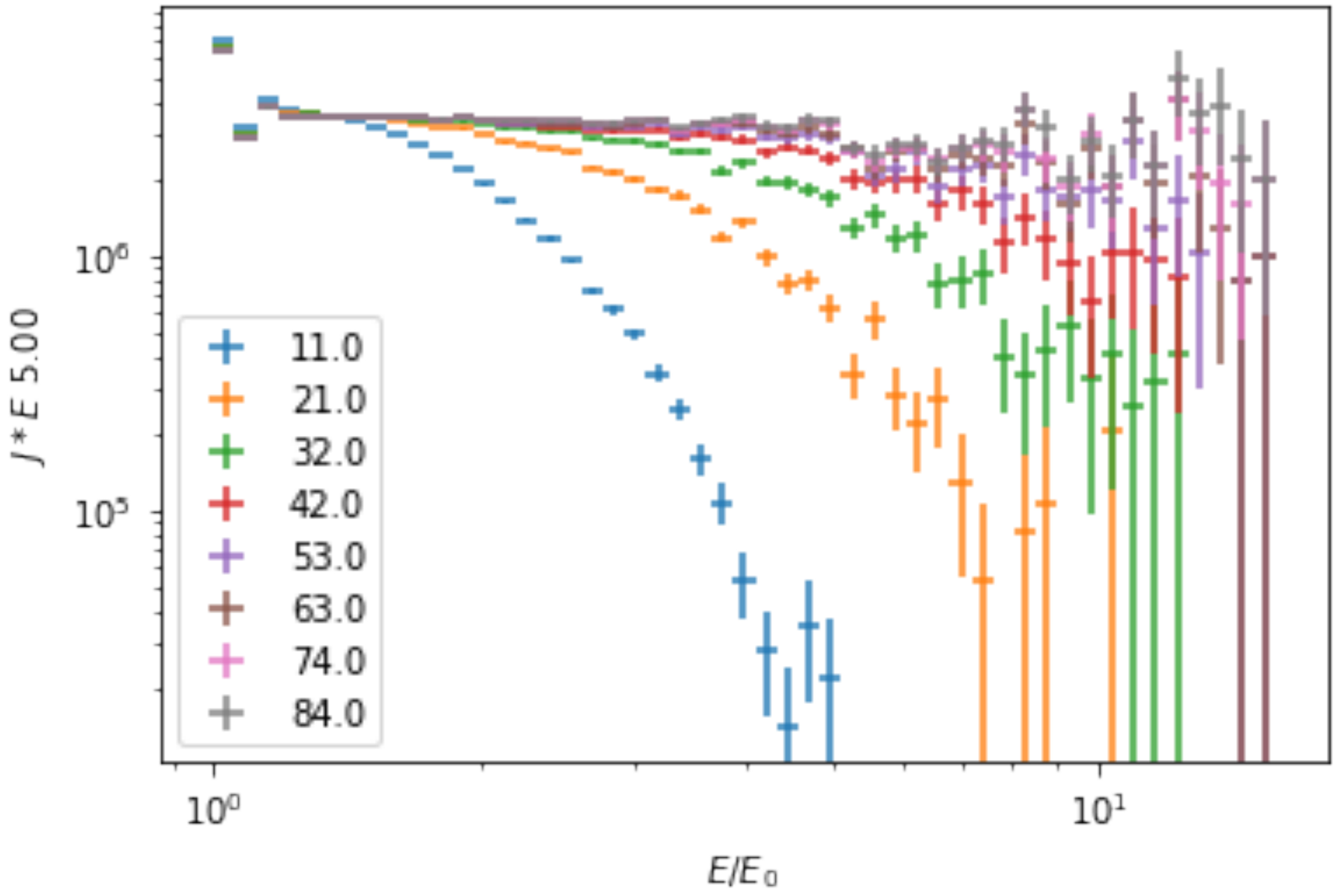

Conclusion

- Diffusive Shock Acceleration can be modelled with CRPropas DiffusionSDE module when shock profile is approximated by continuous advection field
- Resulting spectral index highly depends on the choice of advective step, shock width and diffusive step
- Model acceleration at shocks with finite width
- For ideal shocks, small shock widths and therefore advective step is necessary to produce the correct spectral index
- Any other possibilities to take ideal shocks into account?

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CALCULATION OF DIFFUSIVE SHOCK ACCELERATION OF CHARGED PARTICLES BY SKEW BROWNIAN MOTION

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Time scaling instead of jump conditions

- Velocity and diffusion coefficient defined by continuous functions and jump conditions at the shock:

$$V = V_c(x) + \tfrac{1}{2}\Delta V(0)\ \mathbf{sign}\ (x)\ ,$$

$$\kappa = \kappa_c(x) + \tfrac{1}{2}\Delta\kappa(0)\ \mathbf{sign}\ (x)\ ,$$

- Resulting SDEs:

$$dx(t) = \sqrt{2\kappa}\, dW_x(t) + \Delta\kappa(0)\delta(x)dt + \left[\frac{\partial\kappa_c(x)}{\partial x} + V\right]dt\ ,$$

$$dp(t) = \sqrt{2D}\, dW_p(t) - \frac{1}{3}\,\Delta V(0)p\,\delta(x)dt + \left\{\frac{\partial D}{\partial p} - \frac{1}{3}\left[\frac{\partial V_c(x)}{\partial x}\right]p - k\right\}dt\ .$$

Time scaling instead of jump conditions

- Time scaling to eliminate delta-functions:

$$y = s(x)x \quad \text{with } s(x) = \begin{cases} \alpha\ , & x < 0\ , \\ \frac{1}{2}\ , & x = 0\ , \\ (1-\alpha)\ , & x > 0\ . \end{cases}$$

- Resulting SDE can be solved by the Euler scheme:

$$dy(t) = s(y)\left\{\sqrt{2\kappa}\, dW_x(t) + \left[\frac{\partial \kappa_c(x)}{\partial x} + V\right]dt\right\}\ .$$

- Energy gain depends on the jump conditions and whether the trajectory passes the shock:

$$dp(t) = \sqrt{2D}\, dW_p(t) - \frac{1}{3\Delta\kappa(0)}\, \Delta V(0)p[dx - s^{-1}(y)dy)] + \left(\frac{\partial D}{\partial p} - k\right)dt\ .$$

Conclusion II

- Diffusive Shock Acceleration can be modelled in CRPropa by
  - Approximating velocity profile of the shock (for finite shock widths)
  - Scaling when candidates cross shock front (for ideal shocks)
- Spatial varying Diffusion Coefficient
- Momentum Diffusion for Second Order Fermi Acceleration

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Thank you for your attention!