g-2 from Budapest-Marseille-Wuppertal

Kalman Szabo
Forschungszentrum Jülich & University of Wuppertal

Borsanyi, Fodor, Hoelbling, Kawanai, Krieg, Lellouch, Malak, Miura, Szabo, Torrero, Toth:

[1612.02364, PhysRevD] Slope and curvature ...

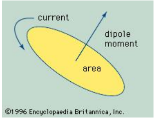
[1711.04980, PhysRevLett] ... magnetic moments of leptons ...

Part I. Intro

From the textbooks

Classical Electrodynamics

$$\vec{\mu} = \frac{e}{2m}\vec{j}$$

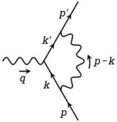


Relativistic Quantum Mechanics

$$\vec{\mu} = g\,\frac{e}{2m}\,\vec{s} \text{ with } g = 2$$

Quantum Electrodynamics

$$a \equiv \frac{g-2}{2} = \frac{\alpha}{2\pi}$$



Experiment

$$a_e = 0.00115965218091(26)$$
$$a_\mu = 0.00116592091(63)$$
$$a_\tau \leqslant 0.013$$

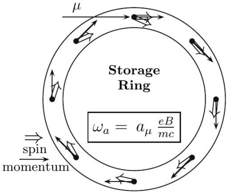
g-2 measurement principle

Feed fast muons into a magnetic field!

$$m\frac{d\vec{v}}{dt} = e\,\vec{v}\times\vec{B} + \ldots$$

$$m\frac{d\vec{s}}{dt} = \frac{g}{2}\,e\,\vec{s}\times\vec{B} + \ldots$$

Precession freq $\omega_a = a_\mu \frac{eB}{m_\mu}$

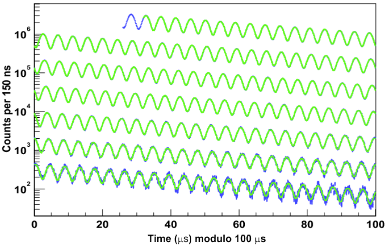


Muons decay:

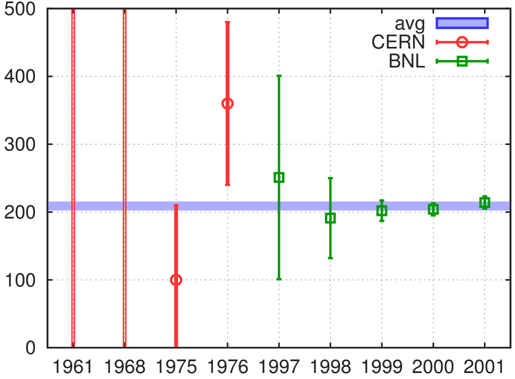
$$\mu^+ \to e^+ + \nu_e + \bar{\nu}_\mu$$

Count positrons:

$$N(t) = N_0 e^{-\frac{t}{\tau}}\,[1 + A\sin(\omega_a t + \varphi)]$$



g-2 experimental summary



$$a_\mu^{\text{exp}} = 11659209.1(6.3) \times 10^{-10}$$

Two new experiments plan to reduce error to $1.6 \cdot 10^{-10}$

- Fermilab E989: already taking data, first results end of 2018, final in 2020
- J-PARC E34: starts data taking in 2020

g-2 from theory

What contributes to a_μ^{theo}?

Knowns

QED $\frac{\alpha}{\pi} \sim 10^{-3}$

QCD $\left(\frac{\alpha}{\pi}\right)^2 \left(\frac{m_\mu}{m_\rho}\right)^2 \sim 10^{-7}$ $\rightarrow$ is it under control?

weak $\frac{\alpha_W}{\pi} \left(\frac{m_\mu}{m_W}\right)^2 \sim 10^{-9}$

Unknowns

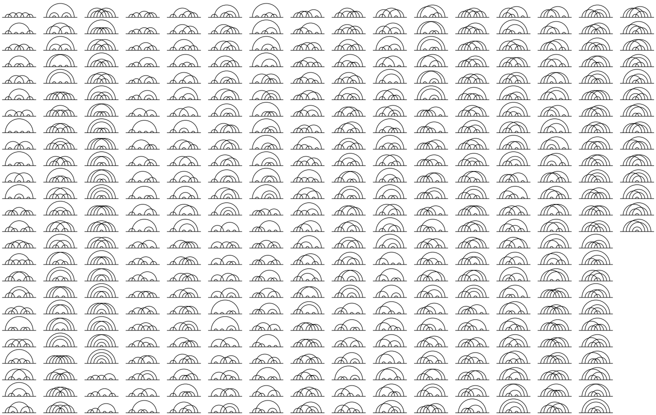
SUSY $\left(\frac{m_\mu}{m_{\text{SUSY}}}\right)^2$ $\rightarrow e \ll \mu \ll \tau$

??? ???

QED contribution [Kinoshita et al '15]

Diagrams with only photons and leptons.

12672 diagrams at five loops



Automated diagram generation, numerical evaluation of integrals, only some diagrams known analytically.

QED contribution

Inputs are m_μ/m_e, m_μ/m_τ and α, α taken from a_e:

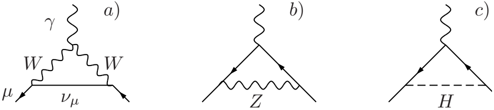
$\alpha^{-1}(a_e) = 137.035999069(96)$ 13 digits precision!

n-loop	$a_\mu^{\rm QED} \times 10^{-10}$
1	11614097.330(0.008)
2	41321.762(0.010)
3	3014.190(0.000)
4	38.081(0.030)
5	0.448(0.140)
total	11658471.811(0.160)

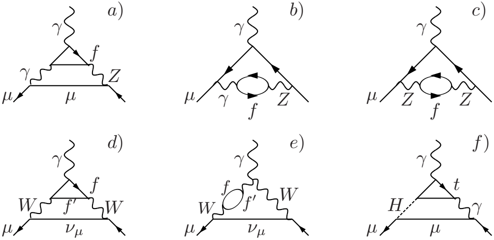
$$a_\mu^{\rm exp} - a_\mu^{\rm QED} = 737.2(6.3) \times 10^{-10}$$

Weak contribution [Gnendiger et al '15]

1-loop $a_\mu^{\text{weak},(1)} = 19.480(1) \times 10^{-10}$



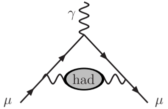
2-loop $a_\mu^{\text{weak},(2)} = -4.12(60) \times 10^{-10}$



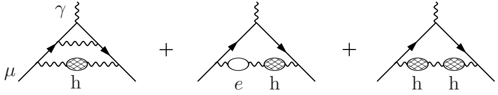
$$a_\mu^{\text{exp}} - a_\mu^{\text{QED}} - a_\mu^{\text{weak}} = 721.8(6.3) \times 10^{-10}$$

QCD contributions

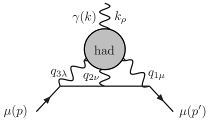
LO hadron vacuum polarization (LO-HVP, $(\frac{\alpha}{\pi})^2$)



NLO hadron vacuum polarization (NLO-HVP, $(\frac{\alpha}{\pi})^3$)

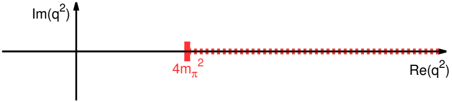


Hadronic light-by-light (HLbL, $(\frac{\alpha}{\pi})^3$)



HVP from $e^+e^- \to$ hadrons [Bouchiat et al '61]

$$\Pi_{\mu\nu}(q) = \left(q_\mu q_\nu - g_{\mu\nu} q^2\right) \Pi(q^2)$$



Use a dispersion relation (analiticity)

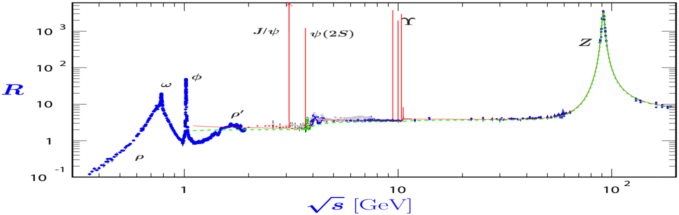
$$\Pi(q^2) = \int_{4m_\pi^2}^{\infty} ds \, \frac{q^2}{s(s+q^2)} \frac{1}{\pi} \mathrm{Im}\Pi(s)$$

and the optical theorem (unitarity)

$$\mathrm{Im}\Pi(s) = -\frac{\sigma(e^+e^- \to \mathrm{had})}{48\pi^2 \alpha(s)^2/(3s)}$$

HVP from experiments

Use $e^+e^- \to$ had data of CMD, SND, BES, KLOE, BABAR, etc:

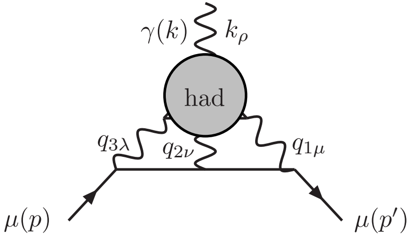


Recent determinations

LO	688.1(4.1)	[Jegerlehner ’17]
LO	692.6(3.3)	[Davier ’16]
LO	694.9(4.3)	[Hagiwara et al ’11]
NLO	-9.87(0.09)	[Kurz et al ’14]
NNLO	1.24(0.01)	[Kurz et al ’14]

$$a_\mu^{\text{exp}} - a_\mu^{\text{QED}} - a_\mu^{\text{weak}} - a_\mu^{\text{HVP}} = 37.8(7.1) \times 10^{-10}$$

HLbL estimates



Only $O(\alpha^3)$ but very complicated.
Involves $\Pi_{\mu\nu\lambda\rho}(q_1, q_2, q_3, k)$ with 47 Lorentz invariants of which 12 contribute to a_μ [Colangelo et al '14]

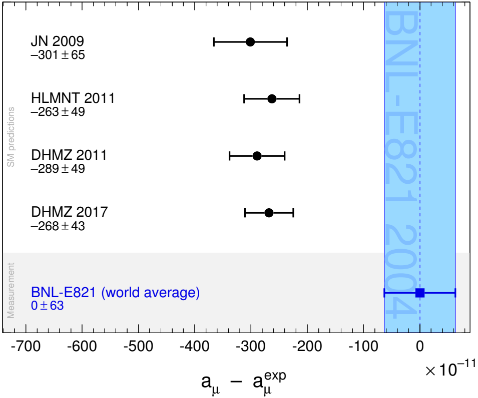
Not fully related to experimental observables → model estimates [Bijnens, Hayakawa, Nyffeler et al] → world average

$a_\mu^{\text{HLbL}} \approx 10.5(2.6) \times 10^{-10}$ [Prades et al '09]

New dispersive approach [Colangelo Lat17] and lattice [Mainz, RBC/UKQCD] in progress.

A $3 \div 4\sigma$ discrepancy

$$a_\mu^{\mathrm{exp}} - a_\mu^{\mathrm{QED}} - a_\mu^{\mathrm{weak}} - a_\mu^{\mathrm{HVP}} - a_\mu^{\mathrm{HLbL}} = 27.3(7.6) \times 10^{-10}$$



Discrepancy is about $2\times$ electroweak contribution.

Outlook

Error budget

$$(7.6) = (6.3)_{\text{exp}}(0.2)_{\text{QED}}(0.6)_{\text{weak}}(3.3)_{\text{HVP}}(2.6)_{\text{HLbL}}$$

exp: Fermilab E989 and J-PARC E34 aim for $(1.5)_{\text{exp}}$

HVP and HLbL: lattice QCD with controlled errors

No new physics requires

- 4% larger HVP, ie. $a_\mu^{\text{HVP}} = 720.0(6.8) \times 10^{-10}$
- 360% larger HLbL, ie. $a_\mu^{\text{HLbL}} = 37.9(7.1) \times 10^{-10}$

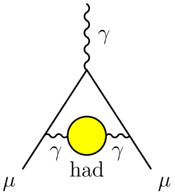
For new physics

assuming central values remain the same

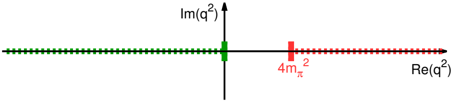
- E989 + same theory errors 6σ
- E989 + HLbL 10% + HVP 0.2% 11σ

Part II. LO-HVP from lattice QCD

a_μ^{HVP} from the lattice [Blum '02]



$$\Pi_{\mu\nu}(q) = \left(q_\mu q_\nu - g_{\mu\nu} q^2\right) \Pi(q^2)$$



Consider the diagram in Euclidean instead ($q_0 \to -i(q_E)_0$):

$$a_\mu^{\text{LO-HVP}} = \left(\frac{\alpha}{\pi}\right)^2 \int_0^\infty dq_E^2\; w(q_E^2)\; \Pi(q_E^2)$$

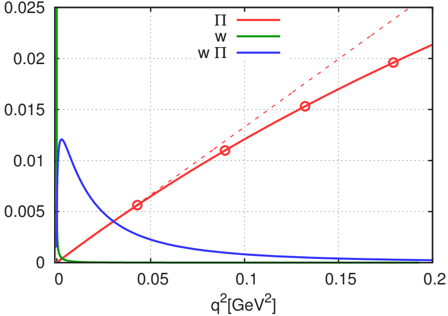
- w known function, describes the non-HVP part
- $\Pi(q_E)$ can be obtained from the lattice current-current correlator
- charge renormalization requires $\Pi(0) = 0$

a_μ^{HVP} integral

$$a_\mu = \int dq^2 \; w(q^2)\Pi(q^2)$$

For small momenta $\Pi \sim q^2, w \sim 1/q^2$

Integrand is peaked around $q = m_\mu/2 \rightarrow \Pi(q^2)$ has to be computed precisely for small momenta.



For small momenta $\Pi(q^2)$ can be nicely approximated by first few derivatives: slope $\Pi'(0)$ - curvature $\Pi''(0)$

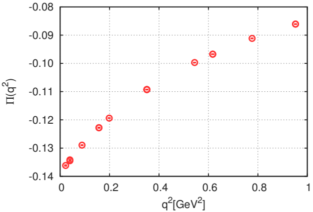
Lattice details

Recipe

1. Measure $\Pi_{\mu\nu}(q) = \sum_x e^{iqx} \langle j_\mu(x) j_\nu(0) \rangle$ on the lattice
2. Extract Π from $\Pi_{\mu\nu} = \left(\hat{q}_\mu \hat{q}_\nu - \delta_{\mu\nu} \hat{q}^2\right) \Pi(q^2) + \ \ldots$

Problems

- non-conserved current requires multiplicative renorm.
- $\Pi(q)$ is only for discrete momenta $\to$ interpolation?



- $\Pi(0)$ is not directly accessible $\to$ extrapolation??

Moments [HPQCD '14]

$$\Pi(q^2) = \Pi'(0)\cdot q^2 + \Pi''(0)\cdot \frac{q^4}{2} + \ \ldots$$

Start from continuum, infinite box:

$$\int dx\ e^{iqx}\langle j_\mu(x) j_\nu(x)\rangle = (q_\mu q_\nu - \delta_{\mu\nu} q^2)\Pi(q^2)$$

and Taylor expand both sides in q. You get

$$\Pi'(0) = \frac{1}{24}\int dt\ t^4 C(t), \quad \Pi''(0) = -\frac{1}{720}\int dt\ t^6 C(t), \quad \ldots$$

C(t) is zero momentum timelike jj-propagator

On the lattice calculate

$$\Pi'(0) = \frac{1}{24}\sum_{t=-T/2+1}^{T/2} t^4 C(t),\ \ldots$$

and use in the Taylor expansion:

- $\Pi(q^2)$ as a continuous function of q^2
- renormalized by construction $\Pi(0) = 0$

Note: Pade instead of Taylor for improved convergence (first 2 orders suffice!)

Master formula [Bernecker et al '11, RBC/UKQCD '14 '16]

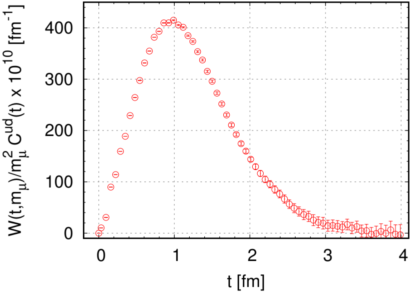
Put back the expansion coefficients into the expansion:

$$\Pi(q^2) = \sum_{t=-T/2+1}^{T/2} \left[\frac{\cos(qt)-1}{q^2} + \frac{t^2}{2}\right] C(t)$$

and finally into the a_μ-integral $a_\mu = \int dq^2\ w(q^2)\Pi(q^2)$

$$a_\mu^{\mathrm{HVP}} = \sum_{t=-T/2+1}^{T/2} W(t, m_\mu a) C(t)$$

a lattice spacing, W is known kinematical function



Contributions

Decompose the current as

$$j_\mu = \frac{2}{3}u_\mu - \frac{1}{3}d_\mu - \frac{1}{3}s_\mu + \frac{2}{3}c_\mu$$

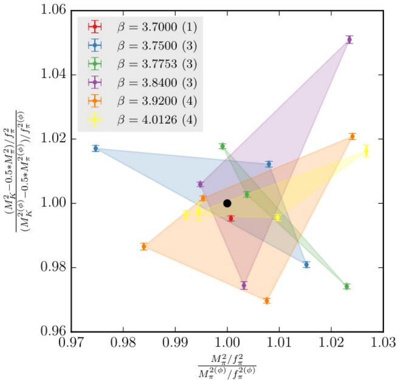
Assuming isospin symmetry the correlator is

$$\langle jj \rangle \sim C(t) = C^{ud} + C^{s} + C^{c} + C^{disc}$$

- **ud** has poor signal for large t and is very sensitive to finite box size → needs high statistics and tricks
- **s** "nothing to see here"
- **c** has large lattice artefacts
- **disc** SU(3) suppressed (vanishes if $m_s = m_{ud}$)

Simulations [1612.02364, 1711.04980]

Landscape



- $N_f = 2+1+1$ smeared staggered
- 15 high statistics simulations
- bracketing physical quark masses
- 6 lattice spacings
- $L \gtrsim 6$ fm

Techniques

conserved EM current - EigCG - Low Mode Averaging - All Mode Averaging [Blum et al '13] - close to 10M/40M conn/disc measurements

Noise reduction

Noise/signal grows $\exp[(M_\rho - M_\pi)t]$.

Consider upper/lower bounds

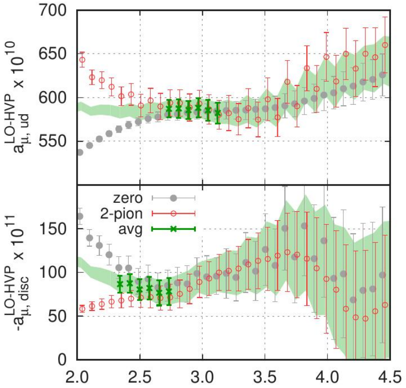
See also [Lehner '15]

Connected $I = 1$

$$0 \leqslant C^{ud}(t) \leqslant C^{ud}(t_c)e^{E_{2\pi}(t-t_c)}$$

Disconnected $I = 0$

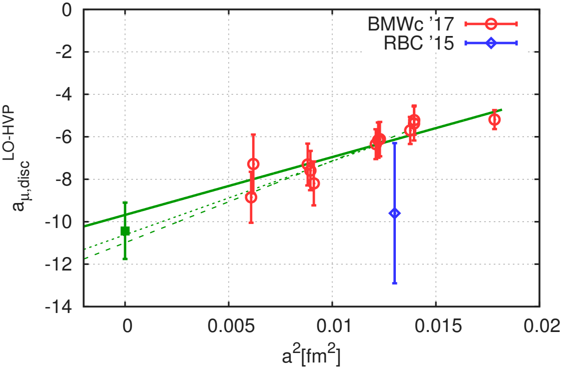
$$0 \leqslant -C^{disc}(t) \leqslant \frac{1}{10}C^{ud}(t_c)e^{E_{2\pi}(t-t_c)}$$



For $t \geqslant t_c$ bounds meet, replace $C(t)$ by average bounds.

Vary t_c for systematics.

Disconnected isn't difficult

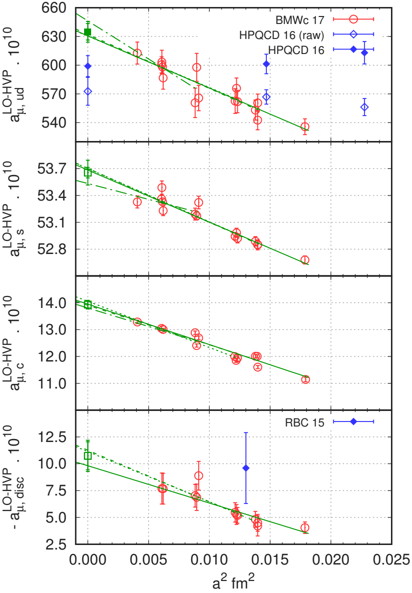


Contrary to earlier expectations

- easy to measure → use LMA and approximate SU3 symmetry [Francis et al '14]
- small → ~ 2% instead of 10% [Juttner, DellaMorte '10]

Also measured charm disconnected, wich is really tiny.

Continuum limits



- With 6 a's, have full control over continuum limit
- Get good χ^2/dof w/ extrapolation linear in a^2 and interpolations, linear in m_π^2 and m_K^2
- Strong continuum extrapolation for a_μ^{ud} due to taste violations and for a_μ^c due to large m_c
- Get continuum systematic from all results and by cutting results with $a \geqslant 0.134,\ 0.111,\ 0.095\,\text{fm}$

Matching to perturbation theory

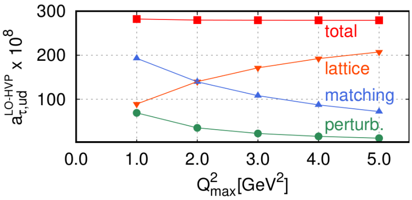
Restrict integral $a_\mu = \int w(q^2)\Pi(q^2)$ to q_{max}! Consider separation

$$a_\mu = a_\mu^{latt} + a_\mu^{pert} + a_\mu^{match}$$

with

$$a_\mu^{latt} = \int_0^{max} w(q^2) \cdot \Pi(q^2)$$

$$a_\mu^{pert} = \int_{max}^{\infty} w(q^2) \cdot [\Pi(q^2) - \Pi(q^2_{max})], \qquad a_\mu^{match} = \left[\int_{max}^{\infty} dq^2 w(q^2)\right] \cdot \Pi(q^2_{max})$$

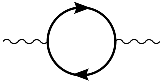


Result is independent of $q^2_{max} \gtrsim 2\text{GeV}^2 \rightarrow$ lattice matches perturbation theory perfectly, ie. $\Pi(q^2 \rightarrow \infty)$ is under control.

Finite volume effect

Work in fixed physical volume $\rightarrow$ FV effect cannot be estimated from simulations. Use estimates now.

Long distance dominated by 2π



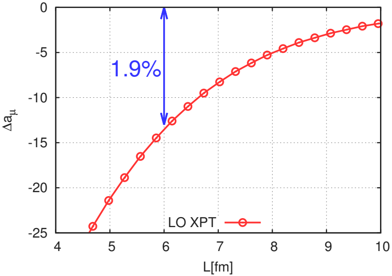
FV effects are exponentially suppressed, but can be large.

Recent results

- **Leading order XPT** [Aubin et al '16] just the $\pi^+\pi^-$ loop - no interaction between pions
- **Interacting pions** [Francis et al '13] determine energy levels and matrix elements of 2-pion states in a finite box [Lellouch-Luscher] from experiment data [Gounaris-Sakurai]

  $\rightarrow$ "Free theory is smaller by factor 1.5-2.0"
- **dedicated FV study** with $L = 9\text{fm}$ [Izubuchi et al '18]

Finite volume effect, etc.



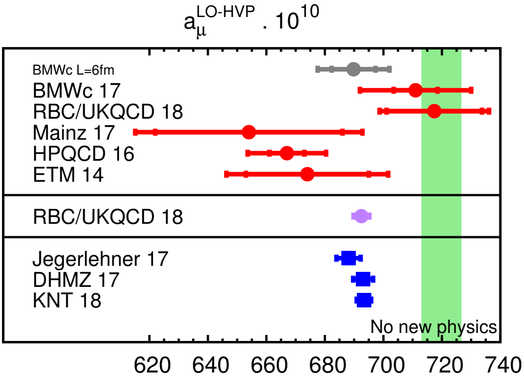
We corrected the results by LO XPT and lack of numerical evidence we assigned 100% error to this correction.

Isospin breaking

We also corrected results for QED and $m_u \neq m_d$ effects, by taking the missing effects from the dispersive approach, again with a conservative error

$$\Delta_{IB} a_\mu = 7.8(5.1) \times 10^{-10}$$

Results



Consistent with both no new physics and phenomenology.

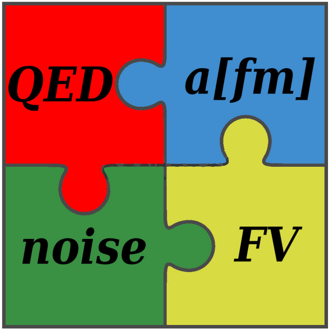
HPQCD gets 2σ smaller, discrepancy in light contribution.

Error budget:

$$(2.7)\ \% = (1.1)_{\text{stat}}(1.1)_{\text{cont}}(0.8)_{\text{scale}}(1.9)_{\text{FV}}(0.7)_{\text{QED}}\ \%$$

Moments [1612.02364], a_e and a_τ have also been determined

Outlook - going for 0.2%

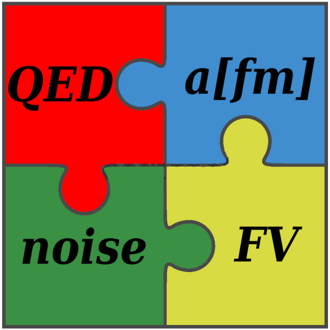


- **QED** compute QED and $m_u \neq m_d$ effects - no high precision is needed, but there are many diagrams [ETM '17, RBC/UKQCD '17 '18]
- **a[fm]** to reach 0.2% error from the scale, need $\lesssim 0.1\%$ error on the scale, since $[\Pi'] = \mathrm{GeV}^{-2}$ - current w_0 determinations have $\sim 0.5\%$
- **FV** understand and control FV effects much better, eg. dedicated studies [Izubuchi et al '18]
- **noise** currently $\sim 1.0\%$, increase statistics

Super(?)computers

JUQUEEN will be shutdown completely no later than Tuesday 22nd of May.

	IBM BGQ	Intel KNL	Nvidia 6× Volta
bandwidth[GB/s]	40	400	5400
comm[GB/s]	20	25	50
comm/band	0.5	0.06	0.009



Thank you!