Power divergences and the gradient flow: A continuum hammer for lattice nails

Chris Monahan

Institute for Nuclear Theory

University of Washington

with Kostas Orginos

and separately with
Andrea Shindler and Matt Rizik

Outline

Hadron structure

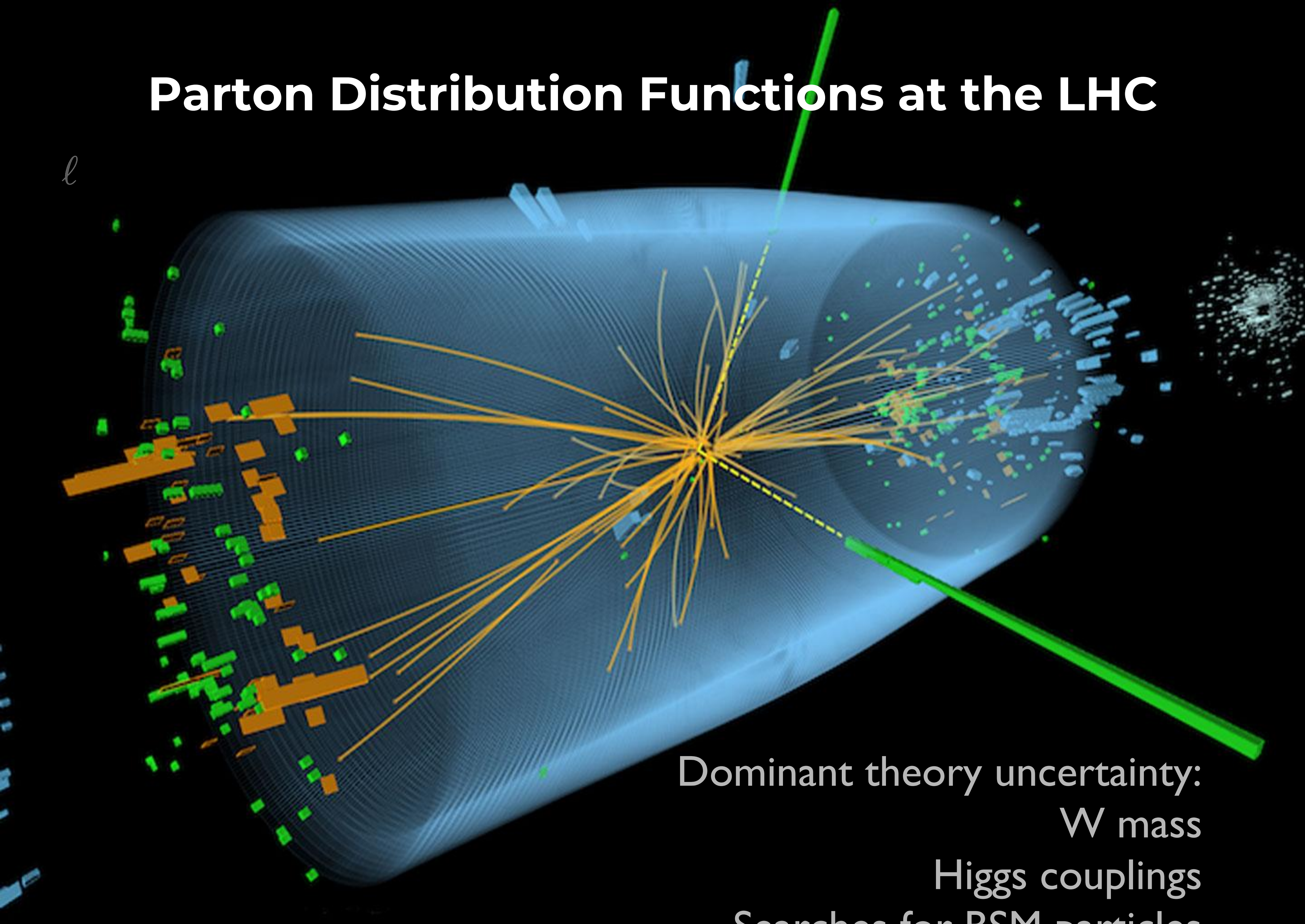
Recent approaches on the lattice and power divergences

- moments
- spacelike distributions

Smeared distributions

- “factorisation”
- perturbative analysis

Parton Distribution Functions at the LHC



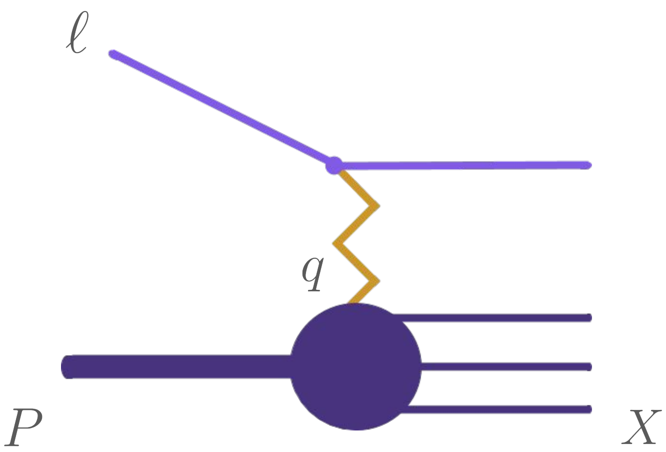
Dominant theory uncertainty:

W mass

Higgs couplings

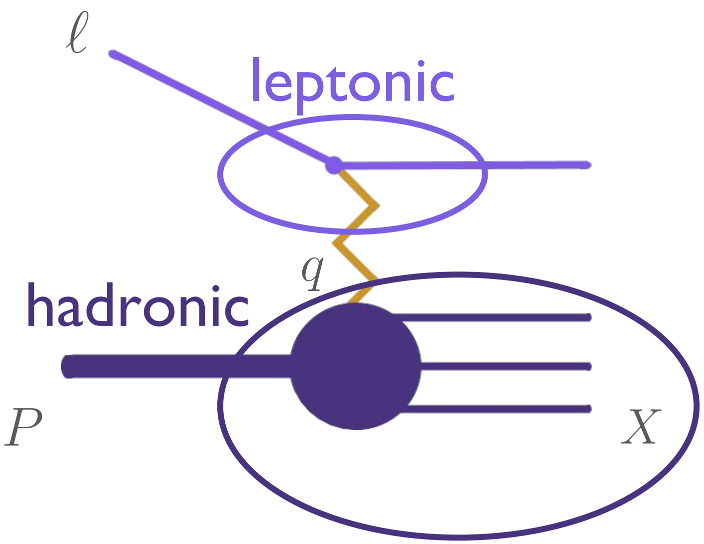
Searches for BSM particles

Deep Inelastic Scattering



$$Q^2 = -q^2 \qquad x = \frac{Q^2}{2P \cdot q}$$

Deep Inelastic Scattering



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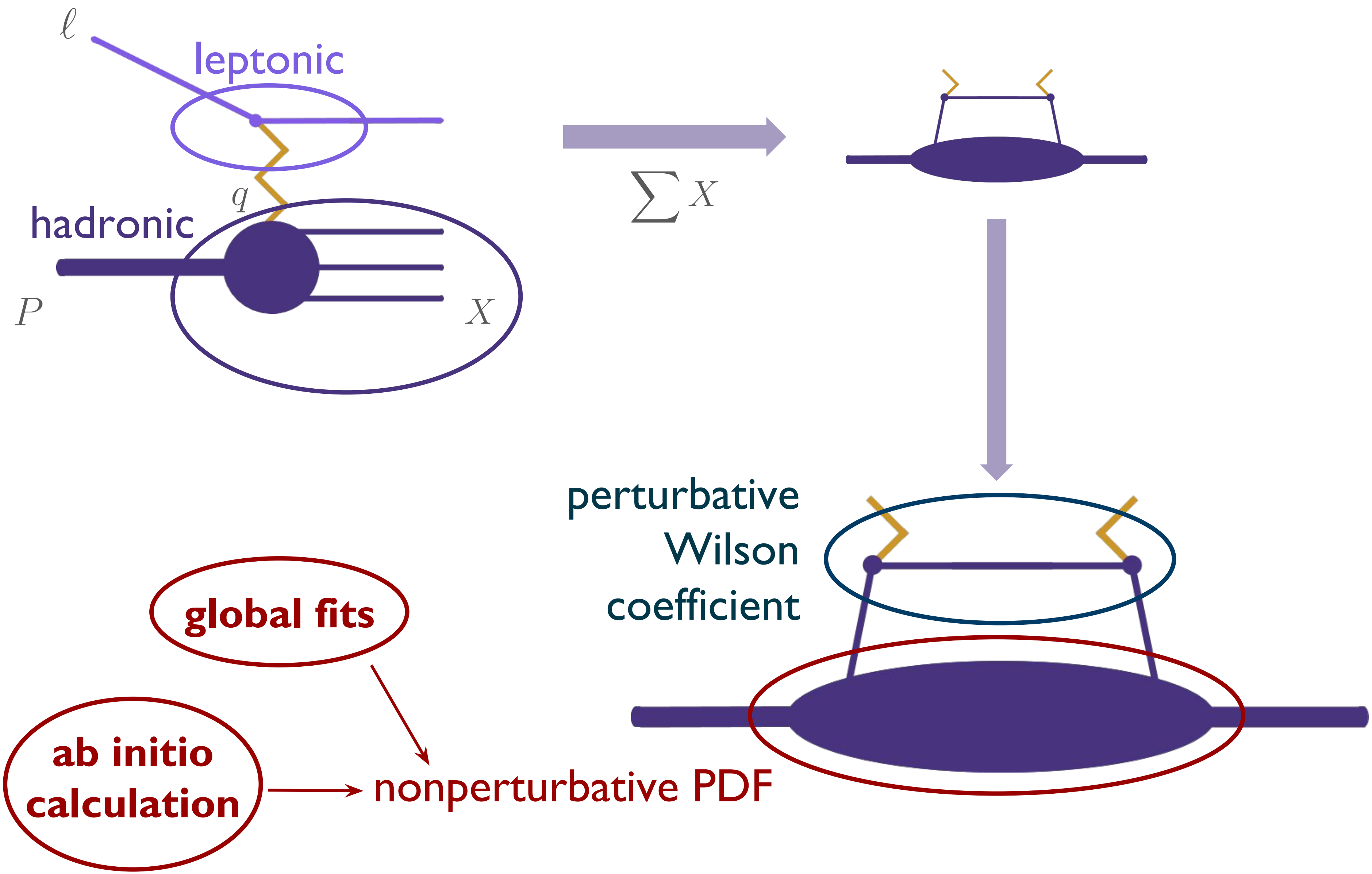


Parton distribution functions

Field theoretic definition

$$f^{(0)}(\xi) = \int_{-\infty}^{\infty} \frac{\mathrm{d}\omega^-}{4\pi} e^{-i\xi P^+ \omega^-} \left\langle P \left| T\, \overline{\psi}(0, \omega^-, \mathbf{0}_\mathrm{T}) W(\omega^-, 0) \gamma^+ \frac{\lambda^a}{2} \psi(0) \right| P \right\rangle_\mathrm{C}$$

where

$$W(\omega^-, 0) = \mathcal{P} \exp \left[-i g_0 \int_0^{\omega^-} \mathrm{d}y^- A^+_\alpha(0, y^-, \mathbf{0}_\mathrm{T}) T_\alpha \right]$$

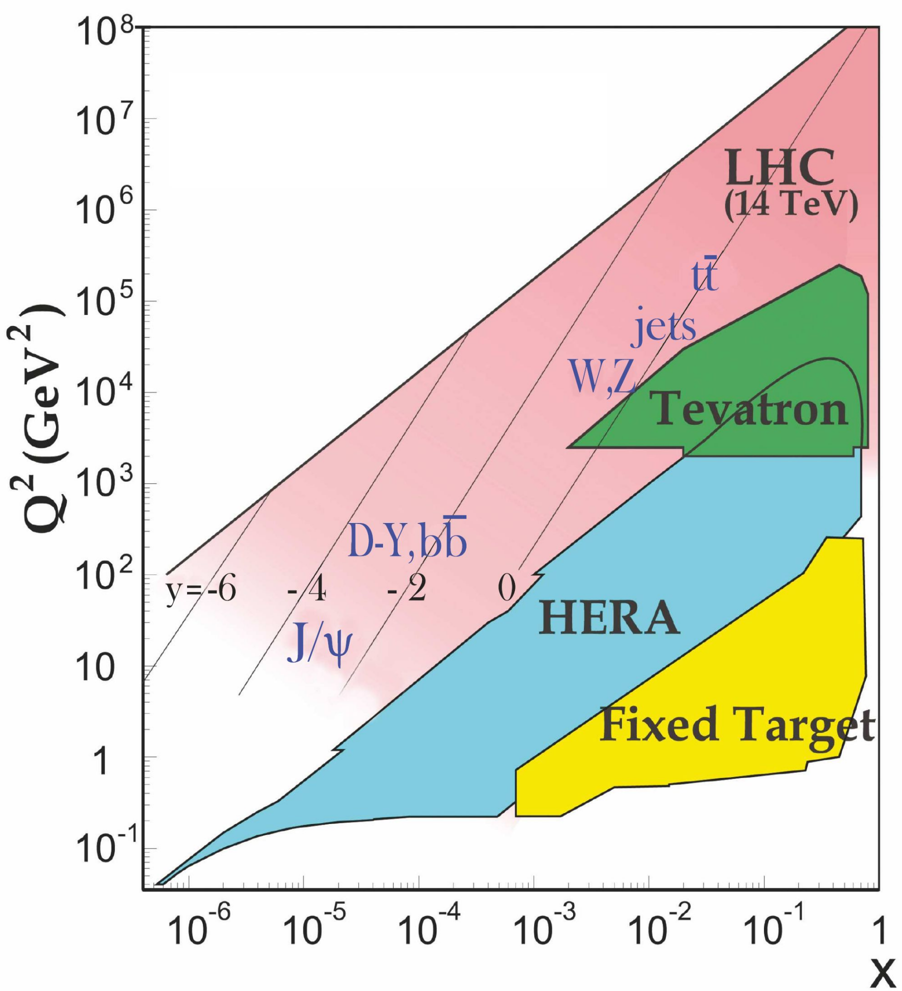
$$\langle P' | P \rangle = (2\pi)^3 2P^+ \delta\left(P^+ - P'^+\right) \delta^{(2)}\left(\mathbf{P}_\mathrm{T} - \mathbf{P}'_\mathrm{T}\right)$$

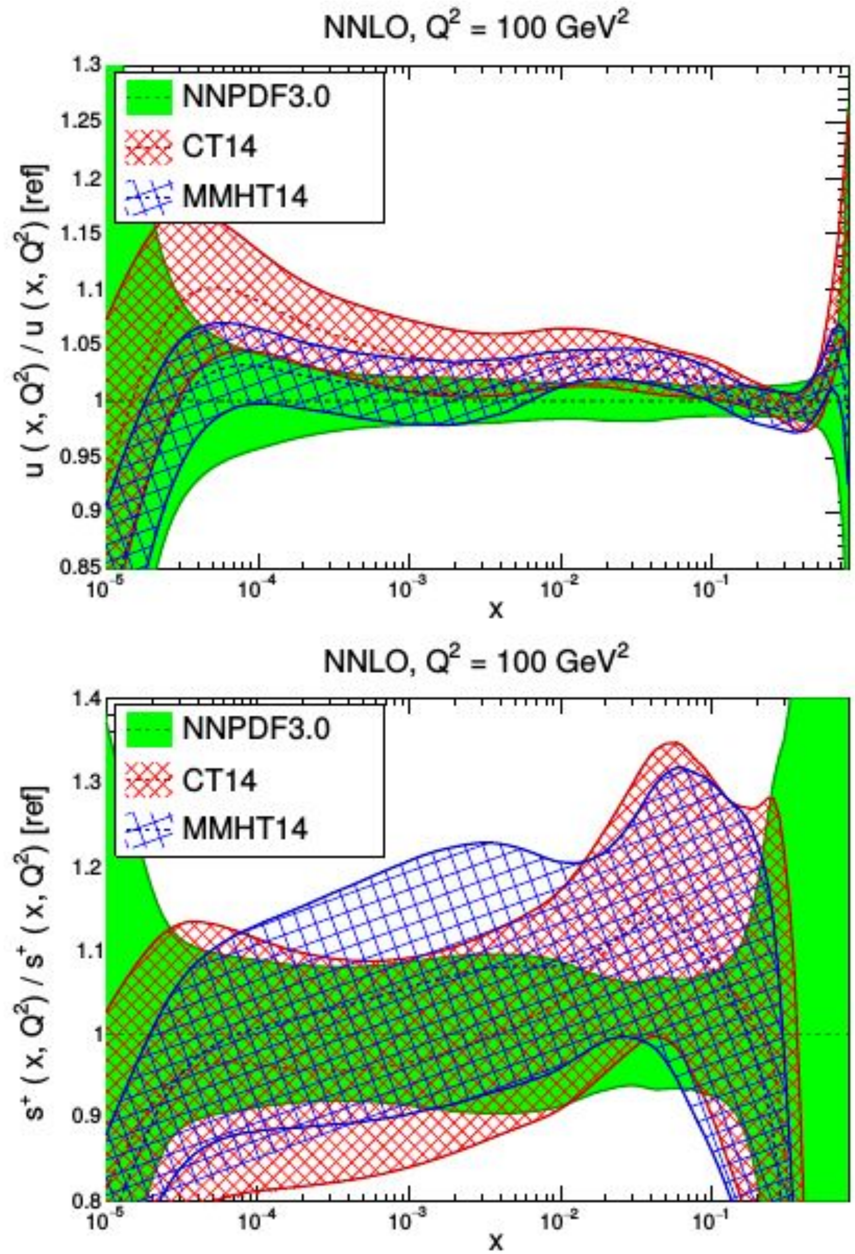
Renormalised PDFs satisfy DGLAP evolution

$$f(\xi, \mu) = \int_x^1 \frac{\mathrm{d}\zeta}{\zeta} \mathcal{Z}\left(\frac{\xi}{\zeta}, \mu\right) f^{(0)}(\zeta) \qquad \mu \frac{\mathrm{d}f(\xi, \mu)}{\mathrm{d}\mu} = \frac{\alpha_s(\mu)}{\pi} \int_x^1 \frac{\mathrm{d}\zeta}{\zeta} f(\zeta, \mu) P\left(\frac{\xi}{\zeta}\right)$$

PDG, Chin. Phys. C 40 (2016) 100001

Global fits





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Ab initio calculations: moments

Mellin moments of PDFs

$$a^{(n)}(\mu) = \int_0^1 \mathrm{d}\xi\, \xi^{n-1}\left[f(\xi,\mu) + (-1)^n \overline{f}(\xi,\mu)\right] = \int_{-1}^1 \mathrm{d}\xi\, \xi^{n-1} f(\xi,\mu)$$

related to matrix elements of twist-two operators

$$\langle P|\mathcal{O}^{\{\nu_1\ldots\nu_n\}}(\mu)|P\rangle = 2a^{(n)}(\mu)\left(P^{\nu_1}\cdots P^{\nu_n} - \mathrm{traces}\right)$$

$$\mathcal{O}^{\{\nu_1\ldots\nu_n\}}(\mu) = Z_{\mathcal{O}}(\mu)\left[i^{n-1}\overline{\psi}(0)\gamma^{\{\mu_1}D^{\mu_2}\cdots D^{\mu_n\}}\frac{\lambda^a}{2}\psi(0) - \mathrm{traces}\right]$$

Limited to three moments by power-divergent mixing

$$\overline{\psi}\gamma_4\gamma_5\overleftrightarrow{D}_4\overleftrightarrow{D}_4\psi \sim \frac{1}{a^2}\overline{\psi}\gamma_4\gamma_5\psi$$

Reconstructions, rather than ab initio calculations

Detmold *et al.*, Eur. Phys. J. C 3 (2001) 1
Detmold *et al.*, Phys. Rev. D 68 (2001) 034025
Detmold *et al.*, Mod. Phys. Lett. A 18 (2003) 2681

Davoudi & Savage, PRD 86 (2012) 054505

Ab initio calculations: moments

Smeared lattice operators should allow higher moments to be computed

$$a^{(n)}(\mu) = \int_0^1 \mathrm{d}\xi\, \xi^{n-1} \left[f(\xi,\mu) + (-1)^n \overline{f}(\xi,\mu)\right] = \int_{-1}^1 \mathrm{d}\xi\, \xi^{n-1} f(\xi,\mu)$$

Construct and fix operator size in continuum limit

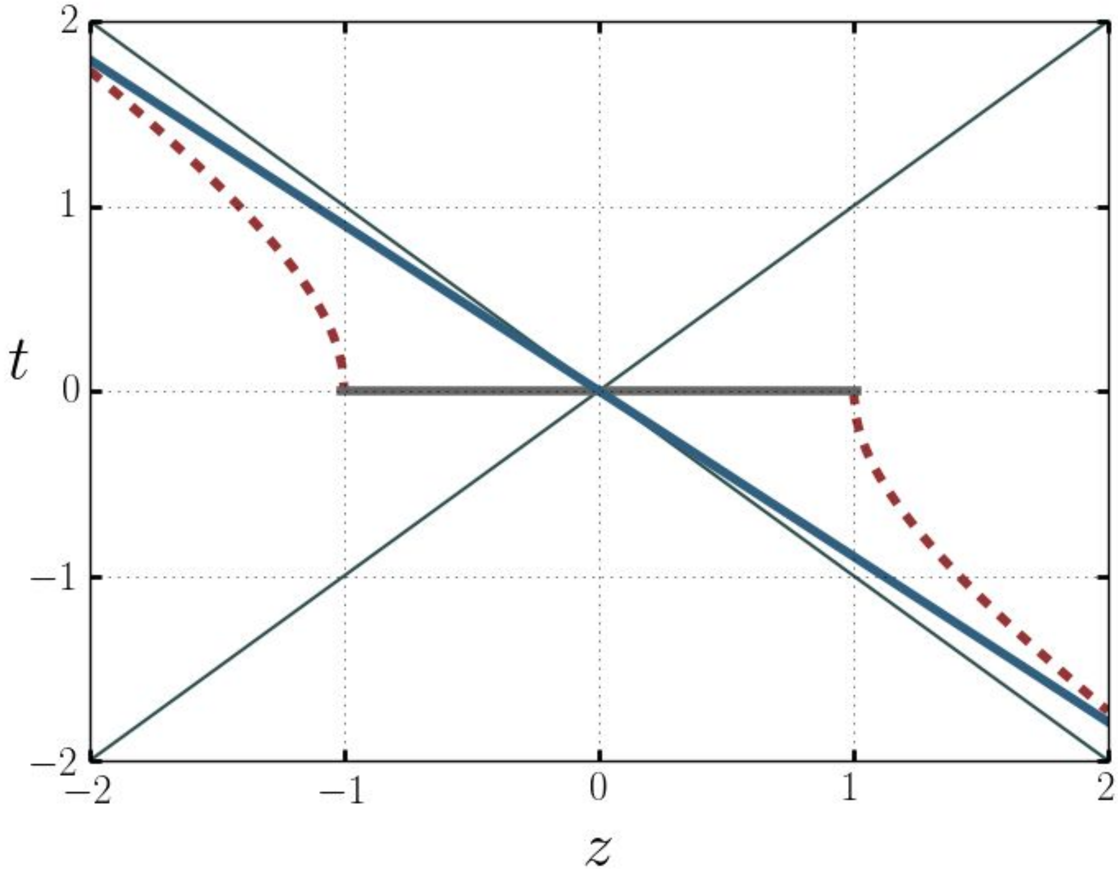
$$\theta_{LM}(\mathbf{x}, a, N) = \frac{3}{4\pi N^3} \sum_{\mathrm{n}}^{|\mathrm{n}|<N} \phi(\mathbf{x})\phi(\mathbf{x}+\mathbf{n}a) Y_{L,M}(\widehat{\mathrm{n}})$$

Some preliminary results presented recently

Flow is a (much more?) natural approach here

CJM & Orginos, PRD 91 (2015) 074513

Ab initio calculations: x-dependence



Ji, PRL 110 (2013) 262002
Radyushkin, PRD 96 (2017) 034025

A panoply of distributions

$$I^{(0)}(\zeta = P\cdot n, n^2) = \frac{1}{2P^\mu}\left\langle P\left|\overline{\psi}(n)W(n,0)\Gamma_\mu\psi(0)\right|P\right\rangle$$

$$W(n(u),0) = \mathcal{P}\exp\left[-ig_0\int_0^u \mathrm{d}v\frac{\mathrm{d}y^\mu}{\mathrm{d}v}A_\mu^a(y(v))T^a\right]$$

Radyushkin, PRD 96 (2017) 034025

A panoply of distributions

PDFs

Quasi PDFs

ξ^-

$$\zeta = -P^+\xi^-$$
$$n^2 = 0$$

P^z

$$\zeta = P^z z$$
$$n^2 = z^2 < 0$$

$$I^{(0)}(\zeta = P \cdot n, n^2) = \frac{1}{2P^\mu} \left\langle P \left| \overline{\psi}(n) W(n,0) \Gamma_\mu \psi(0) \right| P \right\rangle$$

$$W(n(u),0) = \mathcal{P} \exp\left[-ig_0 \int_0^u \mathrm{d}v \frac{\mathrm{d}y^\mu}{\mathrm{d}v} A^a_\mu(y(v)) T^a \right]$$

ζ

Pseudo PDFs

$$\zeta = P^z z$$
$$n^2 = z^2 < 0$$

Fourier transforms generate distribution functions

Factorisation theorems relate distribution functions

Ji, PRL 110 (2013) 262002
Ji et al., NPB 924 (2017) 326
Izubuchi et al., 1801.03917

Example: quasi distributions

Field theoretic definition

Ji, PRL 110 (2013) 262002

$$\widetilde{f}^{(0)}_{j/H}(\xi, P^z) = \int_{-\infty}^{\infty} \frac{\mathrm{d}z}{4\pi} e^{i\xi P_z z} \left\langle P \left| \overline{\psi}(0, z, \mathbf{0}_\mathrm{T}) W(z,0) \Gamma \psi(0) \right| P \right\rangle$$

Factorisation

$$\widetilde{f}_{j/H}(\xi, P^z, \mu_\mathrm{R}) = \int_{-1}^{1} \frac{\mathrm{d}y}{|y|} C^{(\widetilde{f})} \left(\frac{\xi}{y}, \frac{\mu_\mathrm{R}}{P^z}, \frac{\mu}{p^z} \right) f_{j/H}(y, \mu) + \mathcal{O}\left(\frac{M^2}{(P^z)^2}, \frac{\Lambda^2_\mathrm{QCD}}{(P^z)^2} \right)$$

Renormalised quasi PDFs do not satisfy DGLAP evolution

Wilson line operator

- generates a power divergence
- multiplicatively renormalised in coordinate space

Ji & Zhang, PRD 92 (2015) 034006
Ji et al., PRL 120 (2018) 112001

Power divergences

Exponential mass counterterm

$$W^{(0)}(z,0) = Z_q Z_\Psi e^{\delta m z} W^{(\mathrm{R})}(z,0)$$

Ishikawa et al., 1609.02018
Chen et al., NPB 915 1

RI' and RI/MOM schemes

Alexandrou et al., NPB 923 (2017) 324

$$Z_q^{-1} Z_W(z) \frac{1}{12} \mathrm{Tr}\left[V_W(p,z)\left(V_W^{\mathrm{tree}}(p,z)\right)^{-1}\right]_{p^2=\widetilde{\mu}_0^2} = 1$$

$$\left.\frac{\mathrm{Tr}\left[\not{p} V_W(p,z)\right]}{\mathrm{Tr}\left[\not{p} V_W^{\mathrm{tree}}(p,z)\right]}\right|_{p^2=\widetilde{\mu}^2} = 1$$

Chen et al., PRD 97 (2018) 014505

``Reduced'' pseudo distribution

$$\widetilde{p}^{(\mathrm{R})}(\nu, z^2) = \frac{p(\nu, z^2)}{p(0, z^2)}$$

Orginos et al., PRD 96 (2017) 094503

Gradient flow

CJM & Orginos, JHEP 03 (2017) 116
CJM, PRD 97 (2018) 054505

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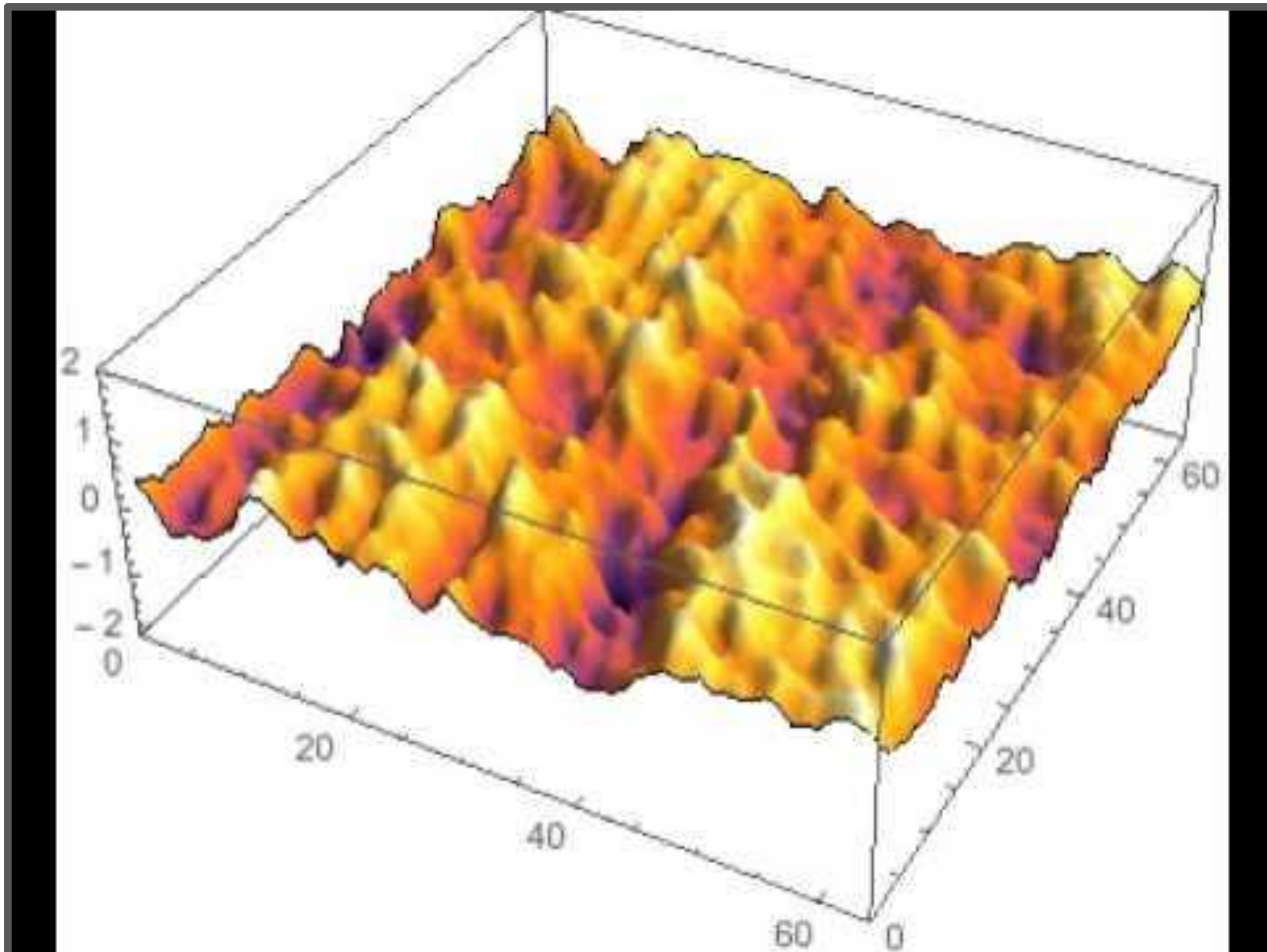


CJM, PoS(Lattice2015) 052

Scalar field theory

Scalar field theory

$$\frac{\partial}{\partial \tau}\overline{\phi}(\tau,x) = \partial^2\overline{\phi}(\tau,x) \qquad \overline{\phi}(\tau\!=\!0,x) = \phi(x) \qquad \widetilde{\overline{\phi}}(\tau,p) = e^{-\tau p^2}\widetilde{\phi}(p)$$

CJM & K. Orginos, PRD 91 (2015) 074513

Exact solution possible with Dirichlet boundary conditions

$$\overline{\phi}(\tau,x) = \int \mathrm{d}^4y \int \frac{\mathrm{d}^4p}{(2\pi)^4} e^{ip\cdot(x-y)} e^{-\tau p^2}\phi(y) = \frac{1}{16\pi^2\tau^2}\int \mathrm{d}^4y\, e^{-(x-y)^2/(4\tau)}\phi(y)$$

Interactions occur at zero flow time (*i.e.* in original "boundary" theory): guarantees that renormalised correlation functions remain finite.

Narayanan & Neuberger, JHEP 0603 (2006) 064
Lüscher, Commun. Math. Phys. 293 (2010) 899

QCD

QCD

$$\frac{\partial}{\partial \tau} B_\mu(\tau, x) = D_\nu\Big(\partial_\nu B_\mu - \partial_\mu B_\nu + [B_\nu, B_\mu]\Big) \qquad D_\mu = \partial_\mu + [B_\mu, \cdot\,]$$

$$\frac{\partial}{\partial \tau}\chi(\tau, x) = D^F_\mu D^F_\mu \chi(\tau, x) \qquad D^F_\mu = \partial_\mu + B_\mu$$

Exact solution no longer possible (even with Dirichlet boundary conditions)

$$B_\mu(\tau, x) = \int \mathrm{d}^4 y \Big\{ K_\tau(x-y)_{\mu\nu} A_\nu(y) + \int_0^\tau \mathrm{d}\sigma\, K_{\tau-\sigma}(x-y)_{\mu\nu} R_\nu(\sigma, y)\Big\}$$

$$K_\tau(x)_{\mu\nu} = \int \frac{\mathrm{d}^4 p}{(2\pi)^4} \frac{e^{ipx}}{p^2} \Big\{(\delta_{\mu\nu} p^2 - p_\mu p_\nu) e^{-\tau p^2} + p_\mu p_\nu\Big\}$$

$$R_\mu(\tau, x) = 2[B_\nu, \partial_\nu B_\mu] - [B_\nu, \partial_\mu B_\nu] - [B_\mu, \partial_\nu B_\nu] + [B_\nu, [B_\nu, B_\mu]]$$

Interactions occur at non-zero flow time: generalised BRST symmetry guarantees renormalised correlation functions remain finite.

Lüscher & Weisz, JHEP 1102 (2011) 51
Luscher, JHEP 04 (2013) 123

Smeared quasi distributions

$$\boxed{\chi(x,\tau) = \sqrt{\frac{-2\,\mathrm{dim}(R)N_f}{(4\pi)^2\tau^2\left\langle\overline{\psi}(x,\tau)\overleftrightarrow{\not{D}}\psi(x,\tau)\right\rangle}}\psi(x,\tau)}$$

Introduce

$$q(x,\sqrt{\tau}P^z,\sqrt{\tau}\Lambda_{\mathrm{QCD}},\sqrt{\tau}M_N) = \int\frac{\mathrm{d}z}{4\pi}e^{ixz\,k^z}\left\langle P\right|\overline{\chi}(z,\tau)\gamma^z e^{-ig\int_0^z \mathrm{d}z'\,B^z(z',\tau)}\chi(0,\tau)\left|P\right\rangle_C$$

Satisfies factorisation relation

$$q(x,\sqrt{\tau}\Lambda_{\mathrm{QCD}},\sqrt{\tau}P^z) = \int_{-1}^{1}\frac{\mathrm{d}y}{y}Z\left(\frac{x}{y},\sqrt{\tau}\mu,\sqrt{\tau}P^z\right)f(y,\mu^2) + \mathcal{O}\left(\sqrt{\tau}\Lambda_{\mathrm{QCD}},\frac{\Lambda^2_{\mathrm{QCD}}}{(P^z)^2}\right)$$

provided

$$\Lambda_{\mathrm{QCD}}, M_N \ll P_z \ll \tau^{-1/2}$$

Matching kernel obeys

$$\mu\frac{d}{d\mu}Z\left(x,\sqrt{\tau}\mu,\sqrt{\tau}P_z\right) = \frac{\alpha_s(\mu)}{\pi}\int_x^{\infty}\frac{dy}{y}Z\left(y,\sqrt{\tau}\mu,\sqrt{\tau}P_z\right)P\left(\frac{x}{y}\right)$$

CJM & Orginos, JHEP 03 (2017) 116
CJM, PRD 97 (2018) 054505

“Factorisation”

Use OPE

$$b_n\left(\sqrt{\tau}P^z, \frac{\Lambda_{\mathrm{QCD}}}{P^z}, \frac{M_N}{P^z}\right) = \frac{c_n(\sqrt{\tau}P^z)}{2P^z}\left\langle P^z \middle| \chi(z,\tau)\gamma_z(iD_z)^{n-1}\frac{\lambda^a}{2}\chi(0,\tau) \middle| P^z \right\rangle_C$$

Operators are not twist-2, but related via

$$b_n\left(\sqrt{\tau}P^z, \frac{\Lambda_{\mathrm{QCD}}}{P^z}, \frac{M_N}{P^z}\right) = C(\sqrt{\tau}\mu, \sqrt{\tau}P^z)a_n(\mu) + \mathcal{O}\left(\sqrt{\tau}\Lambda_{\mathrm{QCD}}, \frac{\Lambda^2_{\mathrm{QCD}}}{(P^z)^2}\right)$$

Introduce a kernel with Mellin moments

$$\left[C_n^{(0)}(\sqrt{\tau}\mu, \sqrt{\tau}P_z)\right]^{-1} = \int_{-\infty}^{\infty} dx\, x^{n-1} Z(x, \sqrt{\tau}\mu, \sqrt{\tau}P_z)$$

This leads to

$$q(x, \sqrt{\tau}\Lambda_{\mathrm{QCD}}, \sqrt{\tau}P^z) = \int_{-1}^{1}\frac{\mathrm{d}y}{y} Z\left(\frac{x}{y}, \sqrt{\tau}\mu, \sqrt{\tau}P^z\right) f(y,\mu^2) + \mathcal{O}\left(\sqrt{\tau}\Lambda_{\mathrm{QCD}}, \frac{\Lambda^2_{\mathrm{QCD}}}{(P^z)^2}\right)$$

“DGLAP”

Introducing a small-flow time expansion

$$b_n^{(s)}(\sqrt{\tau}\Lambda_{\mathrm{QCD}}) = C_n^{(0)}(\sqrt{\tau}\mu, \sqrt{\tau}P_z)a^{(n)}(\mu) + \mathcal{O}\left(\sqrt{\tau}\Lambda_{\mathrm{QCD}}, \frac{\Lambda_{\mathrm{QCD}}^2}{P_z^2}\right)$$

such that

$$\left[\mu\frac{\mathrm{d}}{\mathrm{d}\mu} + \frac{\alpha_s(\mu)}{\pi}\gamma^{(n)}\right] C_n^{(0)}(\sqrt{\tau}\mu, \sqrt{\tau}P_z) = 0 + \mathcal{O}(\sqrt{\tau}\Lambda_{\mathrm{QCD}})$$

Matching kernel satisfies

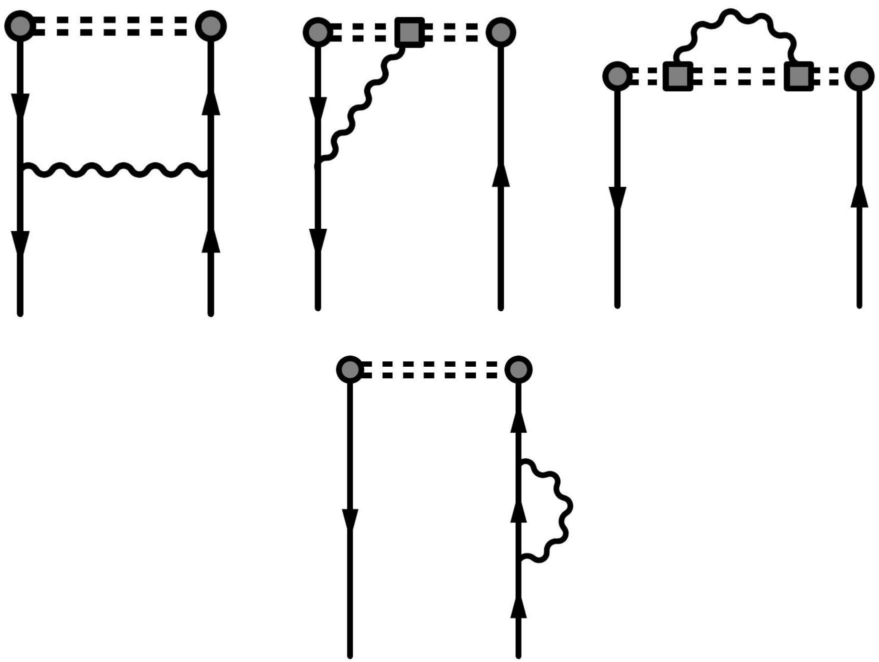
$$\mu\frac{d}{d\mu} Z\left(x, \sqrt{\tau}\mu, \sqrt{\tau}P_z\right) = \frac{\alpha_s(\mu)}{\pi}\int_x^\infty \frac{dy}{y} Z\left(y, \sqrt{\tau}\mu, \sqrt{\tau}P_z\right) P\left(\frac{x}{y}\right)$$

Note

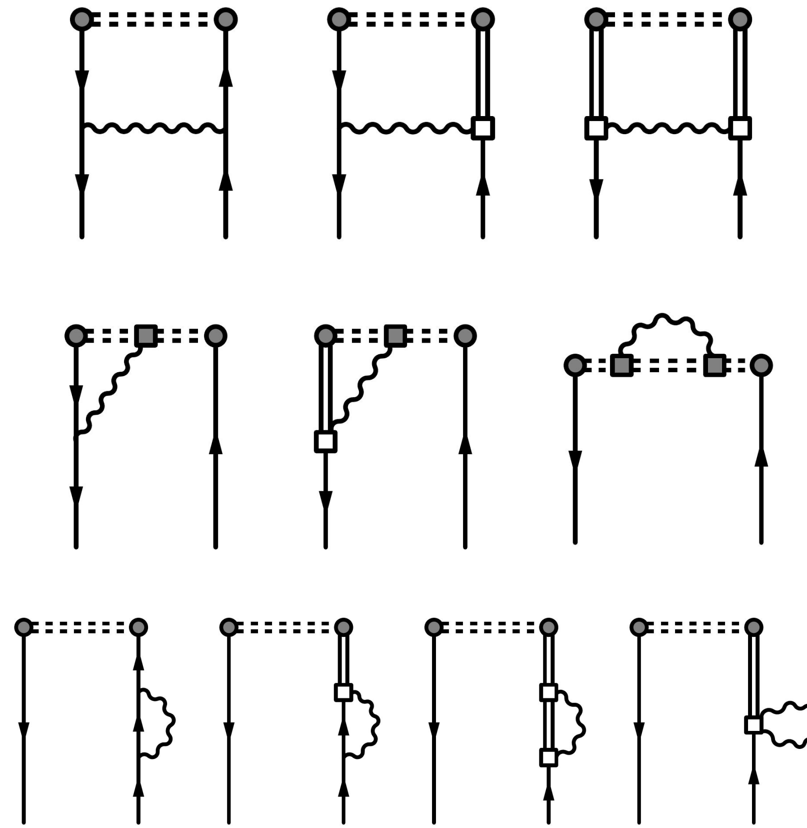
$$\mu\frac{\mathrm{d}\,f(x,\mu)}{\mathrm{d}\mu} = \frac{\alpha_s(\mu)}{\pi}\int_x^1 \frac{\mathrm{d}y}{y} f(y,\mu)P\left(\frac{x}{y}\right) \qquad \left[\mu\frac{\mathrm{d}}{\mathrm{d}\mu} - \frac{\alpha_s(\mu)}{\pi}\gamma^{(n)}\right] a^{(n)}(\mu) = 0 \qquad \int_0^1 \mathrm{d}x\, x^{n-1}P(x) = \gamma^{(n)}$$

Ioffe-time distribution in perturbation theory

Distribution



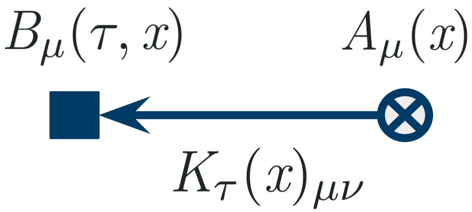
Smeared distribution



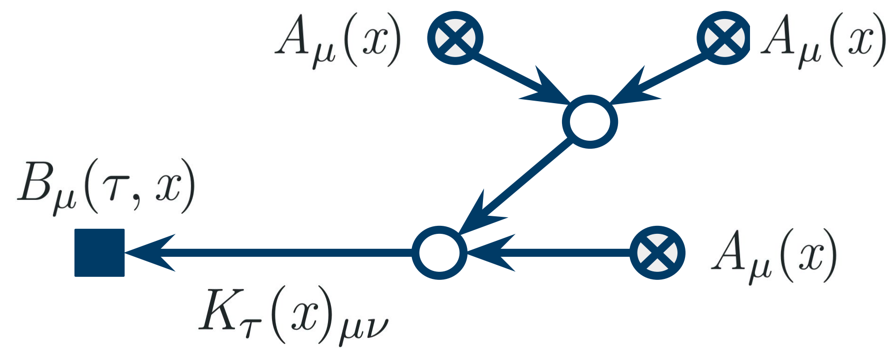
Recall

$$B_\mu(\tau,x) = \int \mathrm{d}^4y \Big\{ K_\tau(x-y)_{\mu\nu} A_\nu(y) + \int_0^\tau \mathrm{d}\sigma\, K_{\tau-\sigma}(x-y)_{\mu\nu} R_\nu(\sigma,y) \Big\}$$

Directed ``tree'' graphs



$+$



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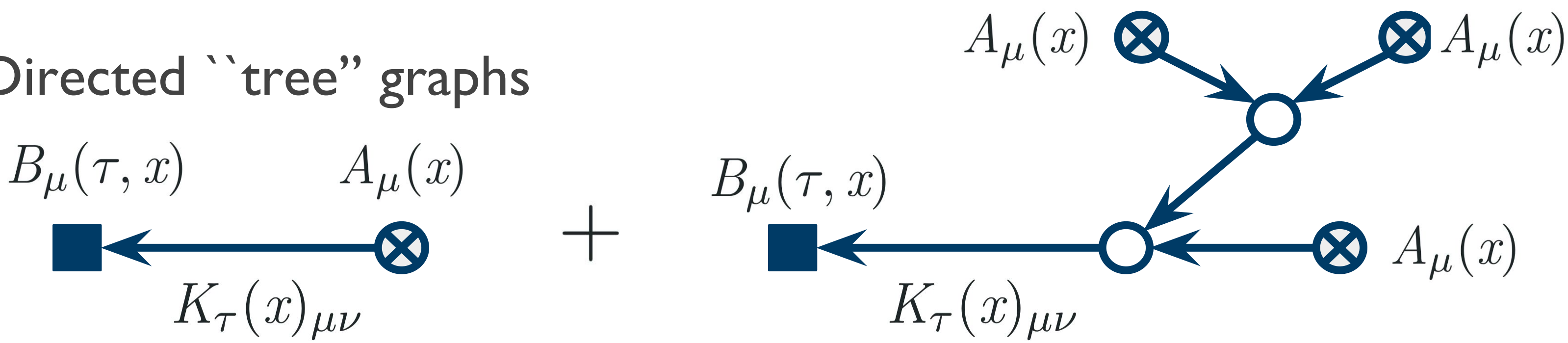
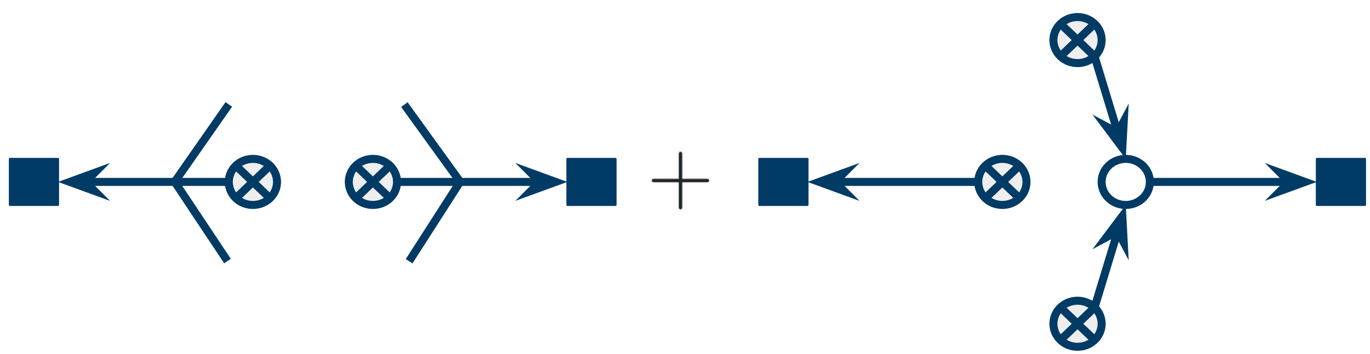


Two-point function

$\left\langle B_\mu^a(\tau,x) B_\nu^b(\sigma,y) \right\rangle =$



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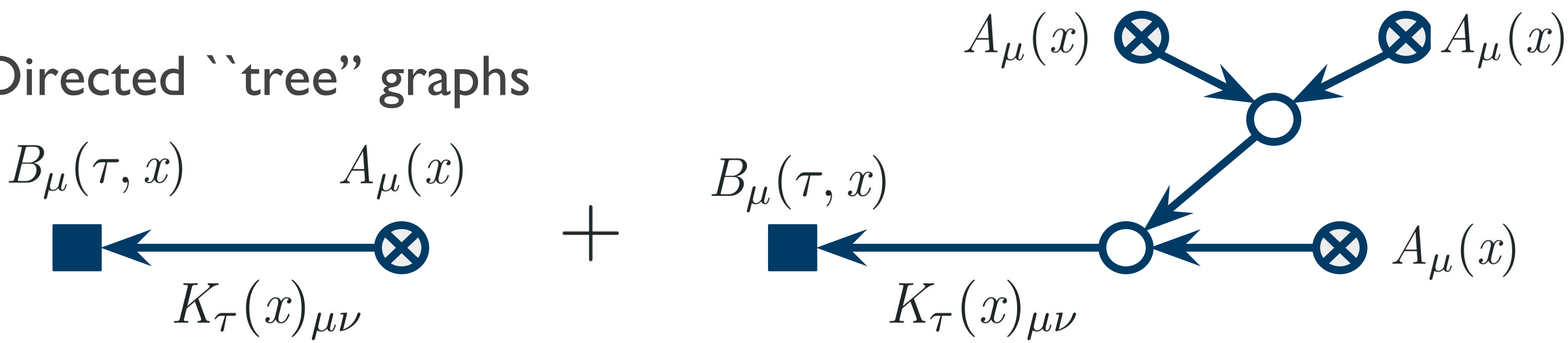
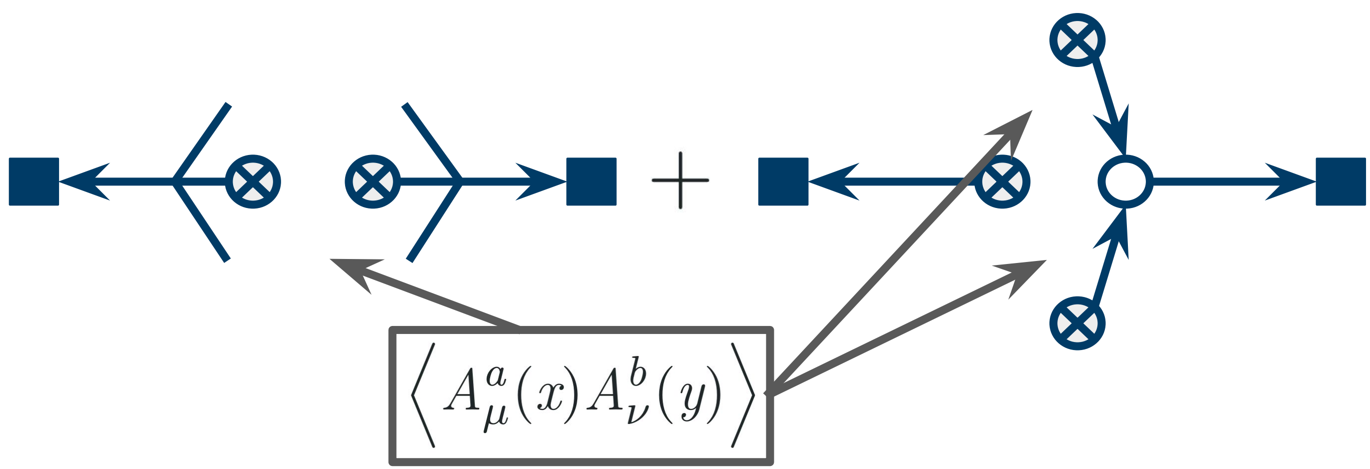
Directed ``tree'' graphs



Two-point function

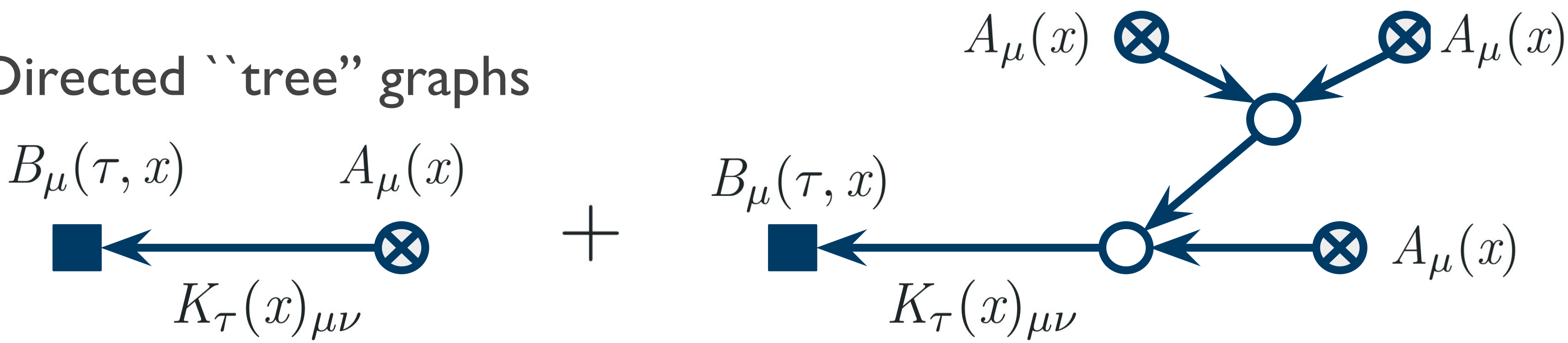
$$\Big\langle B^a_\mu(\tau, x) B^b_\nu(\sigma, y) \Big\rangle =$$



Recall

$$B_\mu(\tau, x) = \int \mathrm{d}^4 y \Big\{ K_\tau(x-y)_{\mu\nu} A_\nu(y) + \int_0^\tau \mathrm{d}\sigma K_{\tau-\sigma}(x-y)_{\mu\nu} R_\nu(\sigma, y) \Big\}$$

Directed ``tree" graphs



Two-point function

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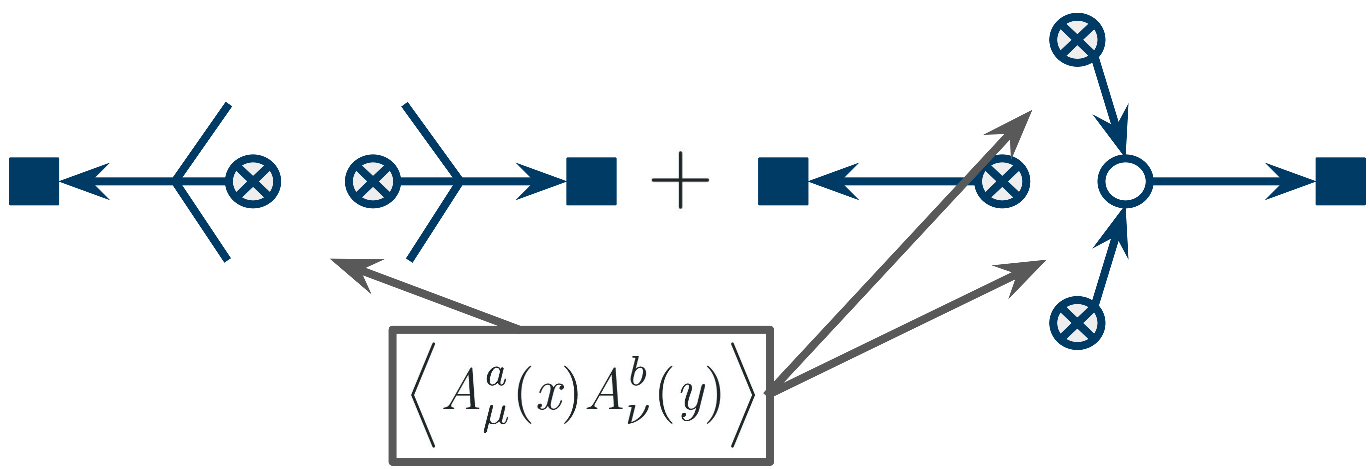

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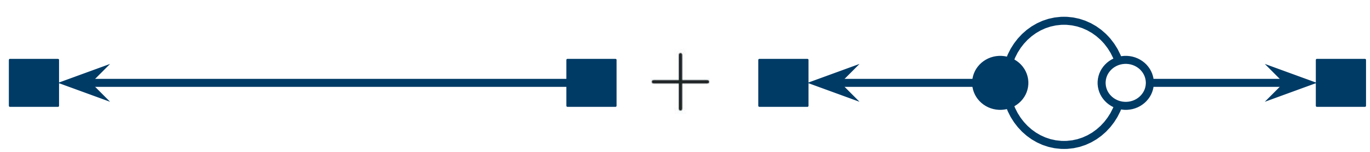
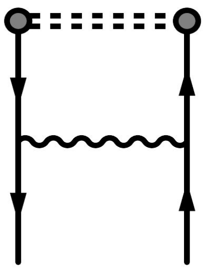
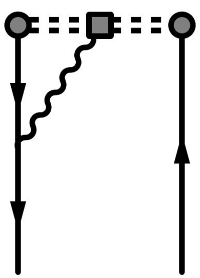


Ioffe-time distribution in perturbation theory

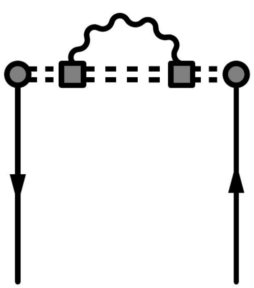


$$= \frac{\alpha_s}{3\pi}\gamma_\alpha\left[\frac{1}{\epsilon_{\mathrm{IR}}} - \gamma_{\mathrm{E}} - \log(\pi\mu^2 z^2) + C^{(\alpha)}(\overline{z}^2) + \mathrm{Ei}(-\overline{z}^2)\right]$$

$$\overline{z}^2 = \frac{z^2}{8t}$$



$$= \frac{2\alpha_s}{3\pi}\gamma_\alpha\left[1 - \gamma_{\mathrm{E}} - \log(\overline{z}^2) + \mathrm{Ei}(-\overline{z}^2) + \frac{1}{\overline{z}^2}\left(e^{-\overline{z}^2} - 1\right)\right]$$



$$= \frac{2\alpha_s}{3\pi}\gamma_\alpha\left[\gamma_{\mathrm{E}} + \log(\overline{z}^2) - \mathrm{Ei}(-\overline{z}^2) - 2\left(e^{-\overline{z}^2} - 1\right) - \sqrt{\pi}\,\overline{z}\,\mathrm{erf}(\overline{z})\right]$$

Here $\mathrm{Ei}(-\overline{z}^2) = -\int_{-\overline{z}}^{\infty} \frac{e^{-t}}{t}\mathrm{d}t \qquad \mathrm{erf}(\overline{z}) = \frac{2}{\sqrt{\pi}}\int_0^{\overline{z}} e^{-t^2}\mathrm{d}t$

Ioffe-time distribution in perturbation theory

At one loop

$$h_\alpha(\overline{z}) = \mathcal{Z}^{(\alpha)}(\overline{z})h_\alpha^{(0)}$$

where

$$\mathcal{Z}^{(\alpha)}(\overline{z}) = 1 + \frac{\alpha_s}{3\pi}\left[C^{(\alpha)}(\overline{z}^2) - \gamma_{\mathrm{E}} + \mathrm{Ei}(-\overline{z}^2) - \log(\overline{z}^2) + 2\sqrt{\pi}\,\overline{z}\,\mathrm{erf}(\overline{z})\right]$$

Two regimes:

Hieda & Suzuki, MPLA 31 (2016) 1650214

1. Local vector-current limit

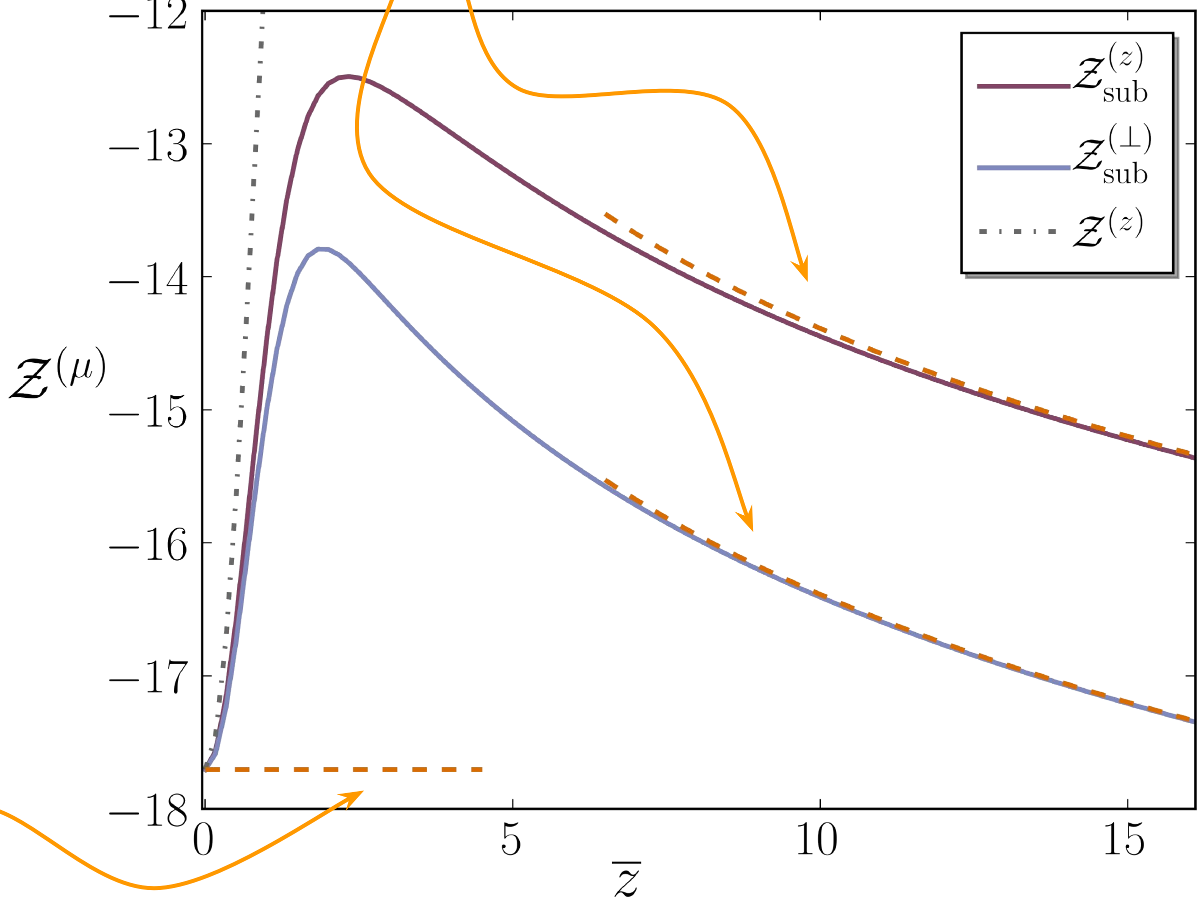
$$\overline{z} \ll 1 \qquad \mathcal{Z}^{(\alpha)}(\overline{z}) \to \mathcal{Z}(\overline{z}) = 1 + \frac{\alpha_s}{3\pi}\left[\frac{1}{2} - \log(432)\right]$$

2. Small flow-time limit

$$\overline{z} \gg 1 \qquad \mathcal{Z}^{(\alpha)}(\overline{z}) \to \mathcal{Z}^{(\alpha)}_{\mathrm{sub}}(\overline{z}) = 1 + \frac{\alpha_s}{3\pi}\left[c^{(\alpha)} - \gamma_{\mathrm{E}} - \log(432) - \log(\overline{z}^2)\right]$$

Ioffe-time distribution in perturbation theory

$$\mathcal{Z}_{\text{sub}}^{(\alpha)}(\overline{z}) = 1 + \frac{\alpha_s}{3\pi}\left[c^{(\alpha)} - \gamma_{\text{E}} - \log(432) - \log(\overline{z}^2)\right]$$



$$\mathcal{Z}(\overline{z}) = 1 + \frac{\alpha_s}{3\pi}\left[\frac{1}{2} - \log(432)\right]$$

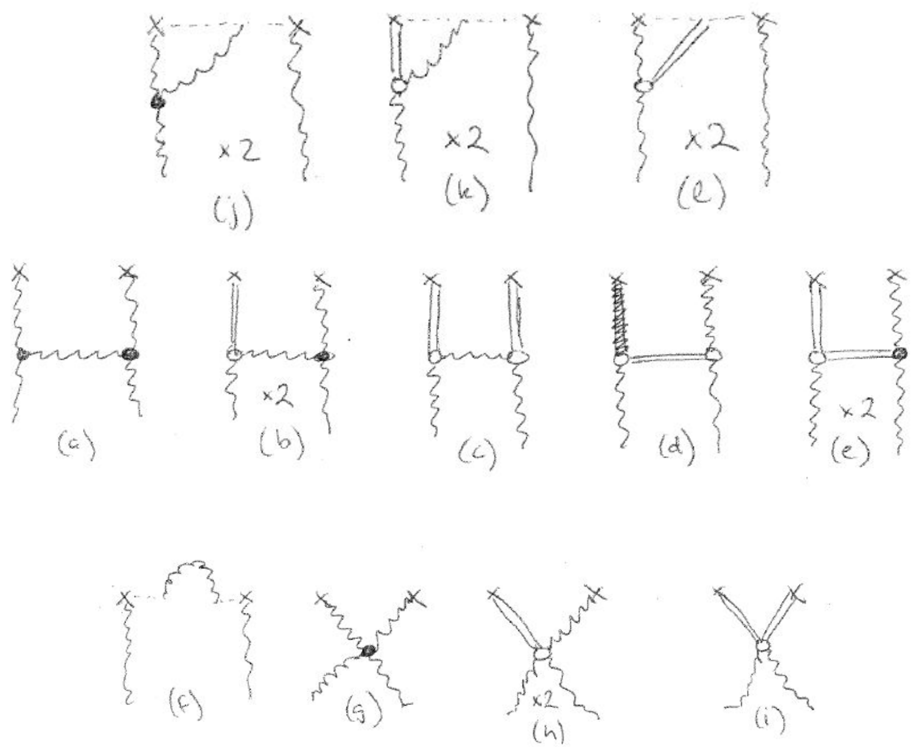
Comments

Gradient flow a natural advantage for gluon case

- suffers extra power divergences
- appears to be possible tension in the literature (?)

Ji et al., PRL 120 (2018) 112001
Wang et al.,1712.09247

Perturbative calculation underway...



Summary

Hadron structure:

- highly relevant to a range of experiments
- recent approaches pose interesting field-theoretic questions

Briceño, Hansen & CJM, PRD 96 (2017) 014502
Briceño, Guerrero, Hansen & CJM, 1805.01034

Recent approaches on the lattice and power divergences

- tamed by the gradient flow?

Smeared distributions

- gradient flow: good for gluons

Thank you

cjm373@uw.edu

Quasi and pseudo distributions

Ji, PRL 110 (2013) 262002
Ji et al., NPB 924 (2017) 326
Izubuchi et al., 1801.03917

Factorisation theorems

$$\widetilde{f}_{j/H}(\xi, P^z, \mu_{\mathrm{R}}) = \int_{-1}^{1} \frac{\mathrm{d}y}{|y|} C^{(\widetilde{f})}\left(\frac{\xi}{y}, \frac{\mu_{\mathrm{R}}}{P^z}, \frac{\mu}{p^z}\right) f_{j/H}(y, \mu) + \mathcal{O}\left(\frac{M^2}{(P^z)^2}, \frac{\Lambda^2_{\mathrm{QCD}}}{(P^z)^2}\right)$$

$$\widetilde{p}_{j/H}(\xi, z^2\mu_{\mathrm{R}}^2) = \int_{-1}^{1} \frac{\mathrm{d}y}{|y|} C^{(\widetilde{p})}\left(\frac{\xi}{y}, \frac{\mu_{\mathrm{R}}^2}{\mu^2}, \mu^2 z^2\right) f_{j/H}(y, \mu) + \mathcal{O}\left(M^2 z^2, \Lambda^2_{\mathrm{QCD}} z^2\right)$$

Examples of ``good lattice cross-sections”

Ma & Qiu, PRL 120 (2018) 022003
Ma & Qiu, 1404.6860