Towards isospin breaking corrections to $(g\text{-}2)_\mu$

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- QED corrections to HLO in massive QED (electroquenched)

  Bussone, DM, Janowski, EPJ Web Conf. 175 (2018) 06005

- General framework for computation of leading order isospin breaking corrections:

  Need for definition of a “scheme”

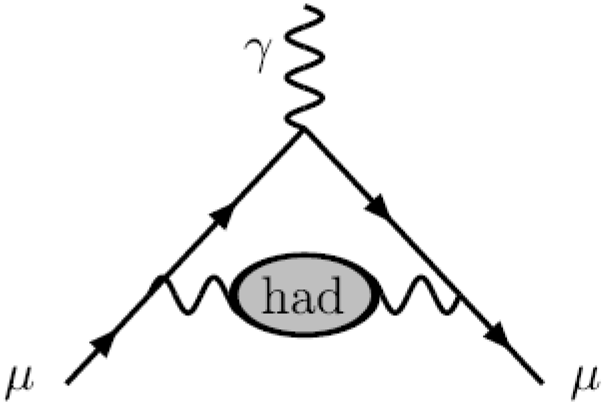
Motivations to study a_μ on the lattice

⋆ 3 sigmas discrepancy between exp and theo [Jegerlehner and Nyffeler, 2009] :

$$a_\mu^{exp} = 1.16592080(63) \times 10^{-3}$$

$$a_\mu^{the} = 1.16591790(65) \times 10^{-3}$$

Contribution	Value	Error
QED incl. 4-loops+LO 5-loops	116 584 718.1	0.2
Leading hadronic vacuum polarization	6 903.0	52.6
Subleading hadronic vacuum polarization	-100.3	1.1
Hadronic light–by–light	116.0	39.0
Weak incl. 2-loops	153.2	1.8



Future experiments will shrink the error!

$\sigma\,(e^+\,e^- \to \text{Had})$-method still the most accurate

(includes all SM contributions)

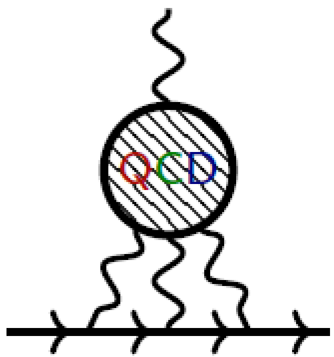
Exp. data with space-like kin. allow for direct comparison with Lattice

[Carloni Calame et al. Phys. Lett. B746:325–329, 2015]

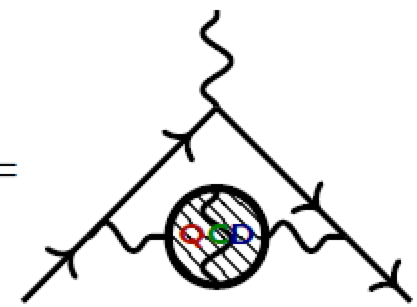
$3\sigma \simeq 4\%$ on $a_\mu^{\rm HLO}$

QED corrections $\approx 1\%$

$a_\mu^{\rm HLbL} =$



$= \quad \mathrm{O}\,(e^7) \quad =$



$= a_\mu^{\rm HLO}(\alpha)$

In this talk we explore EM corrections to the HVP

Can we pin them down? (lattice error $a_\mu^{\rm HLO} \approx 5\%$)

IR regularizations of choice

We do not discuss all the (many) issues with QED here

[Portelli PoS KAON 13 (2013) 023] [Patella PoS LATTICE 2016 (2017)]

$\mathrm{QED_L}$

[Borsanyi et al. Science 347 (2015) 1452]

- Easy implementation
- Non-local constraint
- Power-like finite vol. corr.
- Renorm. issues for op. $d > 4$
- Convincing spectrum results
- Non-comm. $L \to \infty \leftrightarrow a \to 0$

$\mathrm{QED_M}$

[Endres et al. Phys. Rev. Lett. 117 (2016) no.7, 072002]

- Easy implementation
- Local formulation
- Exp. suppressed finite vol. corr.
- m_γ "too small"
  - t-dep. in eff. masses
  - $\underline{p}$ stiffness in eff. en.
- Non-comm. $m_\gamma \to 0 \leftrightarrow L \to \infty$

Non-comm. of limits in $\mathrm{QED_M}$ resembles p and ϵ regimes in χ-PT

In χ-PT: $m\Sigma V$ vs 1

In $\mathrm{QED_M}$: ? vs 1

Remark: Massive photons form Bose-Einstein condensate

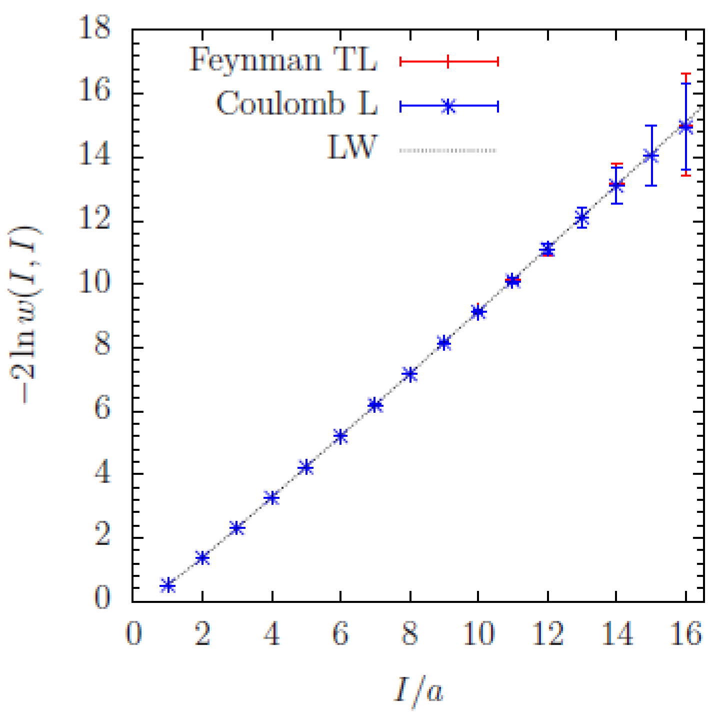
Wilson loops in pure QED: $V = 32^4$

$$w_{\mu\nu}(I,I) = \exp\left(2e^2Q^2\left[C_\mu(I,0) - C_\nu(I,I\hat{\nu})\right]\right), \quad C_\mu(I,x) = ID(x) + \sum_{\tau=1}^{I-1}(I-\tau)D(x+\tau\hat{\mu})$$

$D(x)$ is the infinite lattice massless/massive scalar propagator in coordinate space

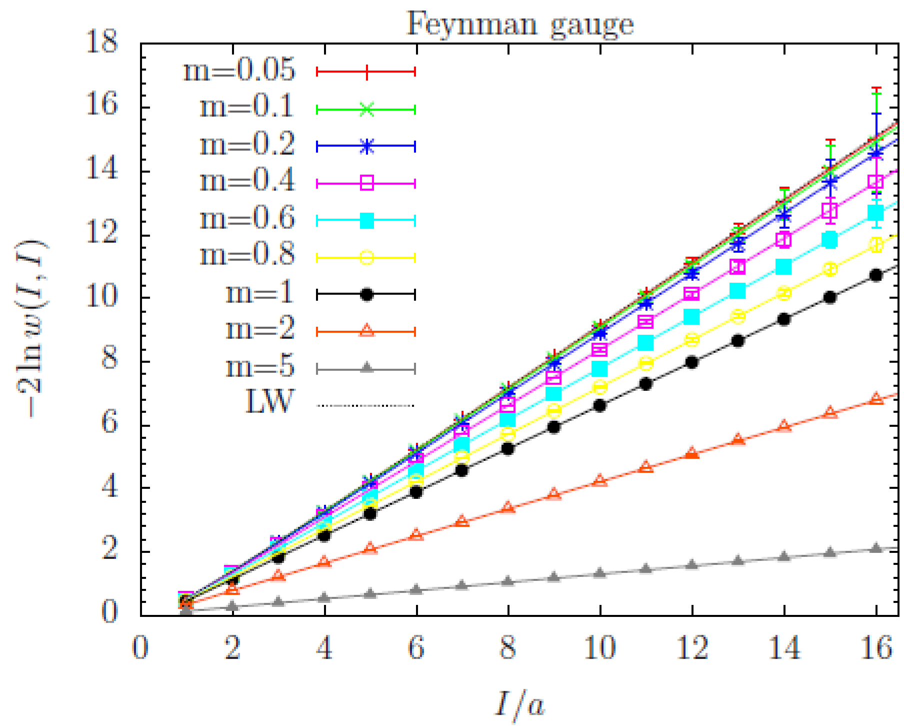
Lüscher-Weisz method

[Luscher and Weisz, Nucl. Phys. B 445 (1995) 429]



Borasoy-Krebs method

[Borasoy and Krebs Phys. Rev. D 72 (2005) 056003]



QCD ensembles

Goal: QED corrections to a_μ^{HLO} in QCD+qQED framework

Dynamical QCD cnfs generated by CLS with $N_f = 2$ degenerate flavors of non-perturbatively $O(a)$ improved Wilson fermions

[Capitani et al. Phys. Rev. D 92 (2015) no.5, 054511]

$\beta = 5.2$, $c_{sw} = 2.01715$, $\kappa_c = 0.1360546$, $a[\mathrm{fm}] = 0.079(3)(2)$, $L/a = 32$

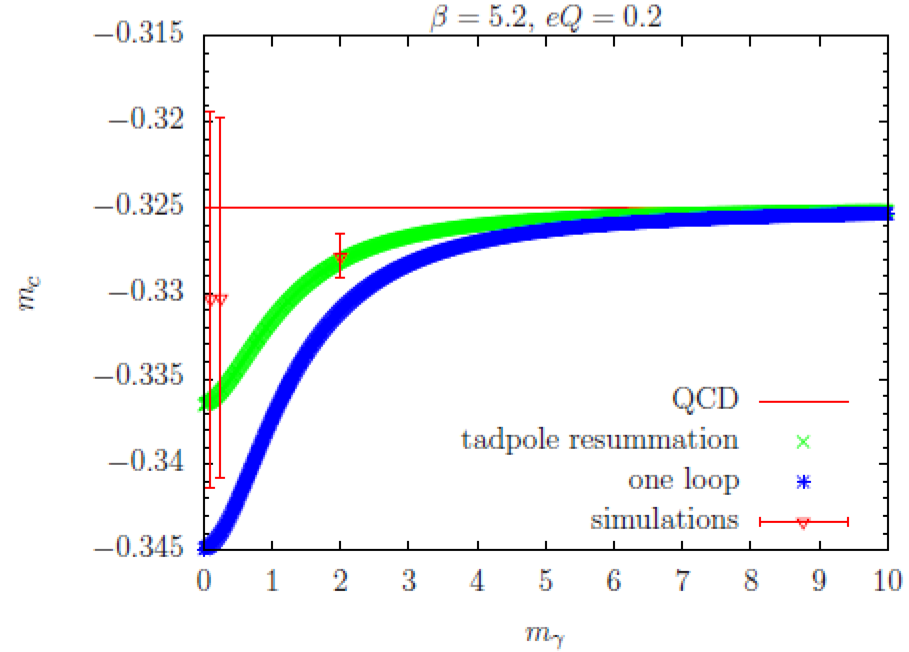
Run	κ	am_π	$m_\pi L$	m_π[MeV]
A3	0.13580	.1893(6)	6.0	473
A4	0.13590	.1459(6)	4.7	364
A5	0.13594	.1265(8)	4.0	316

QED inclusion shifts the critical mass!

Remark: 1% Net effect on m_c translates in O(100%) change in m_q (for $m_0 \simeq m_c^{\mathrm{QCD}}$)
Important for m_π and therefore HVP!

QCD+qQED ensembles

Inclusion of qQED with $\alpha = 1/137$ and physical charges $Q = 2/3, -1/3$



Run	am_γ	$am_{\pi^0=u\bar{u}}$	$am_{\pi^0=d\bar{d}}$	$am_{\pi^\pm}$
A3	0	.2549(9)	.2071(9)	.2330(9)
	0.1	.2556(7)	.2074(8)	.2337(8)
	0.25	.2553(7)	.2072(8)	.2331(8)
A4	0	.2240(8)	.1691(9)	.1994(9)
	0.1	.2252(9)	.1699(9)	.2005(9)
	0.25	.2246(8)	.1700(10)	.1998(9)
A5	0	.2105(7)	.1526(9)	.1849(8)
	0.1	.2114(7)	.1528(9)	.1856(8)
	0.25	.2111(7)	.1531(9)	.1852(8)

Pion masses going from 380 MeV to 640 MeV

Notice: m_c EM shift in A5 gives $m_{\pi^0=u\bar{u}}^{Q(C+E)D} \simeq 2m_\pi^{QCD}$

Notice: Matching between ensembles $m_{\pi^\pm}^{Q(C+E)D}(\mathrm{A5}) \simeq m_\pi^{QCD}(\mathrm{A3})$

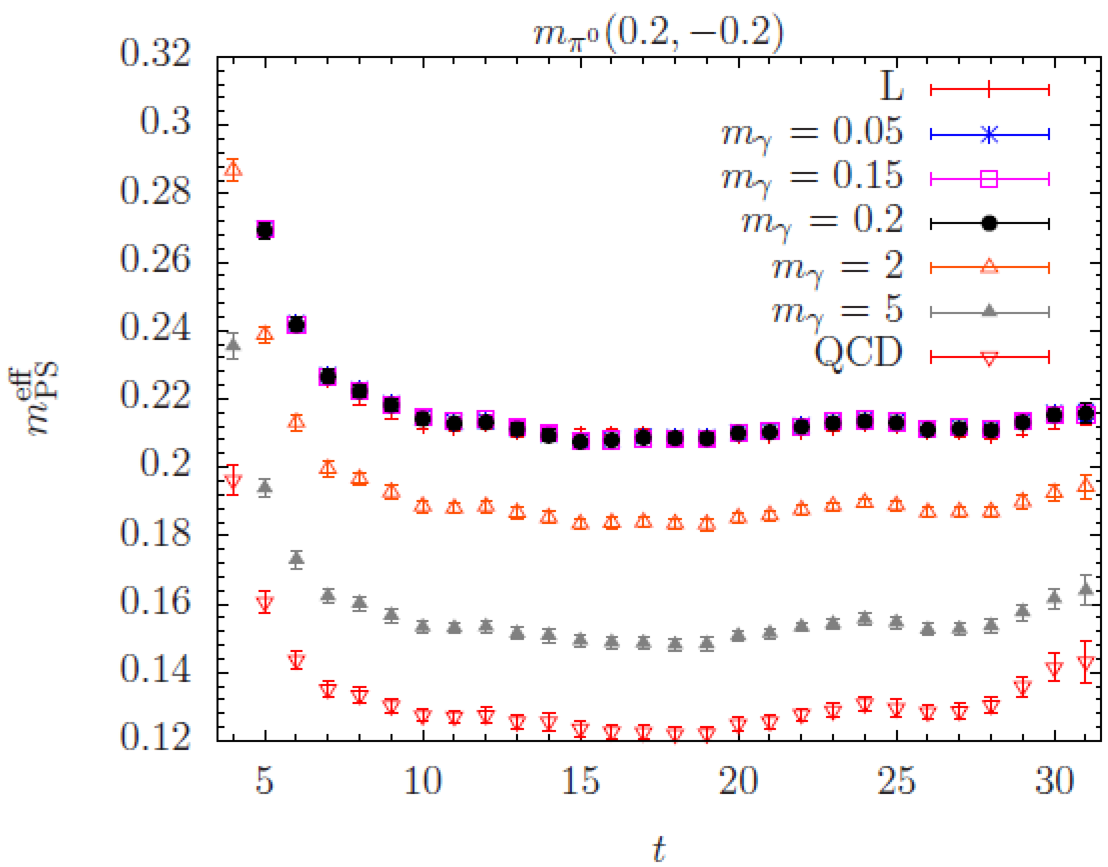
Finite volume and photon mass effects have been checked with PT formulae and negligible within errors

Dependence on m_γ

For $m_\gamma = 0.1$ the coeff. of linear t-term in eff. energies is suppressed

$$(m_\gamma^2 V)^{-1} \simeq 5 \times 10^{-5}$$

not visible in the effective masses for $m_\gamma \in [0.05, 0.1, 0.15, 0.2, 0.25, 2, 5]$



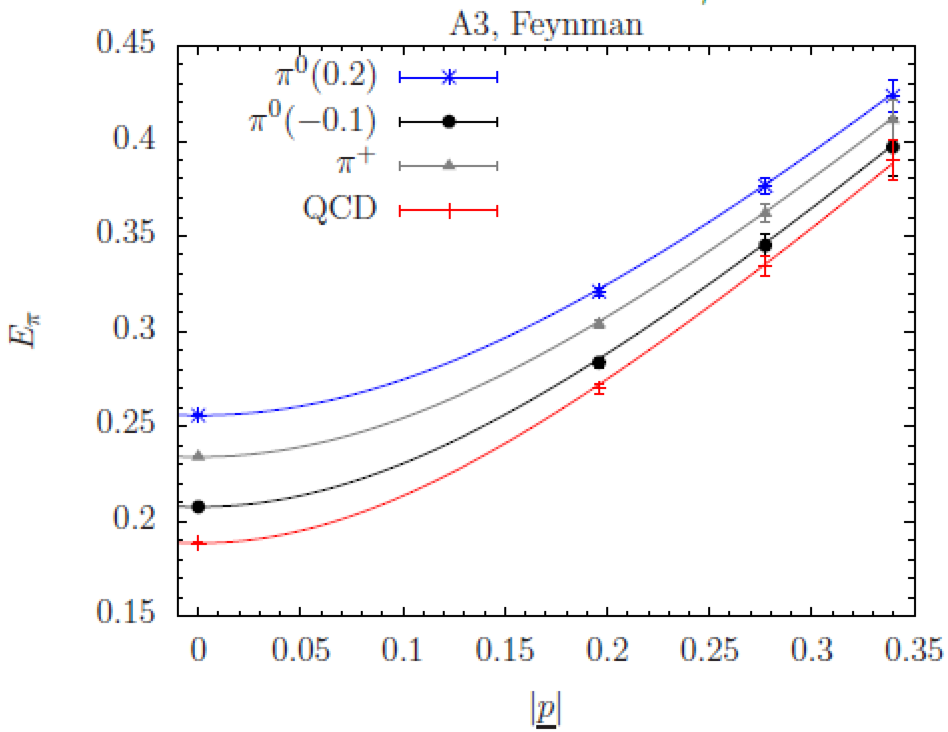
$\mathrm{QED_L}$ is consistent with $\mathrm{QED_M}$ for $m_\gamma \to 0$

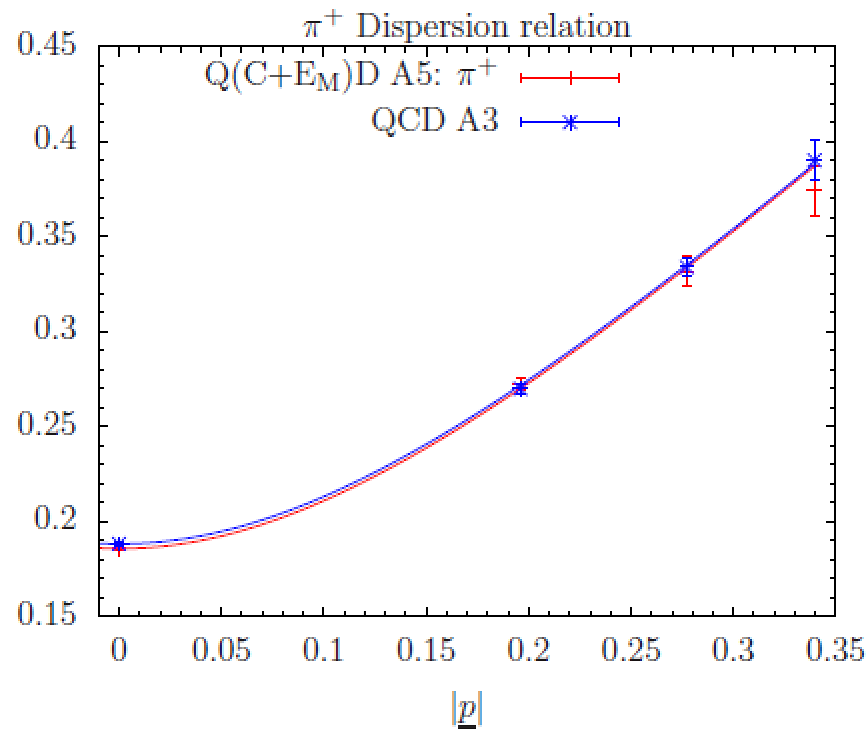
Expectation:
photons decouple for $m_\gamma \to \infty$

[Appelquist and Carazzone Phys. Rev. D 11 (1975) 2856]

Our choices are $m_\gamma = 0.1, 0.25$

Dispersion relation $m_\gamma = 0.1$





No stiffness in $|\underline{p}|$

$$- \quad E_{\rm eff}(t,\underline{p}) \overset{m_\gamma \to 0}{\simeq} \frac{(Q_u - Q_d)^2 e^2}{m_\gamma^2 V} t - \frac{d}{dt} \ln \langle \mathcal{O}(t,\underline{0}) \overline{\mathcal{O}}(0) \delta_{Q^{\rm T},0} \rangle_{\rm TL},$$

All the effective energies agree with the continuum curve (solid lines)

Charged pion mass in A3 QCD matches the one in A5 Q(C+E$_{\rm M}$)D

So far...

$m_\gamma \simeq 0.1$ seems to be a safe choice

- Negligible finite volume effects
- Negligible finite photon mass effects
- No subtle reduction to $\mathrm{QED_{TL}}$
- $\mathrm{QED_L}$ is consistent (for the spectrum and these parameters)

Pion masses in A5 Q(C+E)D "match" A3 QCD ones

- HVP depends strongly on pion masses
- Can give direct access to EM effects in the HVP

HVP

HVP tensor: $\Pi_{\mu\nu}(q) = \int d^4x\, e^{iq\cdot x} \langle V_\mu(x) V_\nu(0) \rangle$

Is the current still conserved in Q(C+E)D formal theory?

$$\mathbf{SU}(2)_{\rm L} \otimes \mathbf{SU}(2)_{\rm R} \otimes \mathbf{U}(1)_{\rm V}$$

$\downarrow$ explicit and spontaneous

$$\mathrm{QCD} : \mathbf{SU}(2)_{\rm V} \otimes \mathbf{U}(1)_{\rm V}$$

$\downarrow$ explicit

$$\mathrm{Q(C+E)D} : \mathbf{U}'(1)_{\rm V} \otimes \mathbf{U}(1)_{\rm V}$$

Combination of **1** and τ^3 in flavor is conserved

$$V_\mu(x) = \overline{\Psi}(x)\gamma_\mu \left[\frac{Q_u}{2}(\mathbf{1} + \tau^3) + \frac{Q_d}{2}(\mathbf{1} - \tau^3) \right] \Psi(x)$$

On the Lattice: 1-point-split current conservation implies $Z_V = 1$
no QED effects to take into account

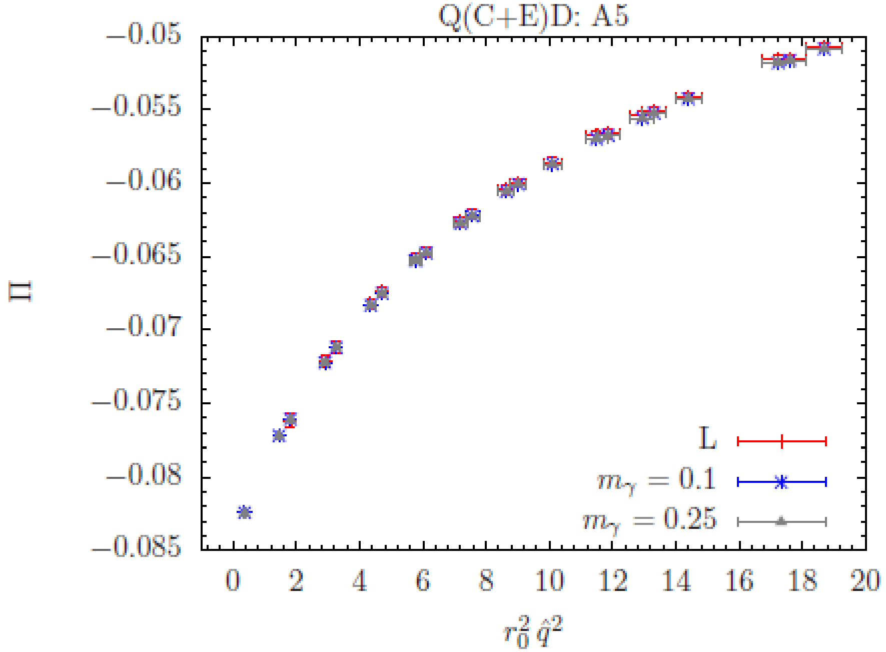
For completeness:

Neglecting quark-disconnected diagrams

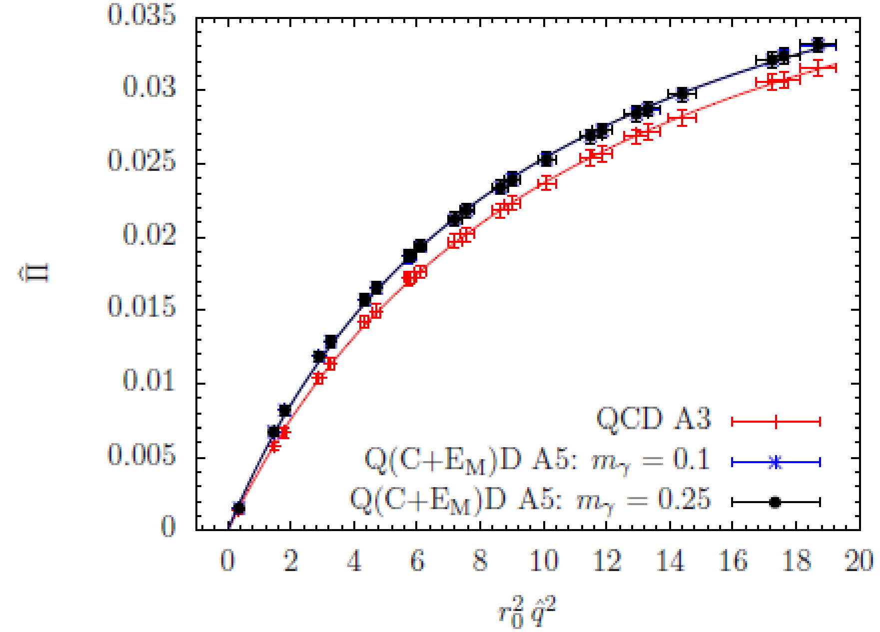
Electroquenched approximation

Scalar HVP

Agreement between $\mathrm{QED_L}$ and $\mathrm{QED_M}$



Matching gives direct access to EM eff.



r_0/a as any other gluonic scale does not receive QED contributions in the quenched approximation

For completeness:

- ZMS modification [Bernecker and Meyer Eur. Phys. J. A 47 (2011) 148]
- Padé fit R_{10} to extract $\Pi(0)$ [Blum et al. JHEP 1604 (2016) 063]
- Point sources are used

Strategy to extract EM effects for a_μ

First strategy

- Fit scalar HVP in Q(C+E)D and compute a_μ
- Fit scalar HVP in QCD and compute a_μ
- After extrapolation to infinite volume, physical point and continumm take the difference between QCD and Q(C+E)D results

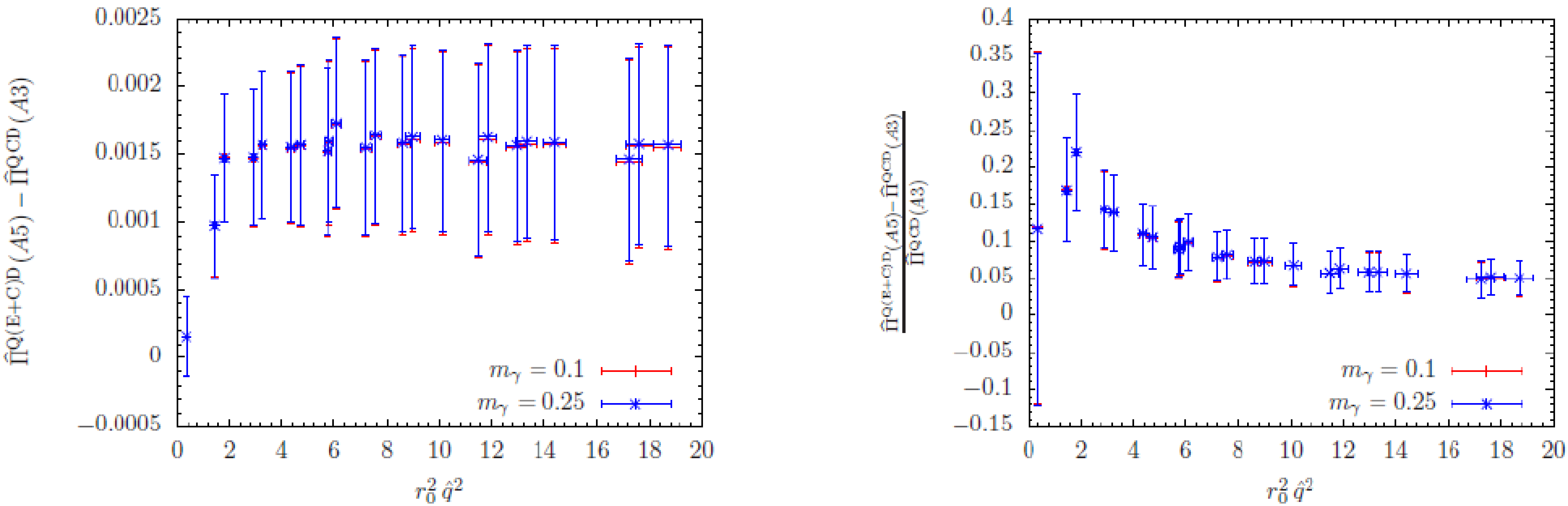
The effect can be washed out by the various systematics...

Second strategy

- Take $\widehat{\Pi}^{\mathrm{Q(C+E)D}} - \widehat{\Pi}^{\mathrm{QCD}} \equiv \delta\widehat{\Pi}$ at fixed pion masses
- Fit $\delta\widehat{\Pi}$ and plug it in $a_\mu^\delta = \int f(q)\delta\widehat{\Pi}$
- Extrapolate to infinite volume, physical point and continuum

Only one fit has to be performed to a slowly varying function

Matching gives direct access to EM eff.



There is a clear signal, integrating up to $r_0 \hat{q}^2 \simeq 20$

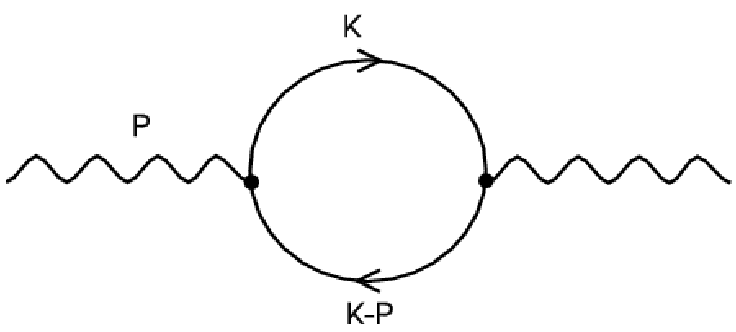
$$a_\mu^\delta \times 10^{10} = 21 \pm 9_{\rm stat}$$

[A. Bussone, MDM, T. Janowski, arXiv:1710.06024]

Still effects to quantify, e.g. in a and m_π (this could be large), so far $m_\pi \approx 460$ MeV, $a \approx 0.8$ fm ... Strong isospin breaking

Renormalization of the photon mass

- We are interested to O(α).
- The renormalization is multiplicative because in the massless limit one recovers gauge invariance and the mass term is not generated.
- To leading order the only continuum diagram contributing is



  that is absent in the electroquenched theory (no quark loops), and so are all the tadpoles. In general they should be proportional to m_γ^2.
- In the electroquenched theory one only needs to scale am_γ with the lattice spacing to keep m_γ fixed.

Setting up a scheme for computing IB corrections

We think of an observable as a function of renormalized parameters. Neglecting the dependece of α_s on α, i.e., the dependence of a on α

$$O = O\left((m_d - m_u)_R(\alpha), (m_d + m_u)_R(\alpha), \alpha\right) \ .$$

Those are clearly not independent, so not suited for an expansion

We can start by fixing
$(m_d + m_u)_R(\alpha) = (m_d + m_u)_{R,phys} = (m_d + m_u)_{PDG} \approx 6.7$ MeV for all values of α. That makes it α-independent by construction.

In χPT EM corrections to $m^2_{\pi^0}$ start at $O(\alpha^2)$ (e.g., [Bijnens and Prades, hep-ph/9610360]) and so do the strong IB corrections in SU(3) χPT. They are due to $\pi^0 - \eta$ mixing (e.g., [Scherer, hep-ph/0210398])

So to leading order in IB corrections fixing $(m_d + m_u)_R(\alpha)$ is equivalent to fixing $m^2_{\pi^0}$ to its physical value.

- One could have fixed directly the PCAC quark masses, although that requires computing O(α) corrections to Z_A and Z_P.
- outside the isospin limit the π^0 correlator receives quark-disconnected contributions. In additin, with $N_f = 3$ one needs to solve the $\pi^0 - \eta$ mixing (e.g. by using at least two interpolating fields and GEVP).

Now we have

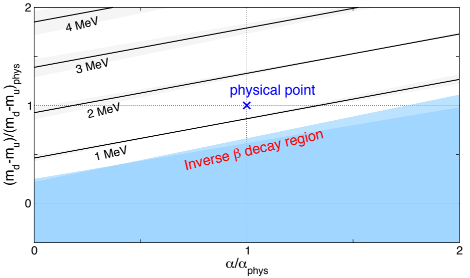
$$O = O\left((m_d - m_u)_R(\alpha), 6.7\ \mathrm{MeV}, \alpha\right)\ .$$

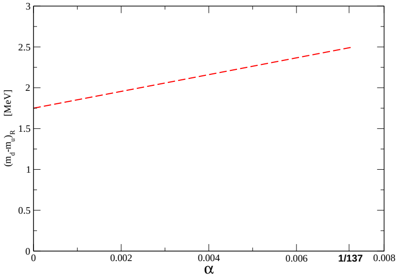
since in the end we are interested in an expansion around $\alpha = 0$ and $\delta m = 0$ it is convenient to rewrite O as function of $(m_d - m_u)|_{\alpha=0}$. Here the *scheme* dependence enters.

We need the splitting as a function of α, but in studying that we must have two prescriptions to fix the quark masses. One is keep m_{π^0} fixed, there's then a scheme dependence of $(m_d - m_u)|_{\alpha=0}$ on the second condition.

Let's look at two examples:
1) keep the neutron proton splitting to its physical value
2) keep the splitting between Σ^+ and Σ^- to its physical value

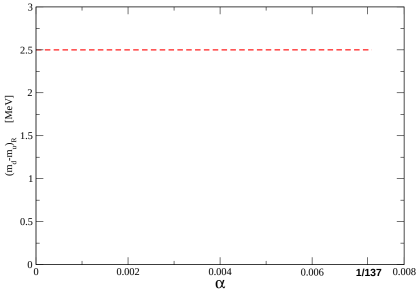
Looking at Fig. 3 in [1406.4088]





say a 30% decrease in the splitting at $\alpha = 0$ compared to the physical value.

Σ^+ is *uus* and Σ^- is *dds*, both have $|\mathrm{charge}| = 1$ so at leading order EM corrections cancel in the splitting, which is entirely due to δm. By tuning quark masses vs α keeping that fixed one gets something like



Both choices are equally good, but this one seems better. The question is what is the difference in $(m_d - m_u)|_{\alpha=0}$ for the two prescriprions. If that were $O(\alpha_{phys})$, the result of the expansion would be ambiguous by $O(\alpha_{phys})$, which would make all the computation meaningless.

The condition:

$$\delta_1 m(\alpha = 1/137) = \delta_2 m(\alpha = 1/137) = \delta m_{phys} \ ,$$

after linearizing the dependence around $\alpha = 1/137$

$$\delta_i m(\alpha) = \delta m_{phys} + \left(\alpha - \tfrac{1}{137}\right) * c_i \ .$$

would only give $\delta_1 m(0) - \delta_2 m(0) = O(\alpha_{phys})$.

In order to obtain a stronger bound we use that in continuum and renormalized perturbation theory the EM corrections to the quark masses are multiplicative as a consequence of chiral symmetry (and the two schemes preserve WI).

We write

$$m_u^i(\alpha) = m_u^i(0) Z_u^i(\alpha) \, , \quad \text{and} \quad m_d^i(\alpha) = m_d^i(0) Z_d^i(\alpha) \, ,$$

with $Z_X^i(\alpha) = 1 + C_X^i \alpha + \cdots$. The mass on the rhs for example is the renormalized QCD mass in the i scheme.
The splitting now reads

$$\begin{aligned} \delta_i m(\alpha) &= \delta_i m(0) \, Z_u^i(\alpha) + (Z_d^i(\alpha) - Z_u^i(\alpha)) m_d^i(0) \, , \\ &= \delta_i m(0) \, (1 + C_u^i \alpha) + C_{(d-u)} \alpha m_d^i(0) \, . \end{aligned}$$

Using the fact that, *numerically,* $\delta m \simeq m_d$*,* one obtains

$$\delta_i m(\alpha) = \delta_i m(0)(1 + C_u^i \alpha) + O(\alpha \delta m) \, .$$

By requiring the two splittings to be the same for $\alpha = 1/137 = \alpha_{phys}$

$$\delta_1 m(0)(1 + C_u^1 \alpha_{phys}) = \delta_2 m(0)(1 + C_u^2 \alpha_{phys}) + O(\alpha \delta m) \, ,$$

which implies

$$\begin{aligned} \delta_1 m(0) - \delta_2 m(0) &= \alpha_{phys} \left(C_u^2 \delta_2 m(0) - C_u^1 \delta_1 m(0) \right) + O(\alpha \delta m) \, , \\ &= \alpha_{phys} (C_u^2 - C_u^1) \delta_1 m(0) + O(\alpha^2) + O(\alpha \delta m) \, . \end{aligned}$$

So, finally

$$\delta_1 m(0) - \delta_2 m(0) = O(\alpha^2) + O(\alpha \delta m) \, .$$

Back to the expansion of O, in conclusion one can either use $\delta_1 m$ or $\delta_2 m$, for leading corrections in α and δm. Now we think of

$$O = O(\delta_i m(0), 6.7\,\mathrm{MeV}\ (\text{or } m_{\pi^0}^2 \text{ fixed to its physical value}), \alpha)\ ,$$

which can be expanded as

$$\begin{aligned} O &= O(0, 6.7\,\mathrm{MeV}, 0) + \alpha_{phys} \left.\frac{\partial O(0, 6.7\,\mathrm{MeV}, \alpha)}{\partial \alpha}\right|_{\alpha=0} + \\ &+ \delta_1 m(0) \left.\frac{O(\delta m, 6.7\,\mathrm{MeV}, 0)}{\partial \delta m}\right|_{\delta m=0} + O(\alpha^2) + O(\alpha\delta m)\ . \end{aligned}$$

- first term should be computed in pure QCD with degenerate up and down quarks.
- for the second one needs to simulate QCD+QED using the same bare masses for the up and down quarks, such that for $\alpha = 0$ the two would be degenerate.
- the third term must be computed in QCD with non-degenerate up and down quarks.

Assuming derivatives of $O(1)$, using $\delta_2 m(0)$ instead of $\delta_1 m(0)$ is an $O(\alpha^2)$, $O(\alpha\delta m)$ effect.

Being pragmatic:

Typical differences between definitions of $\delta m_i(0)$ are around 30%, so in the end one could perhaps use the physical splitting multiplying the derivative above, if interested in IB corrections with that accuracy (30% of 1% is still 3 permil on O)

Conclusions

- Feasibility study for the computation of EM corrections to g-2 using massive QED.
- Already there (and in general in any approach) the definition of a scheme for macthing computations between QCD+QED and QCD seems to be an advantage.
- Setup for the computation of IB corrections. One needs to define a scheme, but at leading order results are 'scheme independent'.
- The setup discussed may be ideal (it involves baryons, disconnected diagrams, and mixing problems). One may have to accept some pragmatic compromises in actual implementations.