Evidence for non perturbative fermion mass generation

Petros Dimopoulos

University of Rome "Tor Vergata"

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• a list of publications on this topic

- **R. Frezzotti and G.C. Rossi**
Nonperturbative mechanism for elementary particle mass generation
PRD 92 (2015) 054505

- **R. Frezzotti and G.C. Rossi**
Dynamical mass generation
PoS LATTICE2013 (2014) 354

- **R. Frezzotti. M. Garofalo and G.C. Rossi**
Nonsupersymmetric model with unification of electroweak and strong interactions
PRD 93 (2016) 105030

- **S. Capitani** *et al.*
Check of a new non-perturbative mechanism for elementary fermion mass generation
PoS LATTICE2016 (2016) 212

- **S. Capitani** *et al.*
Testing a non-perturbative mechanism for elementary fermion mass generation: lattice setup
EPJ Web Conf. 175 (2018) 08008

- **S. Capitani** *et al.*
Testing a non-perturbative mechanism for elementary fermion mass generation: Numerical results
EPJ Web Conf. 175 (2018) 08009

- **F. Pittler**
Spectral statistics of the Dirac operator near a chiral symmetry restoration in a toy model
EPJ Web Conf. 175 (2018)

- **results presented here in collaboration with**

R. Frezzotti, M. Garofalo and G.C. Rossi Università di Roma "Tor Vergata"

B. Kostrzewa, F. Pittler and C. Urbach HISKP (Theory), Universität Bonn

Overview

- Widely accepted the *incompleteness* of the **SM**: a number of fundamental phenomena are not satisfactorily or not at all described by/within it.
- This has motivated a vast variety of ingenious and original **N**ew **P**hysics proposals **BSM**.
- However, this task is proved to be non-trivial perhaps due to the fact that **SM** is a renormalisable theory.
- **SM** describes elemenentary particle masses employing the symmetry breaking $SU(2)_L \times U(1)_Y \to U(1)_{\rm em}$. The *hierarchy* pattern of fermion masses (but also Higgs mass un-natural feature) lack deep understanding.
  Masses are rather accomodated by fitting to experimental data.

Overview

- **Dynamical generation of elementary fermion mass**

  - Similar physics effect which generates $\langle \bar{q} q \rangle \neq 0$
  - where dynamical χSB *triggered* by an explicit χSB term i.e. fermion mass or Wilson term.
  - In (massless) LQCD with Wilson term Non-Perturbative contribution ($\propto \Lambda_{QCD}$) is accompanied by an $1/a$ divergent term.
    - Axial WTI: $\partial_\mu \langle \hat{J}_{5\mu}(x) \hat{O}(0) \rangle = 2(m_0 - \bar{M}(m_0)) \langle \hat{P}(x) \hat{O}(0) \rangle + O(a)$
    - where: $\bar{M}(m_0) = \dfrac{c_0(1-d_1)}{a} + c_1(1-d_1)\Lambda_{\mathrm{QCD}} + d_1 m_0 + O(a)$
    - If we could set

      $m_0 = c_0/a \quad \rightarrow \quad \partial_\mu \langle \hat{J}_{5\mu}(x) \hat{O}(0) \rangle = c_1(1-d_1)\Lambda_{\mathrm{QCD}} \langle \hat{P}(x) \hat{O}(0) \rangle + O(a)$
  - Separation of the two effects requires an infinite fine tuning ($\rightarrow$ *naturalness problem).*

Overview

- **Dynamical generation of elementary fermion mass**
  - *Proposal*: QCD extended to a theory with **enriched symmetry** for tackling *naturalness problem*.
  - elementary fermion mass generation owing to NP mechanism triggered by a Wilson-like (naively irrelevant) chiral breaking term.

- **Simplest toy-model where the mechanism can be realised**:
  - $SU(N_f = 2)$ doublet of strongly ($SU(3)_c$) interacting fermions coupled to scalars via Yukawa and Wilson-like terms
  - physics depends crucially on the phase (Wigner or NG)
  - enhanced symmetry leads to $\langle \Phi \rangle$-independence of fermion masses
  - elementary fermion mass: $m_q = O(\alpha_s)\Lambda_s$

- The intrinsic NP character of the mechanism requires lattice numerical investigation of the toy model thanks to which we can falsify/support it.

Theoretical setup

- Toy-model: QCD$_{N_f=2}$ + Scalar field + Yukawa + Wilson

  $L_{toy} = L_{kin}(Q, A, \Phi) + V(\Phi) + L_Y(Q, \Phi) + L_W(Q, A, \Phi)$, with:

$$
\begin{aligned}
L_{kin}(Q, A, \Phi) &= \frac{1}{4} F^a_{\mu\nu} F^a_{\mu\nu} + \bar{Q}_L \gamma_\mu D_\mu Q_R + \bar{Q}_R \gamma_\mu D_\mu Q_L + \frac{1}{2} \mathrm{Tr}\left[\partial \Phi^\dagger \partial \Phi\right] \\
V(\Phi) &= \frac{1}{2} \mu^2 \mathrm{Tr}\left[\Phi^\dagger \Phi\right] + \frac{1}{4} \lambda \left(\mathrm{Tr}\left[\Phi^\dagger \Phi\right]\right)^2 \\
L_Y(Q, \Phi) &= \eta \left(\bar{Q}_L \Phi Q_R + \bar{Q}_R \Phi Q_L\right) \\
L_W(Q, A, \Phi) &= \rho \frac{b^2}{2} \left(\bar{Q}_L \overleftarrow{D}_\mu \Phi D_\mu Q_R + \bar{Q}_R \overleftarrow{D}_\mu \Phi^\dagger D_\mu Q_L\right)
\end{aligned}
$$

  (where $\overleftarrow{D}_\mu = \overleftarrow{\partial}_\mu + i g_s \lambda^a A^a_\mu$, $D_\mu = \partial_\mu - i g_s \lambda^a A^a_\mu$)

- Q: fermion $SU(2)$ doublet coupled to $SU(3)$ gauge field and to scalar field through Yukawa and Wilson terms.
- $\Phi = (\phi, -i\tau_2 \phi^*)$ and ϕ isodoublet of complex scalar fields.
- b^{-1}: UV cutoff.

Theoretical setup *(contd.)*

- $\chi_L \times \chi_R$ transformations are symmetry of L_{toy}:

$$
\begin{aligned}
\chi_L &: \tilde{\chi}_L \otimes (\Phi \to \Omega_L \Phi) & \chi_R &: \tilde{\chi}_R \otimes (\Phi \to \Omega_R \Phi) \\
\tilde{\chi}_L &: Q_L \to \Omega_L Q_L, & \tilde{\chi}_R &: Q_R \to \Omega_R Q_R, \\
&\quad \bar{Q}_L \to \bar{Q}_L \Omega_L^\dagger & &\quad \bar{Q}_R \to \bar{Q}_R \Omega_R^\dagger \\
&\quad \Omega_L \in SU(2)_L & &\quad \Omega_R \in SU(2)_R
\end{aligned}
$$

- **Exact symmetry** $\chi \equiv \chi_L \times \chi_R$ acting on fermions and scalars $\Rightarrow$ *NO* power divergent mass terms.
- The (fermion) $\tilde{\chi} \equiv \tilde{\chi}_L \times \tilde{\chi}_R$ transformations are *not* a symmetry for *generic* (non-zero) η and ρ.
- P, C, T, gauge invariance are symmetries & power counting renormalisation.

Theoretical setup *(contd.)*

- The shape of $V(\Phi)$ determines crucially the physical implications of the model
- When the scalar potential $V(\Phi)$ has one minimum
  - $\chi_L \times \chi_R$ is realized à la Wigner.
- The (fermion) $\tilde{\chi} \equiv \tilde{\chi}_L \times \tilde{\chi}_R$ transformations generate Schwinger-Dyson Eqs (unrenormalised).
- They get renormalised after considering the operator mixing procedure.
- PT operator mixings $\rightarrow$ *NO* $\tilde{\chi}-$SSB phenomenon occurs $\rightarrow$ *NO* NP fermion mass generation

Theoretical setup *(contd.)*

- **Critical Model**: $\tilde{\chi}$-symmetry restoration occurs when the Yukawa term is compensated by the Wilson term. This takes place (in the Wigner phase) at a certain value of the Yukawa coupling.
- In fact, for $\tilde{J}_\mu^{L,i}$ (or $\tilde{J}_\mu^{R,i}$) get
  $\partial_\mu \langle \tilde{Z}_{\tilde{J}} J_\mu^{L,i}(x) O(0) \rangle = (\eta - \overline{\eta}(\eta; g_s^2, \rho, \lambda)) \langle [\bar{Q}_L \tau^i \Phi Q_R - h.c.](x) O(0) \rangle + O(b^2)$
  (SDE renrm/tion here analogous to chiral SDE renrm/tion in Bochicchio *et al. NPB 1985*)
- *enforce* the current $\tilde{J}_\mu^{L,i}$ (or $\tilde{J}_\mu^{R,i}$) conservation $\Longrightarrow$
  $\eta - \overline{\eta}(\eta; g_s^2, \rho, \lambda) = 0 \quad \rightarrow \quad \eta_{cr}(g_s^2, \rho, \lambda)$.

- The Low-Energy effective action (in the Wigner phase) reads

$$\Gamma^{Wig}_{\mu_\Phi^2>0} = \frac{1}{4}(F \cdot F) + \bar{Q} \not{D} Q + (\eta - \eta_{cr})(\bar{Q}_L \Phi Q_R + \mathrm{h.c.}) + \frac{1}{2}\mathrm{Tr}\left[\partial_\mu \Phi^\dagger \partial_\mu \Phi\right] + \mathrm{V}_{\mu_\Phi^2>0}(\Phi)$$

- in the critical theory ($\tilde{\chi}$ is a symmetry, up to $O(b^2)$)
  - Scalars decoupled (up to cutoff effects) from quarks and gluons.
  - no fermionic mass ($m_Q = 0$ up to $O(b^2)$).

Numerical investigation

- **Lattice simulation details**
  - Lattice discretization, $L_{latt.}$ with naive fermions and $d = 6$ Wilson term: exact χ-symmetry respected.
  - We limit our first study to the quenched approximation
  - Quenching: independent generation of gauge (U) and scalar (Φ) configurations.

    $\Rightarrow$ it is quite certain that the mechanism under investigation, if confirmed, survives quenching.

Numerical investigation

- **Lattice simulation details**

  - To avoid "exceptional configurations" ($\rightarrow$ due to fermions zero modes) introduce twisted mass IR regulator $L_{latt.} + i\mu\bar{Q}\gamma_5\tau^3 Q$.

    (Frezzotti, Grassi, Sint and Weisz, JHEP 2001)

    $\Rightarrow$ at a cost of soft breaking of $\chi_L \times \chi_R$, symmetry recovered after an extrapolation to $\mu \rightarrow 0$.

  - Locally smeared Φ in $\bar{Q}D_{lat}[U, \Phi]Q$ for noise reduction.

Numerical investigation

- ⋆ simulations at **three** values of the lattice spacing
- ⋆ $\beta = 5.75$ ($b = 0.15$ fm), $\beta = 5.85$ ($b = 0.12$ fm) & $\beta = 5.95$ ($b = 0.10$ fm)
- ⋆ $(L/b, T/b) = (16, 32), (16, 40), (20, 48)$ (Wigner phase)
  $(L/b, T/b) = (16, 40), (20, 40), (24, 48)$ (NG phase)
  $L = 2.0 - 2.4$ fm, $T = 4.8$ fm.
- ⋆ use lattice scale $r_0 = 0.5$ fm (motivated from QCD, for illustration)
  Guagnelli, Sommer and Wittig NPB 535 (1998) & Necco and Sommer NPB 622 (2002)
- ⋆ ρ: for checking the validity of the mechanism it is sufficient to set some reasonable value $\neq 0$;
  In this first analysis: $\rho = 1.96$ in the Wigner & NG phase
- ⋆ **statistics:** #configs (gauge $\times$ scalar) 480 (Wigner), 60-80 (NG)

  @ several values of the Yukawa coupling η (and μ).
- ⋆ set same λ_0 in Wigner and NG phases (keep fixed scalar field parameters by imposing conditions on $(r_0 M_\sigma)^2$, λ_R and $(r_0 v_R)^2$ (NG phase)).

▶ Determination of η_{cr} in the Wigner phase

- Renormalised Schwinger-Dyson eqs of $\tilde{V}^3$ and $\tilde{A}^1$-type (in the form of a would be $\tilde{\chi}$-WTI):

$$\partial_\mu \tilde{J}^{V3}_\mu = (\eta - \eta_{cr})\tilde{D}^{V3} + O(b^2)$$

$$\partial_\mu \tilde{J}^{A1}_\mu = (\eta - \eta_{cr})\tilde{D}^{A1} + O(b^2)$$

where

$$\begin{aligned}
\tilde{J}^{V3}_\mu(x) &= \tilde{J}^{R\,3}_\mu(x) + \tilde{J}^{L\,3}_\mu(x) \\
\tilde{J}^{A1}_\mu(x) &= \tilde{J}^{R\,1}_\mu(x) - \tilde{J}^{L\,1}_\mu(x) \\
\tilde{J}^{L/R\,3}_\mu(x) &= \frac{1}{2}\left[\bar{Q}_{L/R}(x-\hat{\mu})\gamma_\mu \frac{\tau_3}{2} U_\mu(x-\hat{\mu}) Q_{L/R}(x) + \bar{Q}_{L/R}(x)\gamma_0 \frac{\tau_3}{2} U^\dagger_\mu(x-\hat{\mu}) Q_{L/R}(x-\hat{\mu})\right] \\
\tilde{D}^{V3}(y) &= \bar{Q}_L(y)\left[\Phi, \frac{\tau^3}{2}\right] Q_R(y) - \bar{Q}_R(y)\left[\frac{\tau^3}{2}, \Phi^\dagger\right] Q_L(y) \\
\tilde{D}^{A1}(y) &= \bar{Q}_L(y)\{\Phi, \frac{\tau^1}{2}\} Q_R(y) - \bar{Q}_R(y)\{\frac{\tau^1}{2}, \Phi^\dagger\} Q_L(y)
\end{aligned}$$

▶ Determination of η_{cr} in the Wigner phase

- A determination of η_{cr} could be obtained employing the "WI" ratio i.e. compute:

$$r_{VWI}(\eta; g_s^2, \lambda_0, \rho, \mu) \equiv \frac{\partial_0 \sum_{\mathbf{x}} \langle \tilde{J}_0^{V3}(\mathbf{x}, x_0) \tilde{D}^{V3}(0) \rangle}{\sum_{\mathbf{x}} \langle \tilde{D}^{V3}(\mathbf{x}, x_0) \tilde{D}^{V3}(0) \rangle}$$

at several values of η (and μ_Q) and extrapolate to

$$r_{VWI}(\eta_{cr}; g_s^2, \lambda_0, \rho, \mu = 0) \to 0 \ .$$

But signal is not sufficiently good due to noise of scalar correlation for euclidian time separation at around 1 fm.

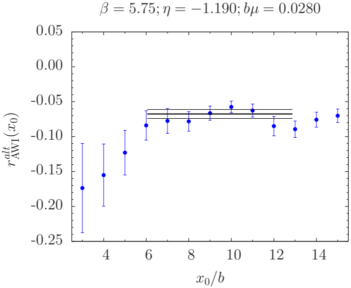
- Alternative determination of η_{cr}:

$$r_{AWI}^{alt}(\eta; g_s^2, \lambda_0, \rho, \mu) = \frac{\sum_{\mathbf{x}} \sum_{\mathbf{y}} \langle P^1(0) [\partial_0 \tilde{J}_0^{A1}](x) \phi^0(y) \rangle}{\sum_{\mathbf{x}} \sum_{\mathbf{y}} \langle P^1(0) D^{A1}(x) \phi^0(y) \rangle}$$

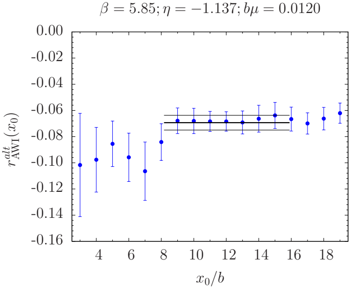
with: $\tilde{D}^{A1}(x) = \bar{Q}_L(x)\{\Phi, \frac{\tau^1}{2}\} Q_R(x) - \bar{Q}_R(x)\{\frac{\tau^1}{2}, \Phi^\dagger\} Q_L(x)$, $P^1 = \bar{Q}\gamma_5\tau^1/2Q$, $\phi^0 = \frac{1}{4}\mathrm{Tr}[\Phi + \Phi^\dagger] = \frac{1}{2}\mathrm{Tr}[\Phi]$, $y_0 = x_0 + \tau$ (τ = fixed (in practice $\sim$ 0.6 fm)).

▶ Determination of η_{cr} in the Wigner phase

$[\beta = 5.75]$

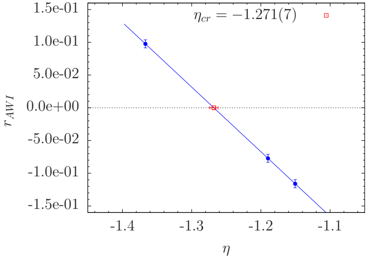


$[\beta = 5.85]$

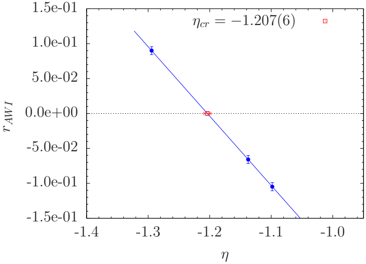


▶ Determination of η_{cr} in the Wigner phase

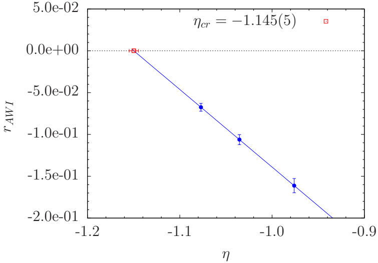
$[\beta = 5.75]$



$[\beta = 5.85]$



$[\beta = 5.95]$



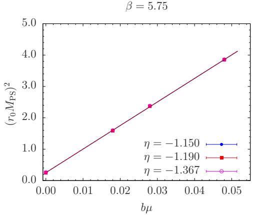
- r^{alt}_{AWI} inter/extrapolation to zero after taking the limit $\mu \to 0$

$\Rightarrow$ A few per mille *statistical* error for η_{cr} determination (Preliminary results!).

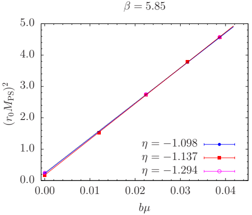
▶ Further investigation in the Wigner phase

- tm μ-term breaks softly χ-symmetry $\rightarrow$ at all η Goldstone bosons have mass vanishing linearly in μ.

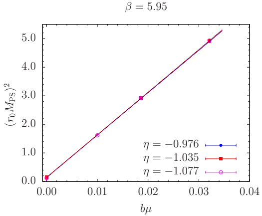
$[\beta = 5.75]$

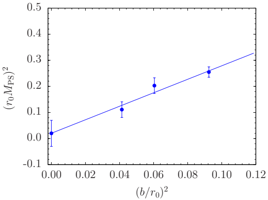


$[\beta = 5.85]$



$[\beta = 5.95]$





▶ Features and properties of the toy-model in NG-phase

- $V(\Phi)$ of mexican hat shape $\to \chi_L \times \chi_R$ realised à la NG.
- $\chi_L \times \chi_R$ spontaneously broken: $\Phi = v + \sigma + i\vec{\tau}\vec{\pi}, \qquad \langle\Phi\rangle = v \neq 0.$
- $L_W(Q, A, \Phi) = \frac{\rho b^2}{2} \left(\bar{Q}_L \overleftarrow{D}_\mu \Phi D_\mu Q_R + \text{h.c.}\right) \underset{a\leftrightarrow b}{\overset{r\leftrightarrow bv\rho}{\sim}} L_W^{QCD}(Q, A) = -\frac{ar}{2} \left(\bar{Q}_L D^2 Q_R + \text{h.c.}\right).$
- In the *critical* theory $\eta = \eta_{cr}$:
  - the (Yukawa) mass term, $v\bar{Q}Q$, gets cancelled.
  - $\tilde{\chi}-$ breaking due to residual $O(b^2)$ effects is expected to trigger dynamical χSB.

▶ Features and properties of the toy-model in NG-phase

$\Longrightarrow$ Look for dynamically generated fermion mass:

- **NP** mass term has to be $\chi_L \times \chi_R$ invariant (and under chiral variation can be accomodated in the $\tilde{\chi}$ WTI's).

  Note that a term like $m[\bar{Q}_L Q_R + \bar{Q}_R Q_L]$ is not $\chi_L \times \chi_R$ invariant.

- At *generic* η, two $\tilde{\chi}$ breaking operators are expected to arise:

  Yukawa induced + dynamically generated ($\leftarrow$ conjecture)

- $\Gamma^{NG} = \ldots + (\eta - \eta_{cr})(\bar{Q}_L \langle \Phi \rangle Q_R + \text{h.c.}) + c_1 \Lambda_s (\bar{Q}_L \mathcal{U} Q_R + \text{h.c.})$ where

  $$\mathcal{U} = \frac{\Phi}{\sqrt{\Phi^\dagger \Phi}} = \frac{(v+\sigma)\mathbb{1} + i\vec{\tau}\vec{\varphi}}{\sqrt{v^2 + 2v\sigma + \sigma^2 + \vec{\varphi}\vec{\varphi}}} \simeq \mathbb{1} + i\frac{\vec{\tau}\vec{\varphi}}{v} + \ldots$$

  and $\Lambda_s \equiv$ RGI NP mass scale.

  – $\mathcal{U}$ is a non-analytic function of Φ, but transforms like Φ under $\chi_L \times \chi_R$; obviously $\mathcal{U}$ can not be defined in the Wigner phase ($\langle \Phi \rangle = 0$) $\rightarrow$ no NP mass or mixings in the Wigner phase.

  – Note that (χ-inv. term): $c_1 \Lambda_s (\bar{Q}_L \mathcal{U} Q_R + \text{h.c.}) \simeq c_1 \Lambda_s \bar{Q} Q + \ldots$

▶ Features and properties of the toy-model in NG-phase

- Work at the same lattice parameters $(\beta,\ \lambda,\ \rho)$ as in the Wigner phase
- Compute WTI quark mass: $m_{AWI} = \dfrac{\partial_0 \langle \widetilde{J}_0^{A\pm}(x) P^{\pm}(y) \rangle}{\langle P^{\pm}(x) P^{\pm}(y) \rangle}$ in the NG-phase (where $P^{\pm} = \bar{Q}\gamma_5 \tau^{\pm} Q$ - pseudoscalar density) & M_{ps} from $\langle P^{\pm}(x) P^{\pm}(y) \rangle$.

▶ Features and properties of the toy-model in NG-phase

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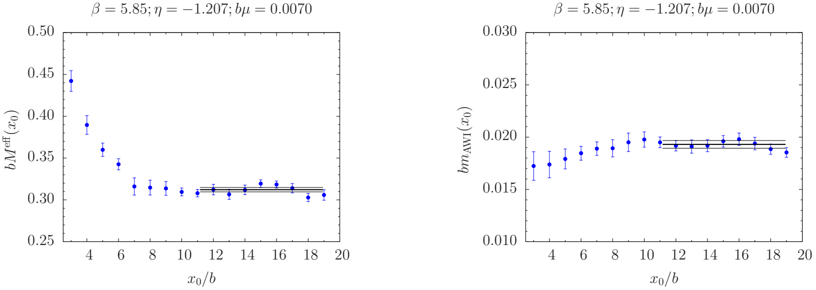
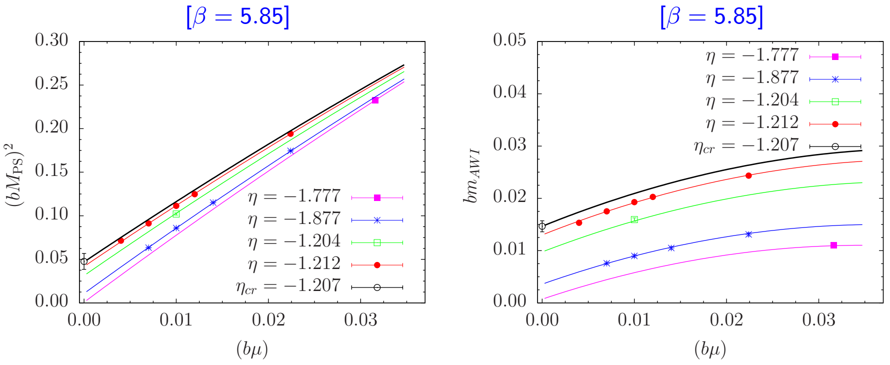


▶ Features and properties of the toy-model in NG-phase

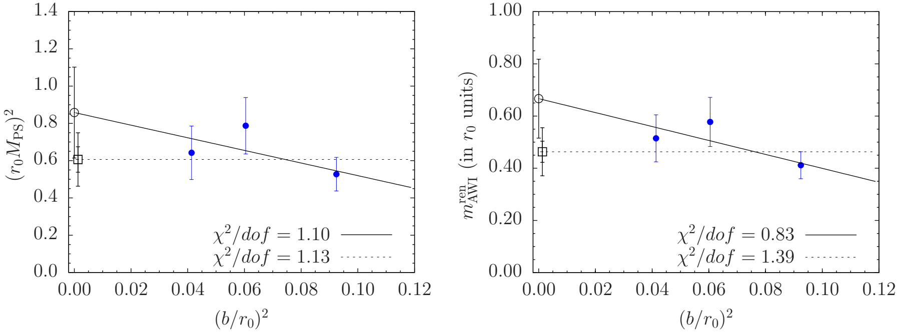
- simultaneous polynomial fits for $M^2_{\rm PS}$ and m_{AWI} in η and in μ (example: $\beta = 5.85$).
- extra/interpolation to η_{cr} (at $\mu \to 0$).



- similar for $M_{\rm PS}$.

▶ Features and properties of the toy-model in NG-phase

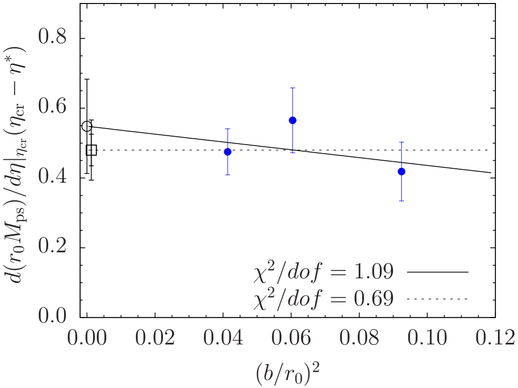
- scaling behaviour of $M^2_{\rm PS}$ and m_{AWI} at η_{cr}



- $m_{AWI} = (\eta - \eta_{cr})v + c_1\Lambda_s \xrightarrow{\eta=\eta_{cr}} m_{AWI} = c_1\Lambda_s$
- m_{AWI} vanishes at $\eta^* = \eta_{cr} - c_1\Lambda_s/v \Rightarrow \eta_{cr} \neq \eta^* \leftrightarrow c_1\Lambda_s \neq 0$

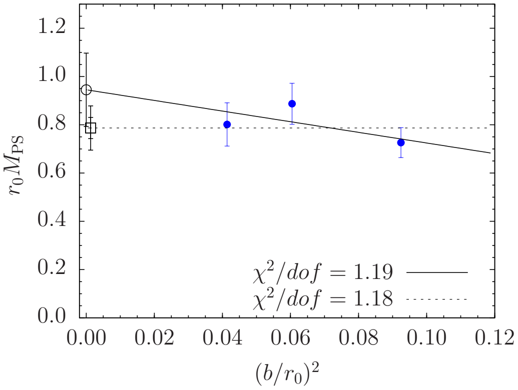
▶ Features and properties of the toy-model in NG-phase

- $c_1 \Lambda_S \neq 0 \Leftrightarrow (\eta_{cr} - \eta^*) \neq 0$
- $(\eta_{cr} - \eta^*)$ has to be renormalised ...
- Consider renormalised quantity: $\mathcal{D}_\eta \equiv d(r_0 M_{PS})/d\eta|_{\eta_{cr}}(\eta_{cr} - \eta^*) \equiv Z_\eta(\eta_{cr} - \eta^*)$



▶ Features and properties of the toy-model in NG-phase

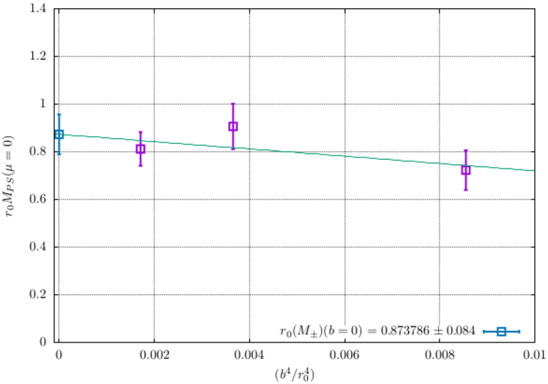
- scaling behaviour of M_{PS} at η_{cr}



- $\sim 6\sigma$ significance from zero for M_{ps} at CL.

▶ Test of no mechanism hypothesis

- It can be shown that in case of no mechanism presence, i.e. $c_1\Lambda_s = 0$, then $M_{\rm PS} \sim O(b^4)$.



No mechanism hypothesis not been supported by the data (8σ from zero).

Conclusions & Outlook

- We have presented a toy-model that exemplifies a *novel* NP mechanism for elementary fermion mass generation.

- The **toy model** is a non-Abelian gauge model with an SU($N_f = 2$)-doublet of strongly interacting fermions coupled to scalars through Yukawa and Wilson-like terms: at the *critical point*, where (fermion) $\tilde{\chi}$ invariance is recovered in Wigner phase (up to UV-effects) the model is *conjectured* to give rise in NG phase to dynamical $\tilde{\chi}$-SSB and hence to non-perturbative fermion mass generation.

- The main physical implications of the conjecture above can be *verified/falsified* by numerical simulations of the toy-model (rather cheap in the quenched approximation).

Conclusions & Outlook

- A study at three values of the lattice spacing ($\sim$ 0.10, 0.12 and 0.15 fm) in the quenched approximation has been presented.

- We have shown that the critical value of the Yukawa coupling in the Wigner phase at which $\tilde{\chi}$ is restored can be accurately determined. Then we explored the effects of dynamical SSB of the (restored) $\tilde{\chi}$-symmetry in the NG phase which look very well compatible with the generation of a non-zero (effective) fermion mass and $M_{PS} \sim O(\Lambda_s)$ at the CL.

- These findings, based on a preliminary analysis, could be checked at a finer value of the lattice spacing in order to get more solid confirmation for the persistence of the dynamical mass generation mechanism in the continuum limit.

Conclusions & Outlook

- A study at three values of the lattice spacing ($\sim$ 0.10, 0.12 and 0.15 fm) in the quenched approximation has been presented.

- We have shown that the critical value of the Yukawa coupling in the Wigner phase at which $\tilde{\chi}$ is restored can be accurately determined. Then we explored the effects of dynamical SSB of the (restored) $\tilde{\chi}$-symmetry in the NG phase which look very well compatible with the generation of a non-zero (effective) fermion mass and $M_{PS} \sim O(\Lambda_s)$ at the CL.

- These findings, based on a preliminary analysis, could be checked at a finer value of the lattice spacing in order to get more solid confirmation for the persistence of the dynamical mass generation mechanism in the continuum limit.

Thank you for your attention!

Extra slides

▶ Lattice action

$$S_{lat} = b^4 \sum_x \left\{ \mathcal{L}_{kin}^{YM,plaq}[U] + \mathcal{L}_{kin}^{sca}(\Phi) + \mathcal{V}(\Phi) + \overline{\Psi} D_{lat}[U,\Phi]\Psi \right\}, \quad \Phi = \varphi_0 1\!\!1 + i\varphi_j \tau^j ,$$

$$\mathcal{L}_{kin}^{sca}(\Phi) + \mathcal{V}(\Phi) = \tfrac{1}{2}\,\mathrm{Tr}\,[\Phi^\dagger(-\partial_\mu^*\partial_\mu)\Phi] + \tfrac{\mu_0^2}{2}\,\mathrm{Tr}\,[\Phi^\dagger\Phi] + \tfrac{\lambda_0}{4}\left(\mathrm{Tr}\,[\Phi^\dagger\Phi]\right)^2 ,$$

$$(D_{lat}[U,\Phi]\Psi)(x) = \gamma_\mu \widetilde{\nabla}_\mu \Psi(x) + \eta F(x)\Psi(x) - b^2\rho \tfrac{1}{2} F(x)\widetilde{\nabla}_\mu\widetilde{\nabla}_\mu\Psi(x)$$

$$-\, b^2\rho\tfrac{1}{4}\left[(\partial_\mu F)(x)U_\mu(x)\widetilde{\nabla}_\mu\Psi(x+\hat{\mu}) + (\partial_\mu^* F)(x)U_\mu^\dagger(x-\hat{\mu})\widetilde{\nabla}_\mu\Psi(x-\hat{\mu})\right],$$

where $F \equiv \varphi_0 1\!\!1 + i\gamma_5\tau^j\varphi_j$. Only derivatives $\widetilde{\nabla}_\mu = \frac{1}{2}(\nabla_\mu + \nabla_\mu^*)$ acting on fermions, with

$$\nabla_\mu f(x) \equiv \tfrac{1}{b}(U_\mu(x)f(x+\hat{\mu}) - f(x)), \qquad \nabla_\mu^* f(x) \equiv \tfrac{1}{b}(f(x) - U_\mu^\dagger(x-\hat{\mu})^f(x-\hat{\mu})).$$

Term $\propto \rho$: Wilson-like, but with $d = 6$ $\Rightarrow$ fermion doublers do not decouple

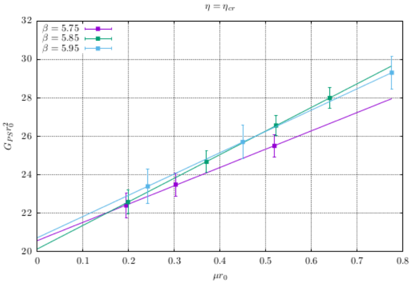
Extension to 2 generations: $\bar{\Psi}_\ell D_{lat}[U,\Phi]\Psi_\ell + \bar{\Psi}_h D_{lat}[U,\Phi]\Psi_h$ in fermionic L_{lat}

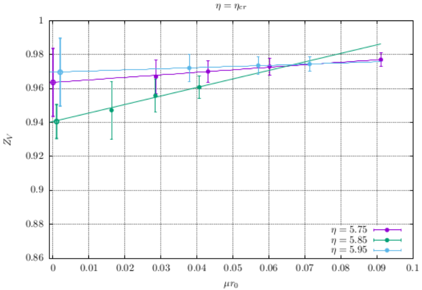
▶ Determination of m_{AWI}^{ren}

- RCs UV quantities: can be calculated either in Wigner of in NG phases
- $1/Z_P^{had} = \langle 0|\bar{Q}\gamma_5\frac{\tau^1}{2}Q|P^1_{meson}\rangle|_{\eta_{cr},\mu\to 0+}\; r_0^2 \equiv G_{PS}^{Wig}\; r_0^2$ eval. in Wigner phase
- $Z_{\widetilde{V}}$: $Z_{\widetilde{V}}\langle 0|\partial_0\widetilde{V}_0^2|P^1_{\rm meson}\rangle|_{\eta_{cr},\mu\to 0+} = 2\mu\langle 0|\bar{Q}\gamma_5\frac{\tau^1}{2}Q|P^1_{\rm meson}\rangle|_{\eta_{cr},\mu\to 0+}$

  evaluated in NG phase
- $Z_{\widetilde{V}} = Z_{\widetilde{A}}$ (at η_{cr})
- $m_{AWI}^{ren} = \dfrac{Z_{\widetilde{V}}}{Z_P^{had}}\, m_{AWI}$

▶ Determination of m_{AWI}^{ren}





▶ Check for finite size effects ($\beta = 5.85$)