Towards a non-perturbative calculation of Weak Hamiltonian Wilson Coefficients

Mattia Bruno
RBC/UKQCD Collaboration

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The RBC & UKQCD collaborations

BNL and RBRC

Mattia Bruno
Tomomi Ishikawa
Taku Izubuchi
Luchang Jin
Chulwoo Jung
Christoph Lehner
Meifeng Lin
Hiroshi Ohki
Shigemi Ohta (KEK)
Amarjit Soni
Sergey Syritsyn

Columbia University

Ziyuan Bai
Norman Christ
Duo Guo
Christopher Kelly
Bob Mawhinney
David Murphy
Masaaki Tomii
Jiqun Tu
Bigeng Wang
Tianle Wang

University of Connecticut

Tom Blum
Dan Hoying
Cheng Tu

Edinburgh University

Peter Boyle
Guido Cossu
Luigi Del Debbio
Richard Kenway
Julia Kettle
Ava Khamseh
Brian Pendleton
Antonin Portelli
Tobias Tsang
Oliver Witzel
Azusa Yamaguchi

KEK

Julien Frison

University of Liverpool

Nicolas Garron

Peking University

Xu Feng

University of Southampton

Jonathan Flynn
Vera Guelpers
James Harrison
Andreas Juettner
Andrew Lawson
Edwin Lizarazo
Chris Sachrajda

York University (Toronto)

Renwick Hudspith

Introduction

Weak decays of hadrons

rich phenomenology
(e.g. CP violation in $K \to \pi\pi$)

QCD $\to$ confinement, light objects

Weak interactions $\to$ short range, heavy mediators

These decays have a natural scale separation

$\downarrow$

build an **effective low-energy theory**

Integrate out heavy degrees of freedom: heavy quarks, weak bosons

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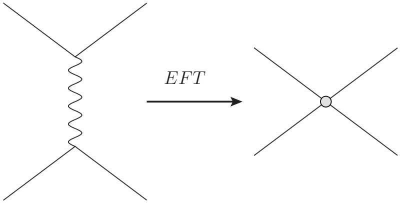
Effective theory

Integrating out weak bosons generates **four-quark vertices**



Current-current diagrams:

$c \to s u \bar{d}$

new divergences in the EFT

$\downarrow$

operator mixing

$$\mathcal{H}_{\rm eff} \propto G_{\rm F} \sum_i C_i Q_i \quad \text{with} \quad i = 1, 2 \text{ in our example}$$

Long distance matrix elements $\langle Q_i \rangle \to$ Lattice

Wilson Coefficients $C_i \to$ PT

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OPE

We use W boson propagator in unitary gauge (Euclidean)

$$W_{\mu\nu}(q) = \frac{1}{q^2 + m_{\mathrm{W}}^2}\left(\delta_{\mu\nu} - \frac{q_\mu q_\nu}{m_{\mathrm{W}}^2}\right) \overset{m_{\mathrm{W}}\to\infty}{\approx} \frac{1}{m_{\mathrm{W}}^2}\left[\delta_{\mu\nu} + O\left(\frac{q^2}{m_{\mathrm{W}}^2}\right)\right]$$

Four-quark operators Q_i are first terms in the expansion

$$\mathcal{H}_{\mathrm{eff}} \propto G_{\mathrm{F}}\left[\sum_i C_i Q_i + \sum_i \frac{c_i^{(d)}}{m_{\mathrm{W}}^{d-6}} O_i^{(d)}\right] \quad , \quad d \geq 8$$

$O_i^{(d)}$ can be gauge-invariant operators

if we fix the (QCD) gauge $O_i^{(d)}$ can be gauge-noninvariant operators

$O_i^{(d)}$ depend on momenta p_i of external states

In the limit $p_i/m_{\mathrm{W}} \to 0$, $\forall i$, only Q_1 and Q_2 survive

Perturbative results

[Buchalla, Buras, Lautenbacher '95]

By matching the full and effective theory at one loop in $\overline{\mathrm{MS}}$:

$$C_1 = \alpha_s(b_1 + c_1 \log(m_W^2/\mu^2))$$
$$C_2 = 1 + \alpha_s(-b_2 + c_2 \log(m_W^2/\mu^2))$$

b_1, b_2 positive coefficients

μ is the matching scale $\to$ large logs

Initial conditions C_1 and C_2 (NDR)

$$C_1(m_W) \approx 0.44\alpha_s(m_W)$$
$$C_2(m_W) = 1 - 0.15\alpha_s(m_W)$$

Anomalous Dimension Matrix (ADM)

U solution of RG equations

$$\vec{C}(\mu) = U(m_W, \mu)\vec{C}(m_W)$$

Resummation of large logs at scale μ

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$\Delta S = 1$ amplitude breakdown

Perturbation theory realm

Matching EFT $\leftrightarrow$ SM at m_{W}

$\downarrow$

Running $m_{\mathrm{W}} \rightarrow m_{\mathrm{b}}$ in $N_{\mathrm{f}} = 5$ theory

$\downarrow$

Matching $\mathcal{H}_{\mathrm{eff}}^{N_{\mathrm{f}}=5}(m_{\mathrm{b}}) \leftrightarrow \mathcal{H}_{\mathrm{eff}}^{N_{\mathrm{f}}=4}(m_{\mathrm{b}})$

$\downarrow$

Running $m_{\mathrm{b}} \rightarrow m_{\mathrm{c}}$ in $N_{\mathrm{f}} = 4$ theory

$\downarrow$

Matching $\mathcal{H}_{\mathrm{eff}}^{N_{\mathrm{f}}=4}(m_{\mathrm{c}}) \leftrightarrow \mathcal{H}_{\mathrm{eff}}^{N_{\mathrm{f}}=3}(m_{\mathrm{c}})$

$\downarrow$

Running $m_{\mathrm{c}} \rightarrow \mu$ in $N_{\mathrm{f}} = 3$ theory

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Lattice QCD realm

Non-Pert. renormalization of Operators at μ

$\downarrow$

Computation $\langle \mathcal{H}_{\mathrm{eff}}^{N_{\mathrm{f}}=3} \rangle$

Motivations

Great progress in last decade on matrix elements from Lattice QCD

- $K \to \pi\pi$ isospin 0 and 2 channels [RBC/UKQCD, '16]
- very precise $\Delta B = 1$ decays [Fermilab,MILC, '16]

Systematic errors of this calculations:

1. matrix elements: statistics, finite volumes, finite masses $\to$ improvable

2. connection to PT: finer lattice spacings $\to$ improvable

3. running to high scales in PT: errors controlled by varying μ

3. matching conditions: higher loops needed to quantify errors

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$K \to \pi\pi^{I=0}$ AMPLITUDE BREAKDOWN

Perturbation theory realm

Matching EFT $\leftrightarrow$ SM at m_{W} $\to$ 6% full basis, 3% C_1, C_2 for $\mathrm{Im}\, A_0$

$\downarrow$

Running $m_{\mathrm{W}} \to m_{\mathrm{b}}$ in $N_{\mathrm{f}} = 5$ theory

$\downarrow$

Matching $\mathcal{H}_{\mathrm{eff}}^{N_{\mathrm{f}}=5}(m_{\mathrm{b}}) \leftrightarrow \mathcal{H}_{\mathrm{eff}}^{N_{\mathrm{f}}=4}(m_{\mathrm{b}})$ $\to$ 0.5% error

$\downarrow$

Running $m_{\mathrm{b}} \to m_{\mathrm{c}}$ in $N_{\mathrm{f}} = 4$ theory

Using lattice here could buy 6-8%

$\downarrow$

Matching $\mathcal{H}_{\mathrm{eff}}^{N_{\mathrm{f}}=4}(m_{\mathrm{c}}) \leftrightarrow \mathcal{H}_{\mathrm{eff}}^{N_{\mathrm{f}}=3}(m_{\mathrm{c}})$ $\to$ 1% error

$\downarrow$

Running $m_{\mathrm{c}} \to \mu$ in $N_{\mathrm{f}} = 3$ theory

---

Lattice QCD realm

Non-Pert. Oper. renormal. at μ — Connection to PT 15% syst. error

$\downarrow$

Computation $\langle \mathcal{H}_{\mathrm{eff}}^{N_{\mathrm{f}}=3} \rangle$ — [RBC/UKQCD, '16, '17]

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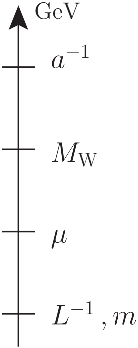
Window problem

μ is the matching scale:



$am_W \ll 1$ for discretization effects

$\mu \ll m_W$ for higher order operators

$\mu \gg m\,, \mu L \gg 1$ for infrared effects

Present study is focused on unphysically small $m_W \approx 2\ \text{GeV}$

Non-perturbative effects $O(\Lambda_{QCD}/m_W)$

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Systematic uncertainties

Wilson Coefficients are ultraviolet quantities

related to $p \gtrsim m_W \rightarrow$ potentially large discretization errors ✓

independent from infrared regulators, up to

finite volume effects ✓

finite quark mass effects ✓

non-perturbative effects ✓

We study all these effects

current available lattices $m_W \approx 2$ GeV

neglect disconnected ($\rightarrow$penguin) diagrams (for larger operator basis)

[Dawson, Martinelli, Rossi, Sachrajda, Sharpe, Talevi, Testa '98]

Seminal ideas for a non-perturbatively defined weak hamiltonian

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RI/(S)MOM scheme

[Martinelli, Pittori, Sachrajda, Testa, Vladikas 95]

Given renormalized amputated Green's function Λ^R
Regularization Independent conditions (RI-MOM)

$$\Lambda^R|_{p^2=\mu^2} = Z_{\mathrm{q}}^{-n/2}\ Z\ \Lambda^{\mathrm{bare}}|_{p^2=\mu^2} = \Lambda^{\mathrm{tree}}$$

The renormalization scheme is defined by the choice of the external states:

- we use off-shell external quark states
  with momentum p_i, $i = 1, 2, 3, 4$
  with masses $m_i = m$, $\forall i$
  with Projectors P_i to project onto definite spin-color states
- we use Landau gauge

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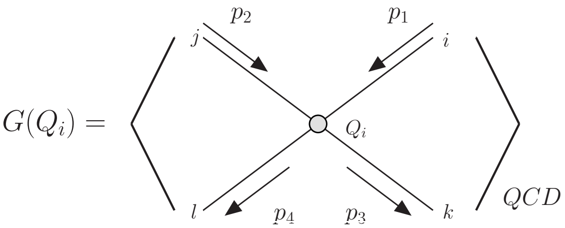
Lattice Observables - EFT



Green's function $G(Q_i)$
$Q_1 = (\bar{s}_i c_j)_{\mathrm{V-A}} \otimes (\bar{u}_j d_i)_{\mathrm{V-A}}$
$Q_2 = (\bar{s}_i c_i)_{\mathrm{V-A}} \otimes (\bar{u}_j d_j)_{\mathrm{V-A}}$

RI schemes
$p_1 = p_3 = p$, $p_2 = p_4 = -p$
$p_1 \neq p_2 \neq p_3 \neq p_4$, $p_i^2 = p^2$

$\Lambda(Q_i)$: amputated $G(Q_i)$ with quark propagators $S(p_i, m_i)$

Projectors: $P_1 = \delta_{il}\delta_{kj}\ (\Gamma_1 \otimes \Gamma_2)$, $P_2 = \delta_{ij}\delta_{kl}\ (\Gamma_1 \otimes \Gamma_2)$ [RBC/UKQCD '10]

We define $M_{ij} = P_j\left[\,\Lambda(Q_i)\,\right]$

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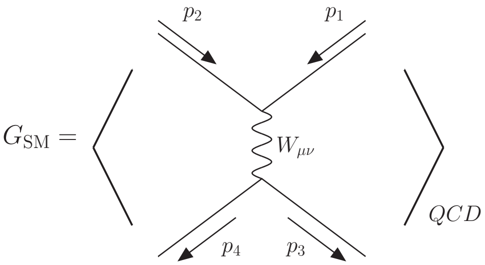
Lattice Observables - Full theory



W boson in unitary gauge

RI schemes:

$p_1 = p_3 = p,\ p_2 = p_4 = -p$

$p_1 \neq p_2 \neq p_3 \neq p_4,\ p_i^2 = p^2$

Weak vertex factor $\propto g_2$

$\Lambda_{\rm SM}$: amputated $G_{\rm SM}$ with quark propagators $S(p_i, m_i)$

3. Define $W_i = P_i(\Lambda_{\rm SM})$

4. Note that $W_i^{\rm RI}(\mu) \propto Z_{\rm q}^{-2}(\mu)\ Z_V^2\ W_i^{\rm lat}|_{p^2=\mu^2}$

Z_V: vector bilinear operator renormalization factor

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Matching procedure

Matching equation for RI conditions

$$\frac{G_F}{\sqrt{2}}\, C_i^{\rm RI}(\mu) M_{ij}^{\rm RI}(\mu) = W_j^{\rm RI}(\mu) = \frac{g_2^2}{8} Z_q^{-2} Z_V^2 W_j^{\rm lat}$$

CKM matrix elements simplify

$G_F/\sqrt{2}$ and $g_2^2/8$ simplification $\rightarrow$ $1/m_W^2$

$$C_i^{\rm RI}(\mu) = m_W^2 \left(W_j^{\rm lat} [M^{\rm lat}]_{jk}^{-1} \right) \left([Z^{\rm RI}(\mu)]_{ki}^{-1} Z_V^2 \right)$$

Bare lattice Wilson Coefficients: $C_k^{\rm lat} = m_W^2 W_j^{\rm lat} [M^{\rm lat}]_{jk}^{-1}$

1. The matching procedure on the lattice
   study effects of higher order operators $O(p^2/m_W^2)$
   study infrared/non-perturbative effects in limit $p^2 \rightarrow 0$

2. Renormalization of the lattice theory to RI (or $\overline{\rm MS}$)

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Lattice Setup

Ensembles $N_\mathrm{f} = 2 + 1$ Shamir Domain-Wall fermions

$a^{-1} \approx 1.78$ GeV ≈ 0.11 fm

$L \approx 1.8$ fm and 2.6 fm

$a^{-1} \approx 2.38$ GeV ≈ 0.08 fm

$L \approx 2.6$ fm

NEW Ensembles $N_f = 2 + 1$, $N_f = 2 + 2$ Möbius Domain-Wall fermions

$a^{-1} \approx 3$ GeV ≈ 0.07 fm

$L \approx 2.2$ fm

$a^{-1} \approx 4$ GeV ≈ 0.05 fm

$L \approx 1.6$ fm

Bare amplitudes with external p between 0.2 and 1.0 GeV

RI/SMOM scheme with external p between 1.4 and 2.4 GeV

Artificially small $m_\mathrm{W} \rightarrow 0.6 < am_\mathrm{W} < 1.3$

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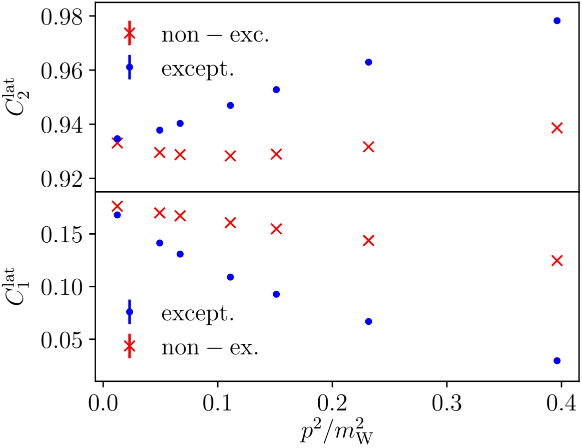
Higher order operators



Excellent statistical precision

Different external states
↓
different p^2 behaviors

Polynomial fits in $\frac{p^2}{m_W^2}$

Projectors: $VA + AV$
$m_W \approx 1.7$ GeV

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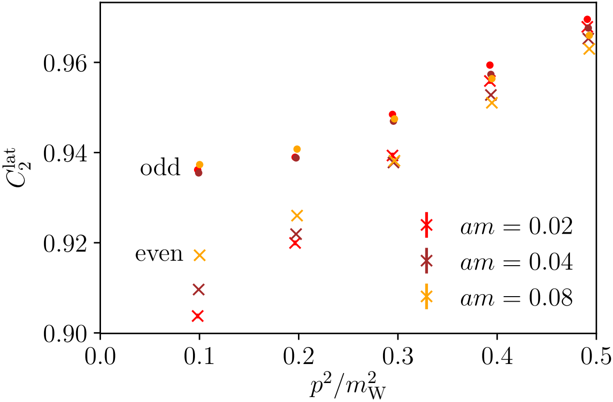
Quark mass dependence - I

Projectors: (parity even and odd) $VV + AA$ and $VA + AV$

parity odd $\rightarrow$ suppression of quark mass effects $\rightarrow$ $\mathcal{CPS}$ symmetry

parity even $\rightarrow$ large contaminations from wrong chiralities



Goldstone-pole $\dfrac{1}{p^2 + M^2}$

$p \approx \Lambda_{\rm QCD}$ spontaneous breaking of chiral symmetry

Domain-Wall excellent chiral symmetry

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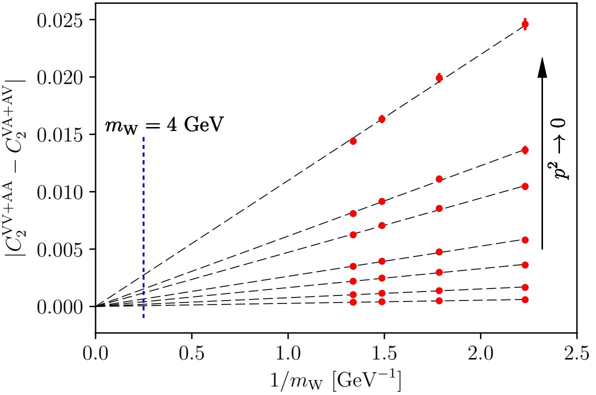
Quark mass dependence - II

From $m_{\rm W} \to \infty$ expansion of propagator only powers of $1/m_{\rm W}^2$

observed $1/m_{\rm W}$ from non-perturbative effects (e.g. condensates)



Example of such operator

$$\frac{\bar{q}q}{p^2 m_{\rm W}} O^{6\text{-dim}}$$

$p^2 \gg \Lambda_{\rm QCD}^2$ vanishes

Systematic error

$$|C_i^{\rm VV+AA} - C_i^{\rm VA+AV}|$$

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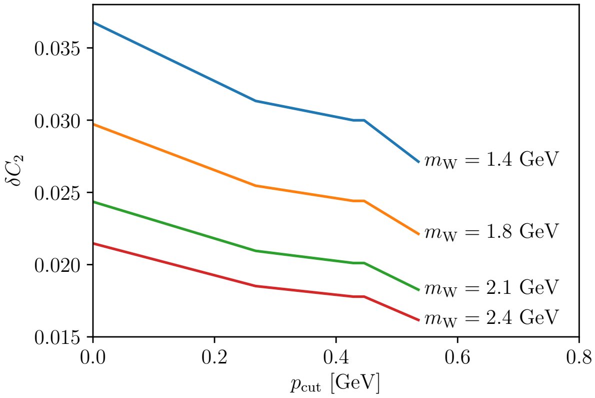
Systematic error

Excluding point with $p^2 < p^2_{\mathrm{cut}}$ from fits



p_{cut} suppresses $O(\Lambda_{\mathrm{QCD}})$ contaminations

p_{cut} controls systematic error $\delta C_i \approx 5 - 10\times$ statistical

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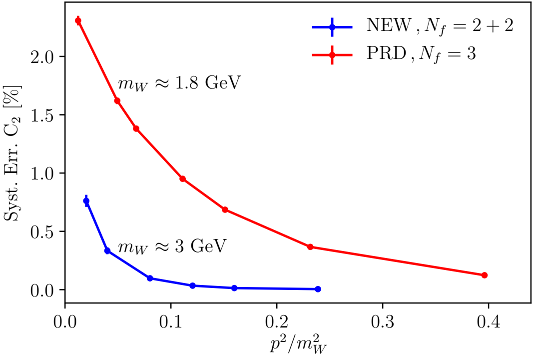
New improvements

Ongoing measurements on finer lattices → higher m_{W}



larger separation
$\Lambda_{\mathrm{QCD}} < m_{\mathrm{W}}$

smaller $O(p/\Lambda_{\mathrm{QCD}})$ effects

better precision

similar improvement for C_1

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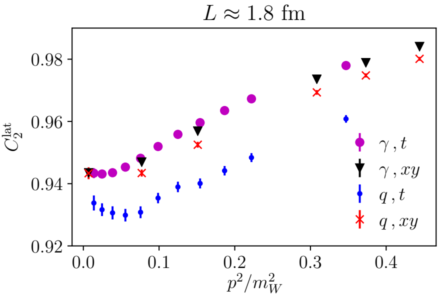
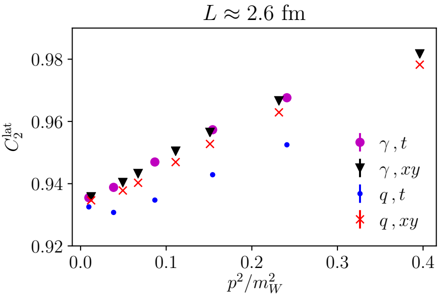
Finite volume effects - I

Momentum injected along time (t) or spatial (xy) directions

time extent is $2\times$ spatial extent

Projectors $VA + AV$: γ and $\not{q}$ schemes [RBC/UKQCD '10]





Breaking of universality at $p^2 = 0$
is a finite volume effect

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Finite volume effects - II

Origin of finite volume error?

m_W vs. L

for fixed p, cutoff a^{-1} and $m_W L$

.

$L \approx 1.8$ fm ✓ finite vol.err.
$L \approx 2.6$ fm ✗ finite vol.err.

$m_W L$ does not govern finite vol.errors otherwise same effect on both lattices

m_W vs. Λ_{QCD}

for fixed p, cutoff a^{-1} and L
we vary m_W

$L \approx 1.8$ fm ✓ finite vol.err. $\propto 1/m_W$
$L \approx 2.6$ fm ✗ finite vol.err.

non-perturbative condensate

$1/m_W$ behavior → non-pert. condensate → strong influence by box size

Finite volume error from QCD
not from weak boson

Renormalization

$$\tilde{Z}(\mu) \equiv Z_V^2 [Z^{\mathrm{RI}}(\mu)]^{-1} [Z^{\mathrm{RI}\to\overline{\mathrm{MS}}}(\mu)]^{-1}$$

$$C_i^{\overline{\mathrm{MS}}}(\mu) = C_j^{\mathrm{lat}} [\tilde{Z}(\mu)]_{ji}$$

Implement mass-less renormalization condition

$$\lim_{m\to 0} [Z^{\mathrm{RI}}(\mu)]^{-1} Z_V^2$$

1. compute $\tilde{Z}$ for $am = 0.02$ and 0.04 at one value of μ
2. Negligible quark mass dependence of $\tilde{Z}$

We use two intermediate RI schemes (γ and $\not{q}$)

1. (this work) 1-loop matching $Z^{\mathrm{RI}\to\overline{\mathrm{MS}}}$ for RI/SMOM for γ and $\not{q}$
2. difference of two schemes is $O(\alpha_s^2)$

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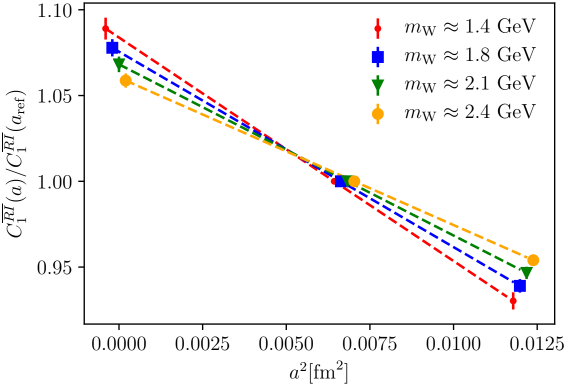
Discretization errros

Remove lattice artifacts with $a \approx 0.11$ fm and 0.08 fm



C_1: $10 - 17\%$ a^2 errors

C_2: $\lesssim 1\%$ errors

Dependence on am_W not relevant

$am_W \approx 1$ under control

$a_{\rm ref} = 0.08$ fm

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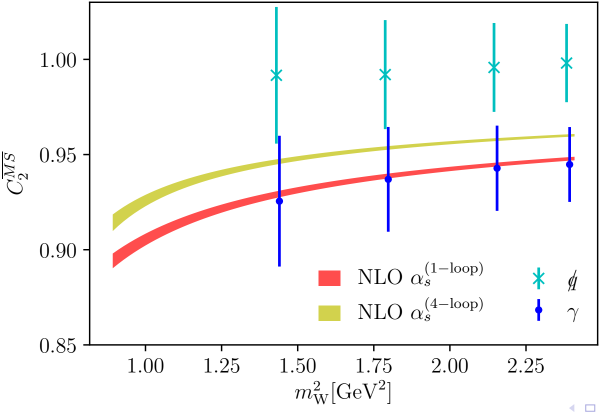
Results - C_2

Large error from matching RI $\to \overline{\text{MS}}$

Error dominated by systematics (90%) over statistical (10%)

Systematics correlated $\to$ fit $\to$ predict 1-loop coefficient

analytic results from [Buchalla, Buras, Lautenbacher '95]



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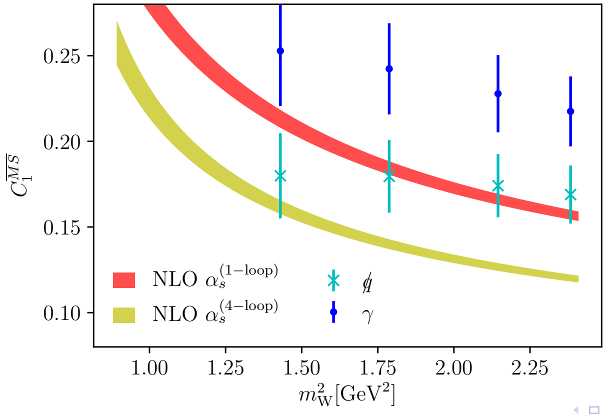
Results - C_1

Large error from matching RI $\rightarrow \overline{\text{MS}}$

Error dominated by systematics (90%) over statistical (10%)

Systematics correlated $\rightarrow$ fit $\rightarrow$ predict 1-loop coefficient (1 σ)

analytic results from [Buchalla, Buras, Lautenbacher '95]



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Towards the Standard Model

With our strategy can we reach the Standard Model?

1. W boson mass of 80 GeV

   fit our data in RI scheme $\rightarrow$ predict higher loop coefficients
   $\rightarrow$ run α_s up to 80 GeV $\rightarrow$ estimate or bound Wils.Coeff.

2. EFT with 5-flavors in the sea

   need simulations with more dynamical quarks in the sea

3. integrating out top quark

   future studies, difficult problem on lattice

With ongoing second calculation

higher m_{W} systematic errors below 1% $\rightarrow$ precise fits $\rightarrow$ **1.** ✓

$N_{\mathrm{f}} = 2+1$ vs. $N_{\mathrm{f}} = 2+2$ $\rightarrow$ flavor dependence $\rightarrow$ **2.** ✓

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Conclusions

We have developed a method to compute (weak) Wilson coefficients to all-orders in α_s in RI scheme

- controlled quark mass and finite volume errors
  - discretization effects removed with 2 lattice spacings
- excellent statistical precision
- account for non-perturbative contributions
- possibility to bound perturbative error

Outlook:

- push towards higher values of $m_{\mathrm{W}} \to$ reduce systematics
- study flavor dependence
- extend the basis of operators (e.g. $\Delta S = 1$ or BSM)

Thanks for the attention!

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