

Investigation of holographic cosmological models using lattice simulations

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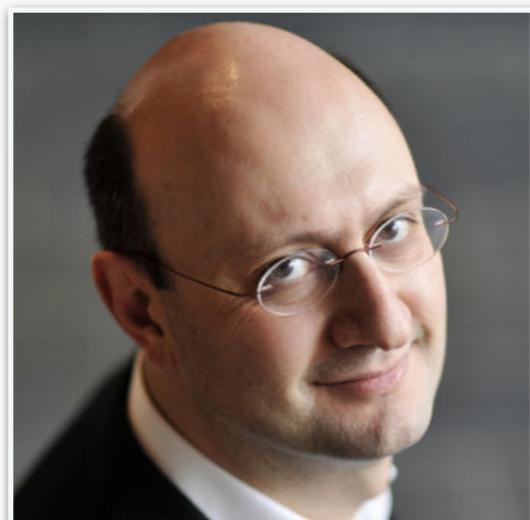
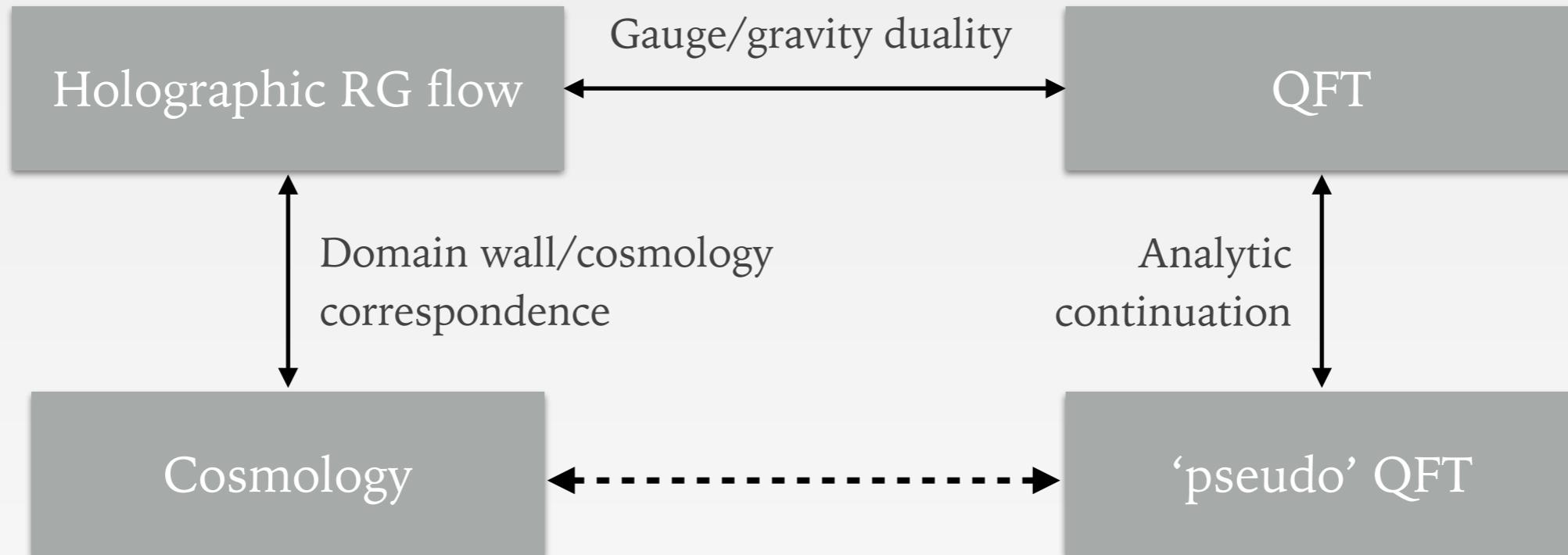


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- Holographic phenomenology for cosmology
- A first step: scalar $SU(N)$ model
- Non-perturbative lattice simulations
- Renormalisation of the energy-momentum tensor
- Conclusion & perspectives

Holographic cosmology

General principle



P.L. McFadden and K. Skenderis
[PRD 81(2) 2010]
[J. Phys. Conf. Ser. 222(1) 2010]
[JCAP 05 2011]

Holographic CMB spectrum

- Dual theory ansatz: 3D SU(N) gauge theory with arbitrary content of massless scalars & fermions.
- Super-renormalisable theory, with a dimension 1 coupling constant g_{YM} .
- CMB scalar power spectrum

$$\Delta_{\mathcal{R}}^2(q) = -\frac{q^3}{4\pi^2} \frac{1}{\langle T(q)T(-q) \rangle} \quad (T = T_{\mu\mu})$$

Holographic CMB spectrum

- At 2-loop, universal form

$$\Delta_{\mathcal{R}}^2(q) = \frac{\Delta_0^2}{1 + \frac{gq_*}{q} \log \left| \frac{q}{\beta g q_*} \right|}$$

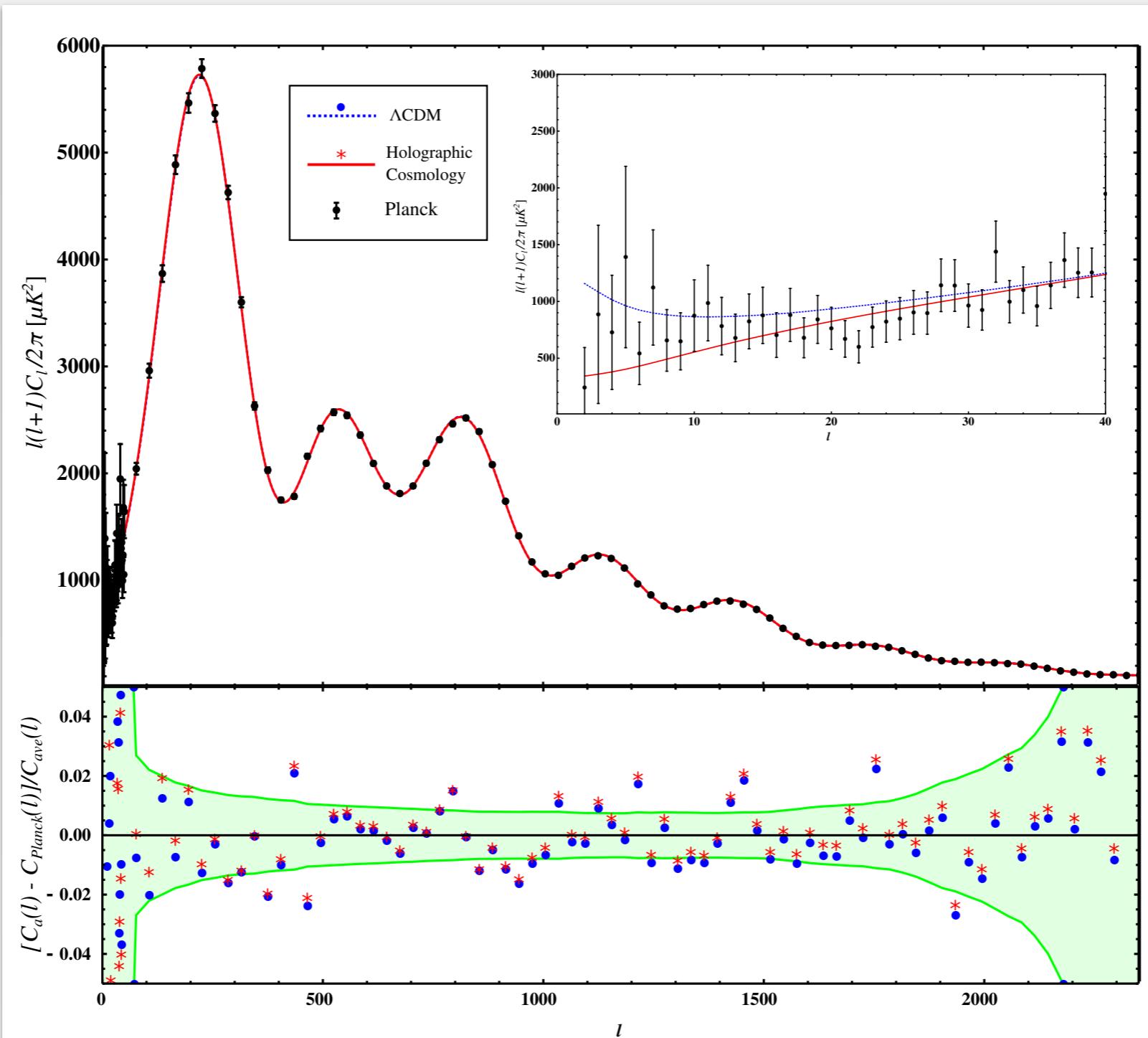
3 cosmological parameters

- Reminder: in Λ CDM

$$\Delta_{\mathcal{R}}^2(q) = \Delta_0^2 \left(\frac{q}{q^*} \right)^{n_s - 1}$$

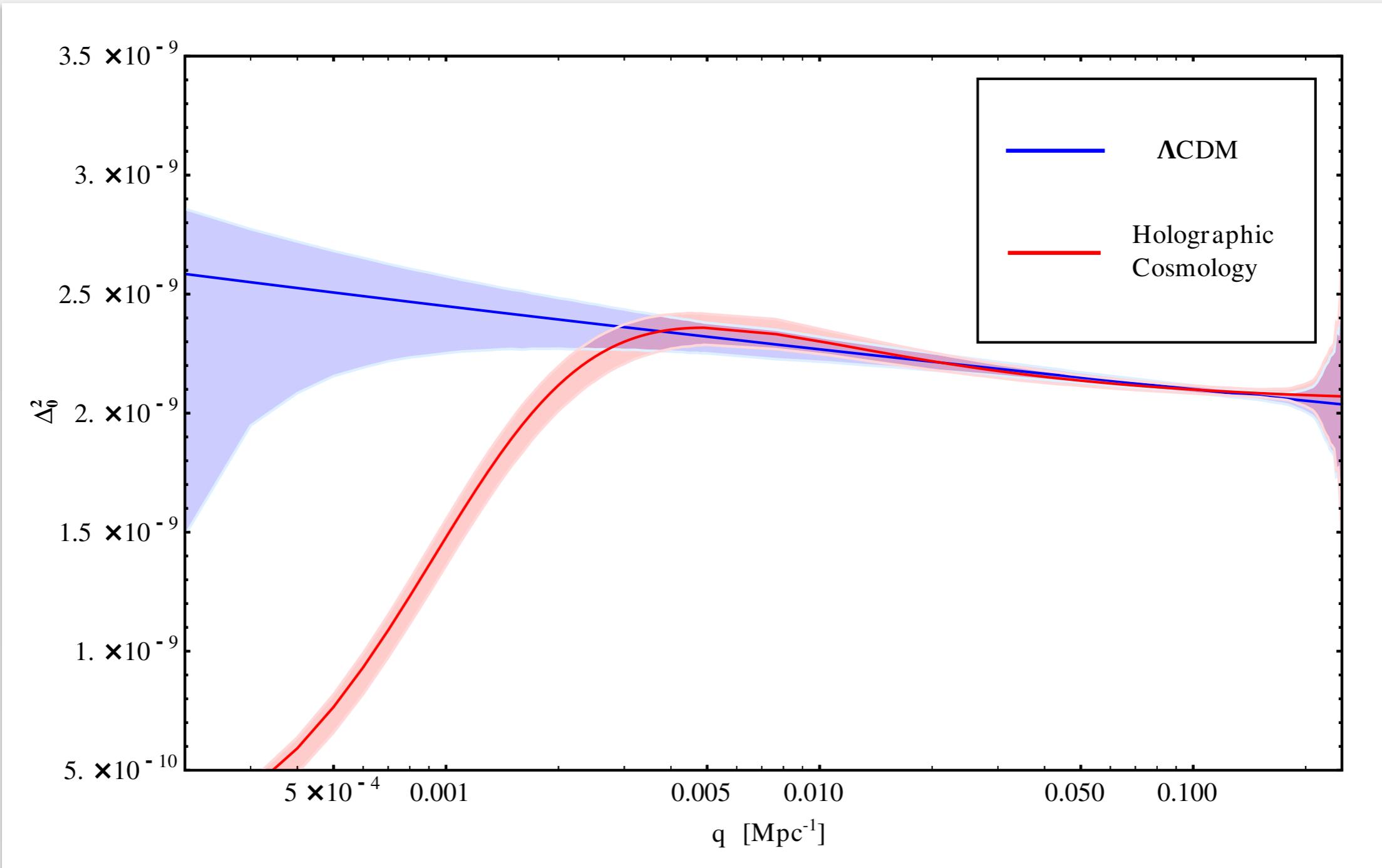
2 cosmological parameters (3 with running)

Confrontation to Planck data



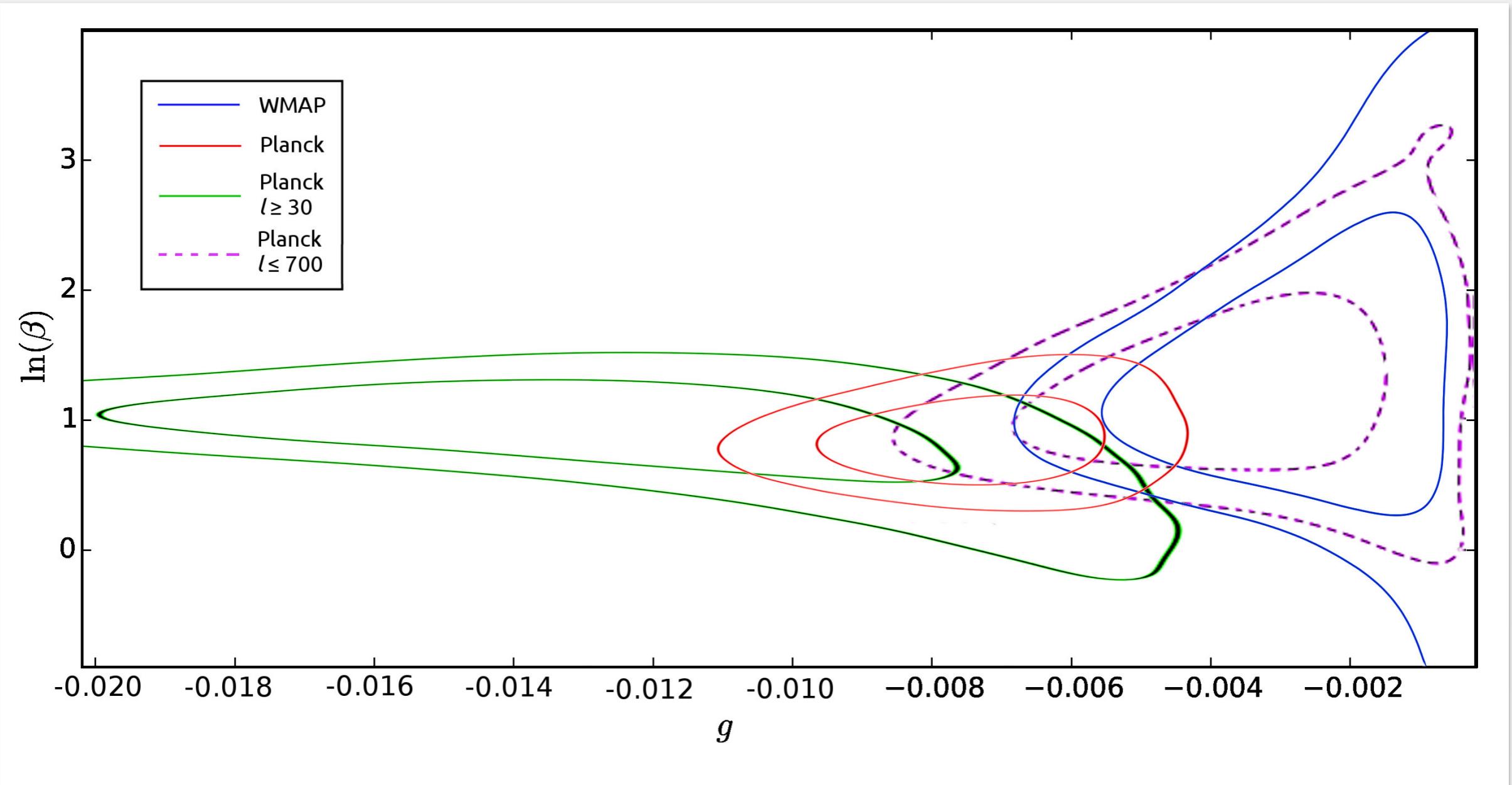
[Afshordi et al., PRL 118(4) & PRD 95(1), 2017]

Confrontation to Planck data



[Afshordi et al., PRL 118(4) & PRD 95(1), 2017]

Confrontation to Planck data



[Afshordi *et al.*, PRL 118(4) & PRD 95(1), 2017]

Conclusions of Planck analysis

- Competitive with Λ CDM, 3 cosmological parameters.
- Model selection: no fermions, large N and large number of nearly conformal scalars.
- The dual theory become non-perturbative around $l \sim 35$
Low-multipole region cannot be trusted.
- The dual theory has IR divergences which are believed to be an artefact of perturbation theory.
- Clear motivations for a non-perturbative calculation.

A first step: scalar $SU(N)$ model

Definition

- 3D ϕ^4 theory with ϕ a $\mathfrak{su}(N)$ -valued field

$$\mathcal{L} = \text{tr}[(\partial_\mu \phi)(\partial_\mu \phi) + \lambda \phi^4] + \text{c.t.}$$

- $[\phi] = \frac{1}{2}$ & $[\lambda] = 1$: super-renormalisable.
- Perturbative regime given by $\lambda/\Lambda_{\text{ext.}} \rightarrow 0$.
- With t'Hooft scaling

$$\mathcal{L} = \frac{N}{g} \text{tr}[(\partial_\mu \phi)(\partial_\mu \phi) + \phi^4] + \text{c.t.} \quad ([\phi] = 1 \quad [g] = 1)$$

Properties

- Linear UV divergence in the mass, no coupling renormalisation.
- UV deg. of div. **decreases** by 1 at each order in PT.
- IR deg. of div. **increases** by 1 at each order in PT.
- IR divergences are believed to be an artefact of PT regulated by the coupling g .

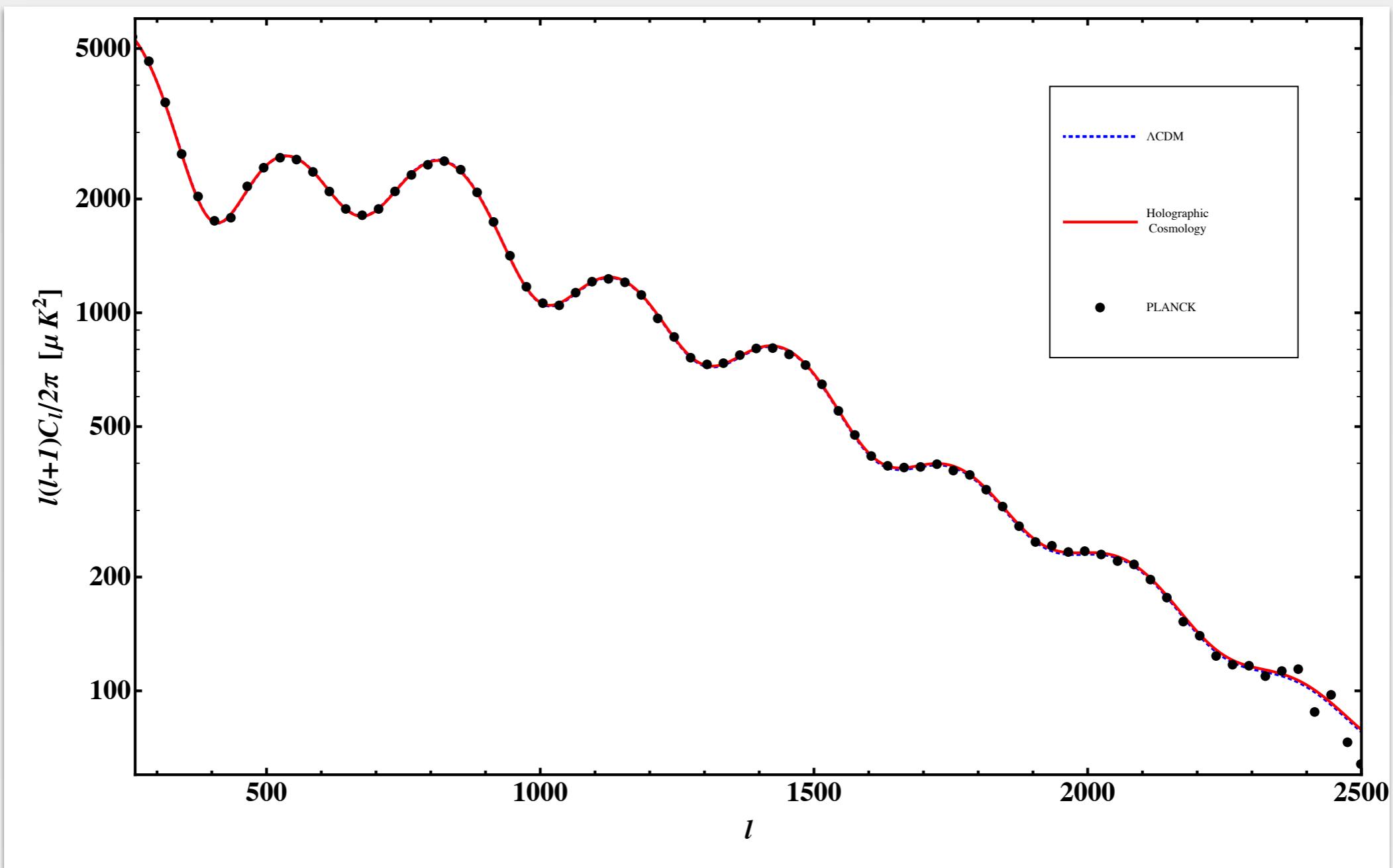
2-loop example

$\text{tr}(\phi^2)$ 2-pt function:

$$\begin{array}{c} \text{Diagram: } \text{A circle with two black dots on the boundary.} \\ \sim \frac{g}{|p|} \end{array}$$
$$\begin{array}{c} \text{Diagram: } \text{Two circles connected by a horizontal line segment between their rightmost points.} \\ + \quad \text{Diagram: } \text{A circle with two black dots on the boundary, with a small loop attached to the top edge.} \\ + \quad \text{Diagram: } \text{A circle with two black dots on the boundary, with a small square box on the top edge.} \\ \sim \frac{g^2}{|p|^2} \end{array}$$

- Expansion driven by $g_{\text{eff.}} = g/|p|$.
- Leading large- N corrections at $O(1/N^2)$.

Just a toy model?



PT with $l > 260$ compatible with Λ CDM.
What about low multipoles?

Non-perturbative lattice simulations

Lattice field theory

$$\mathcal{L} = \frac{N}{g} \text{tr}[(\delta_\mu \phi)(\delta_\mu \phi) + \delta_{m^2} \phi^2 + \phi^4]$$

- δ_μ : forward derivative — δ_{m^2} : mass counter-term.
- No coupling running, continuum limit given by $ag \rightarrow 0$ while keeping the renormalised mass at 0.
- Mass counter-term linearly divergent $\delta_{m^2} \sim \frac{g}{a}$.

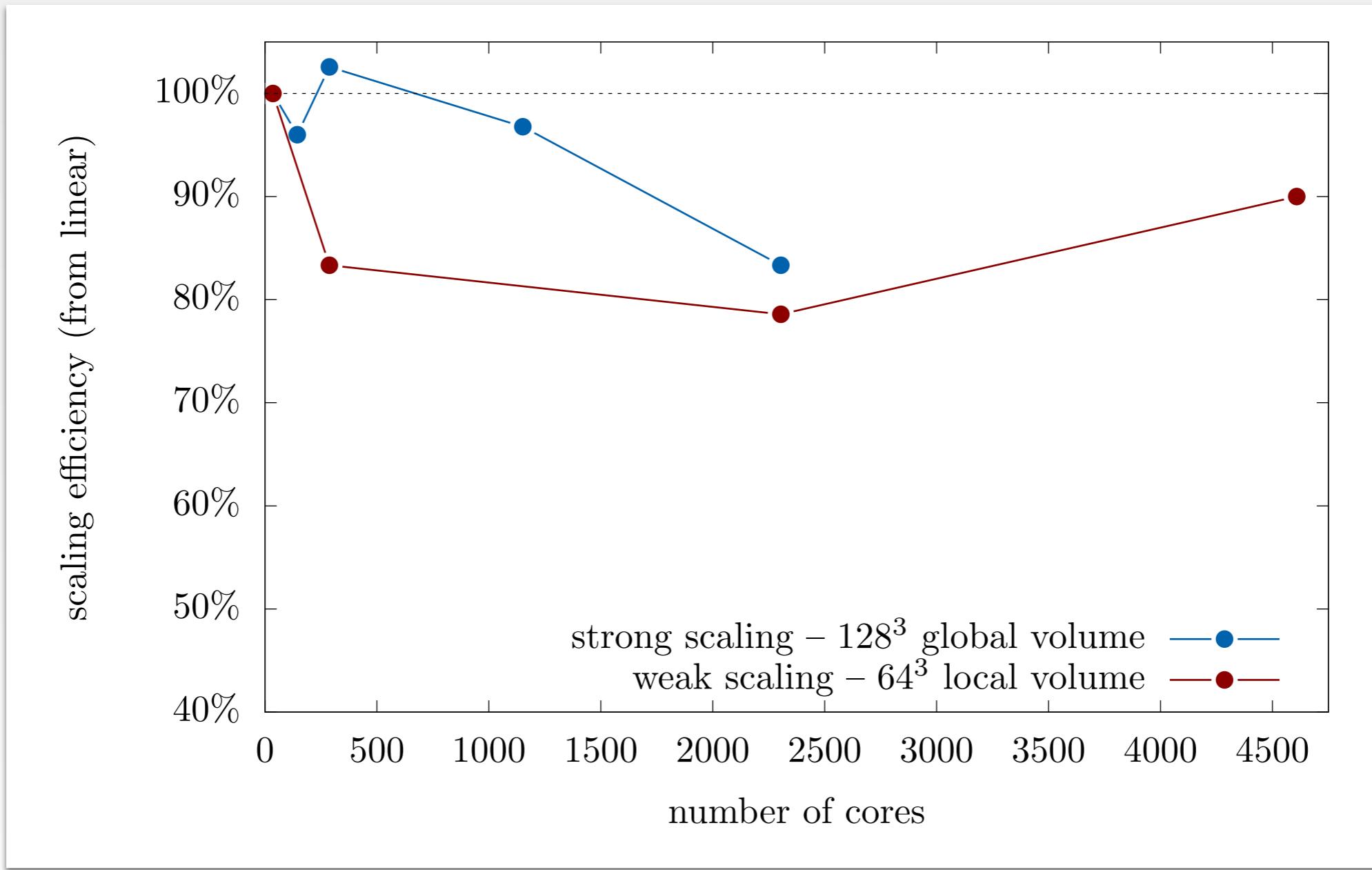
Numerical setup

- HMC implemented using Grid.

<https://www.github.com/paboyle/grid> [P. Boyle, A.P. et. al, arXiv:1512.03487]

- Grid: highly-optimised data parallel C++ library.
- 100% vectorised on many architectures including Intel AVX2 & AVX512. GPU support experimental.
- Full performance on Intel OPA networks.

Numerical setup



LLNL Quartz cluster (Intel Xeon Haswell + OPA)

Critical mass determination

- The mass renormalises through **linear UV divergence**.
- Critical mass: value of the bare mass for which the propagator pole is 0 (infinite correlation length).
- Finite-volume: no phase transition. But we can look for the critical mass through **finite-size scaling**.
- Good probe for FSS: **Binder Cumulant**.

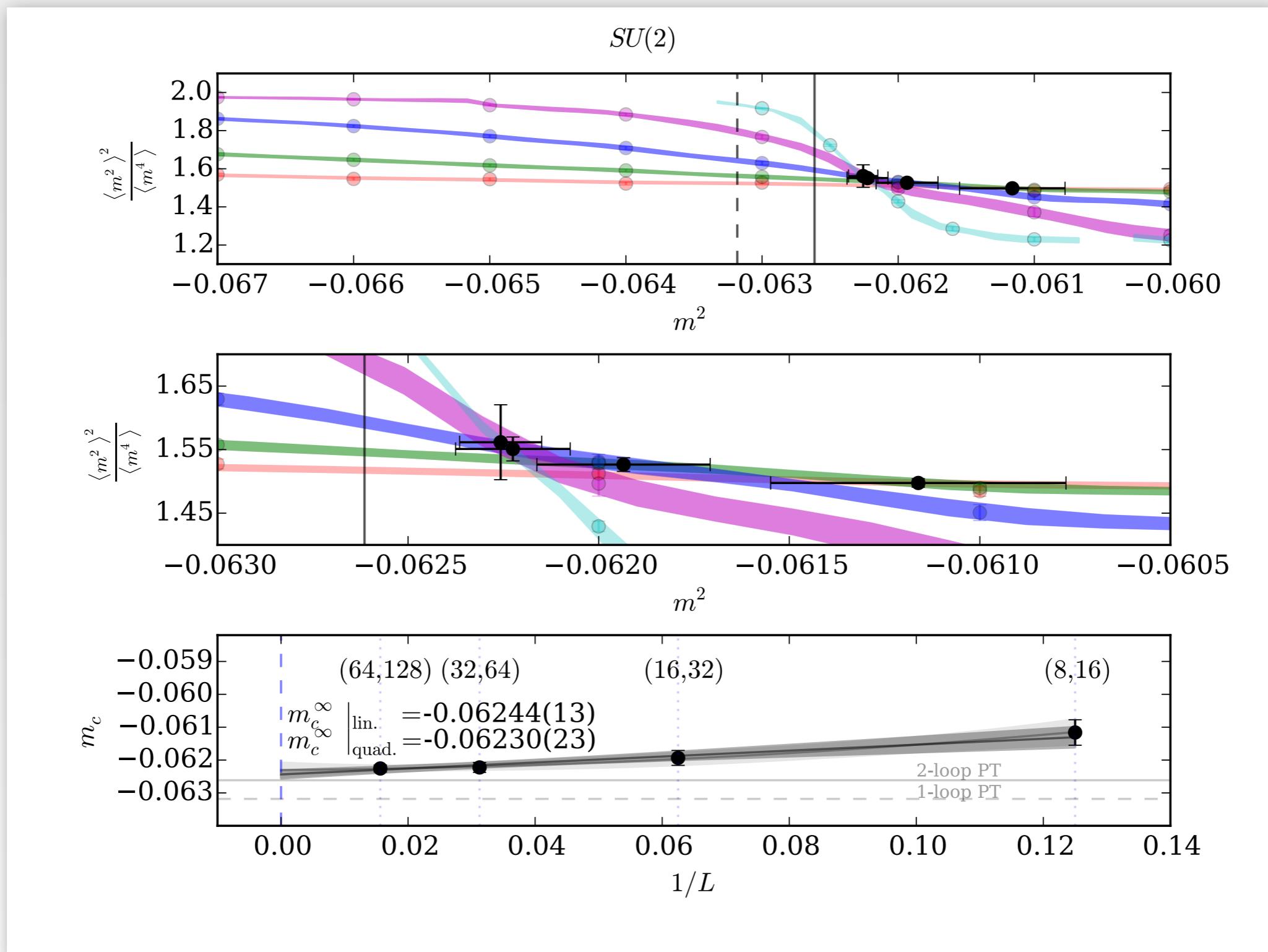
Critical mass determination

- Good probe for FSS: Binder Cumulant.

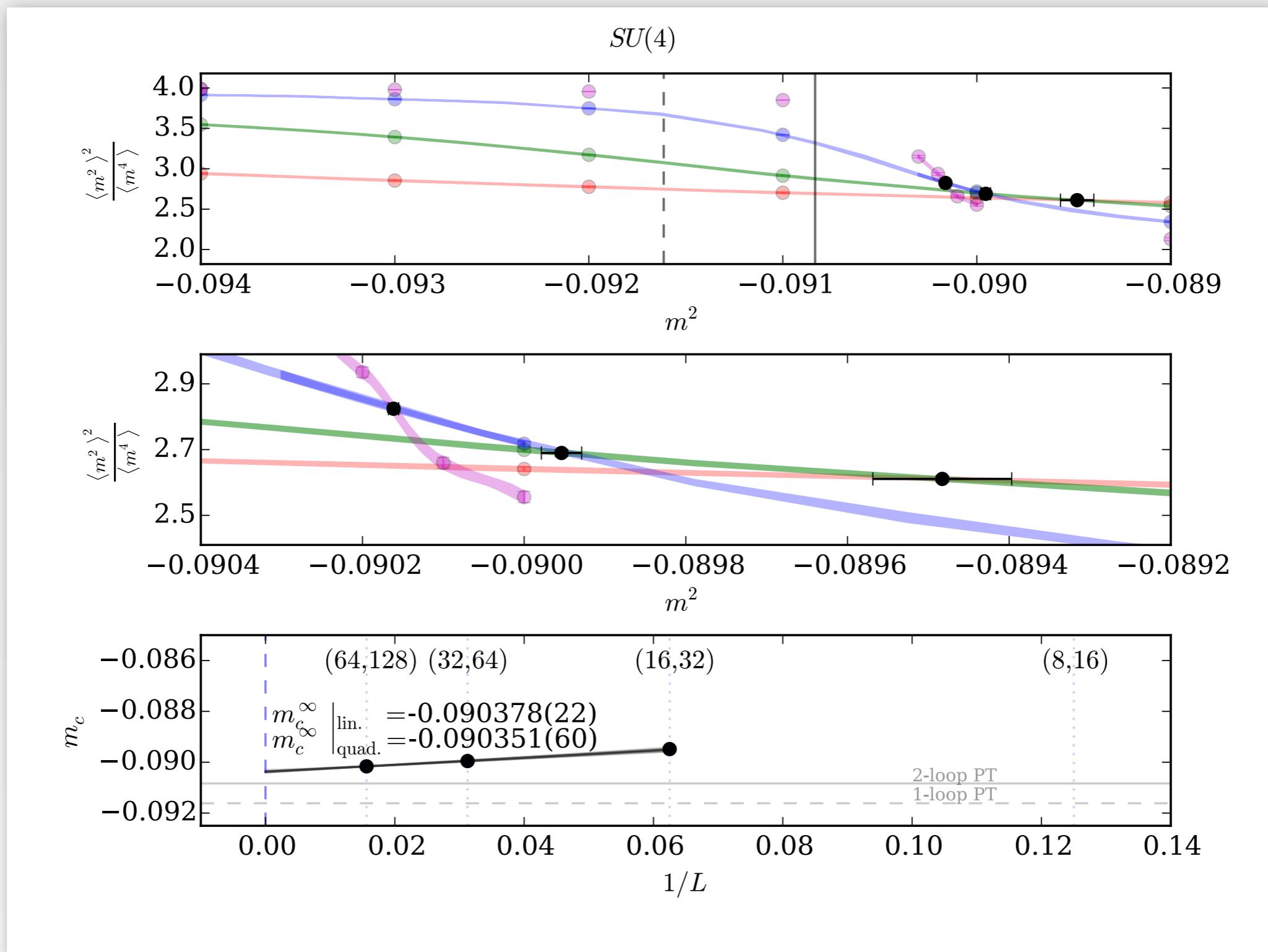
$$B = \frac{\langle \text{tr}(M^4) \rangle}{\langle \text{tr}(M^2) \rangle^2} = g((m_0^2 - m_c^2)L^{1/\nu}), \quad M = \frac{1}{L^3} \int d^3x \phi(x)$$

- Weakly dependent on volume at the critical point.
- Strategy: find m_0^2 which equate B for two different volumes and extrapolate to infinite volume.

Critical mass: N=2 ag=0.2



Critical mass: N=4 ag=0.2



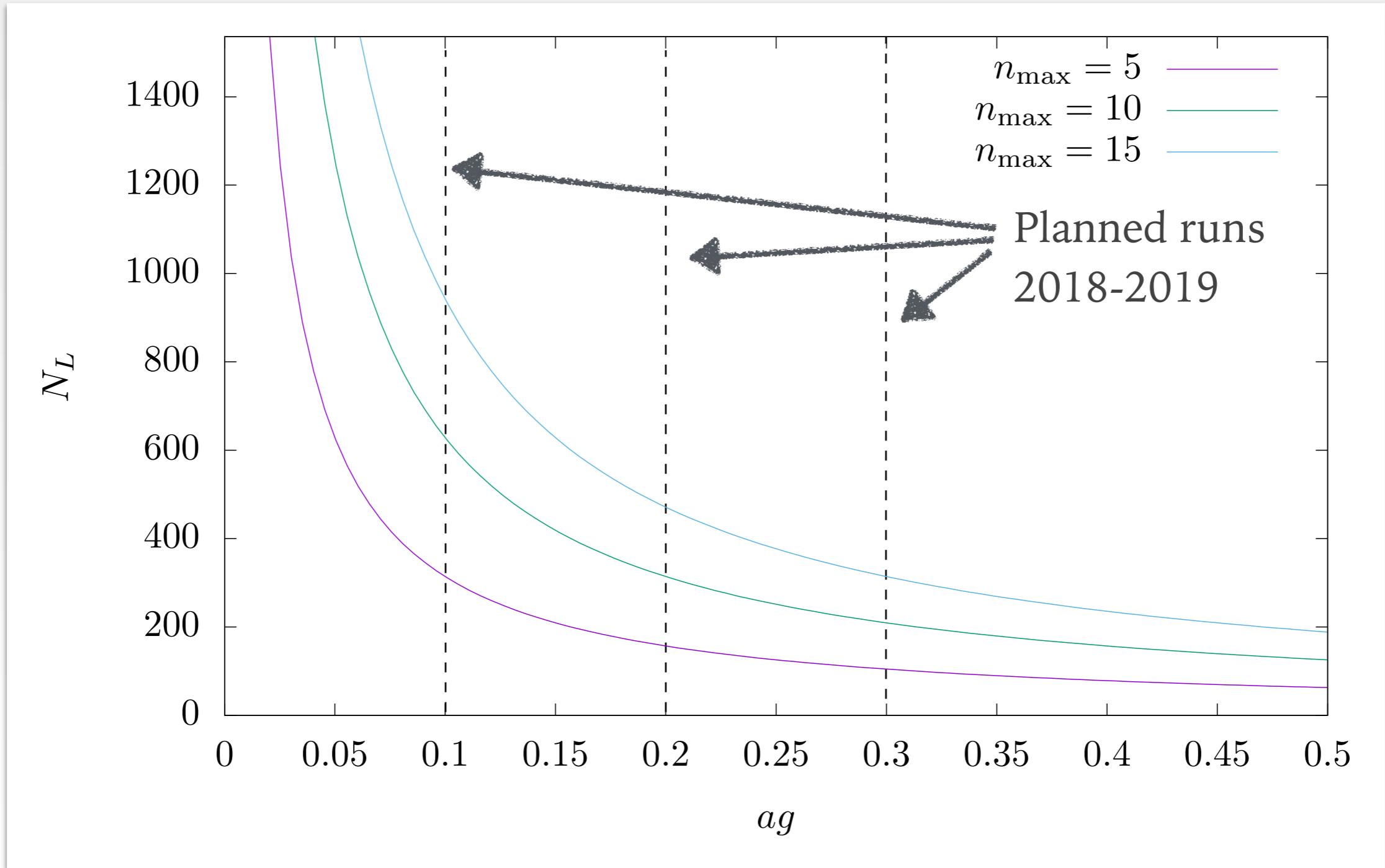
The non-perturbative window

- The perturbative expansion breaks down for $g/|p| \sim 1$.
- Challenge: one wants

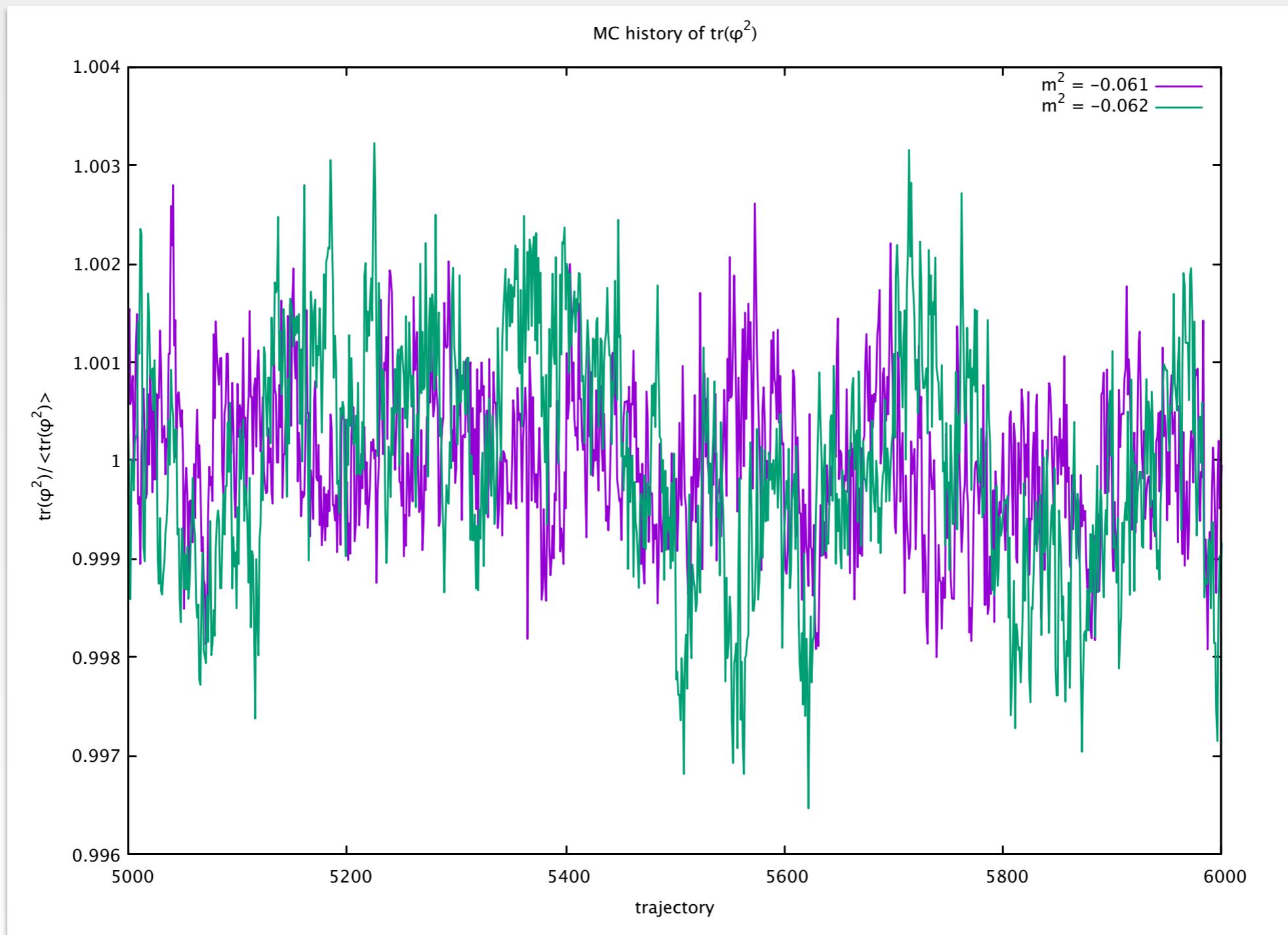
$$\frac{agN_L}{2\pi} \gg 1 : \text{non-perturbative low modes}$$

$ag \ll 1$: continuum scaling regime

The non-perturbative window

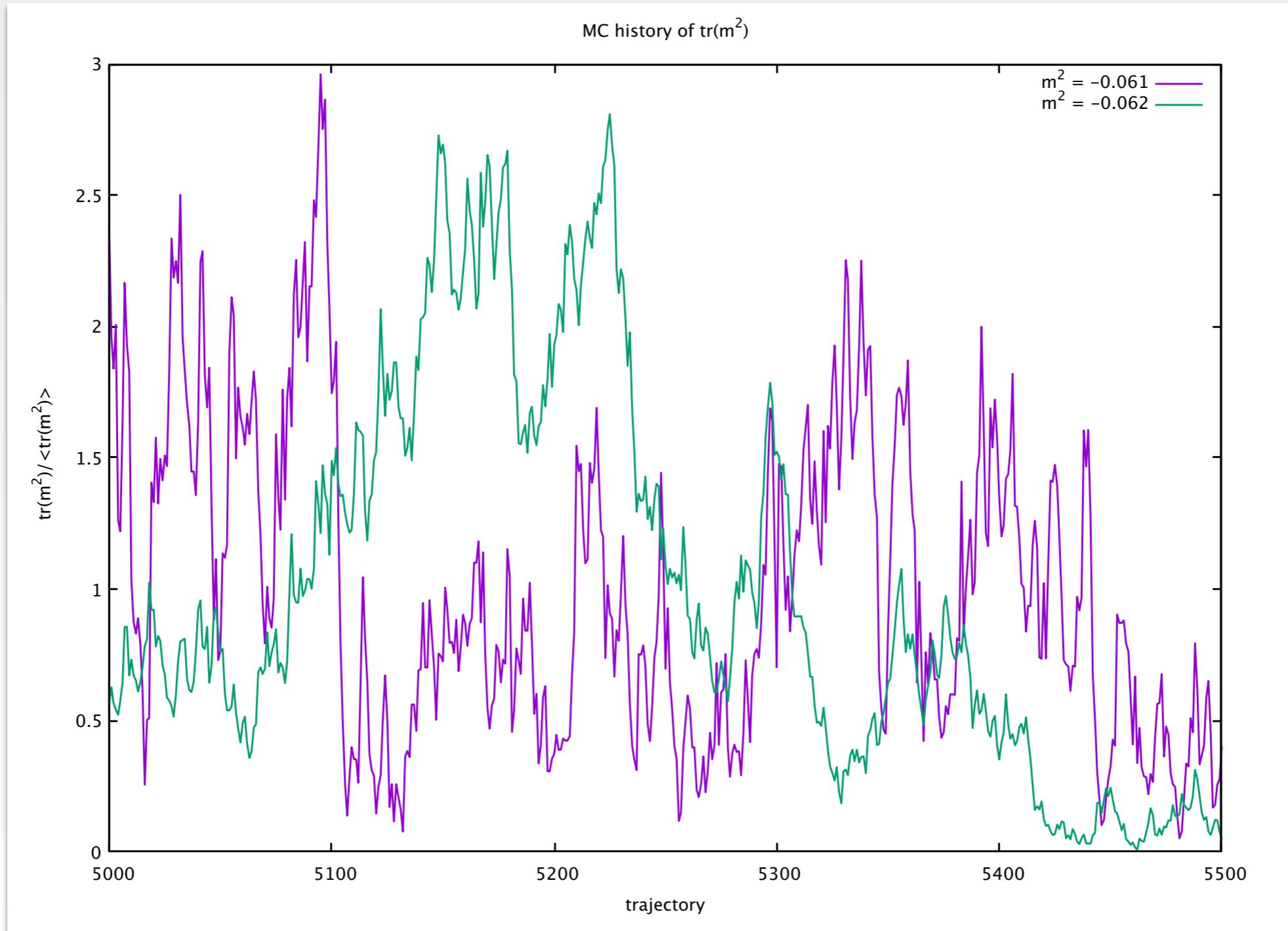


HMC near the critical point



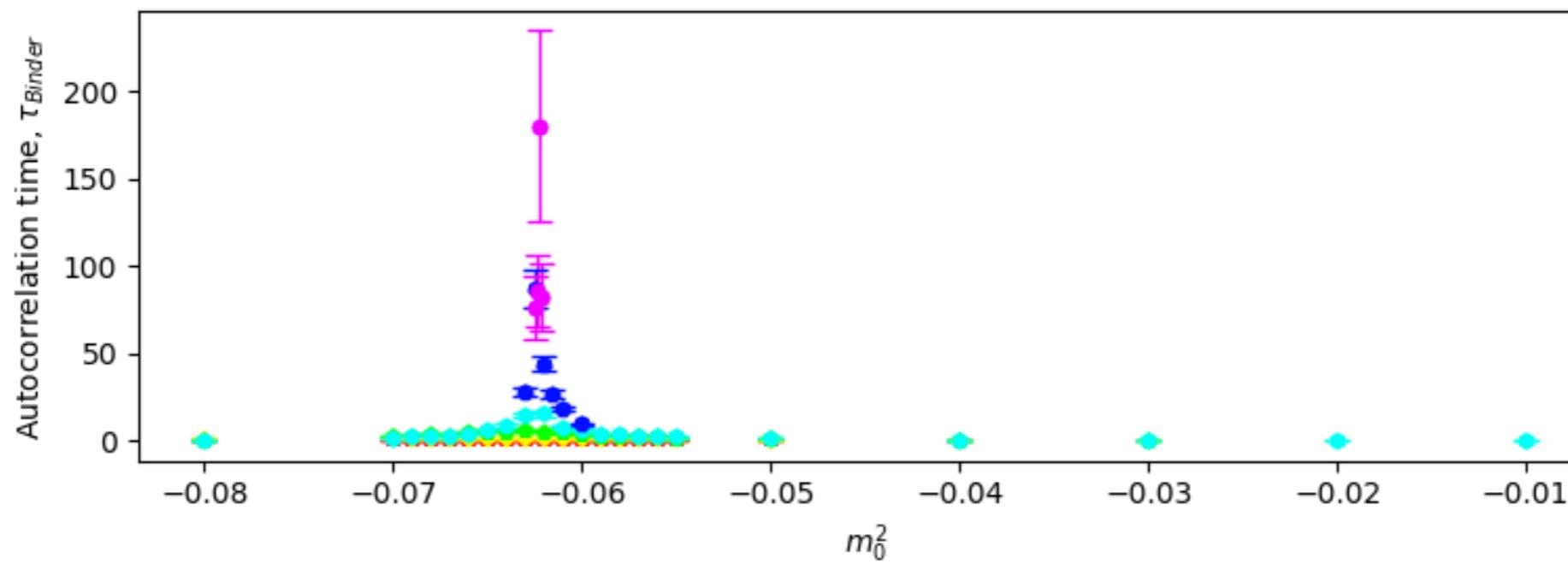
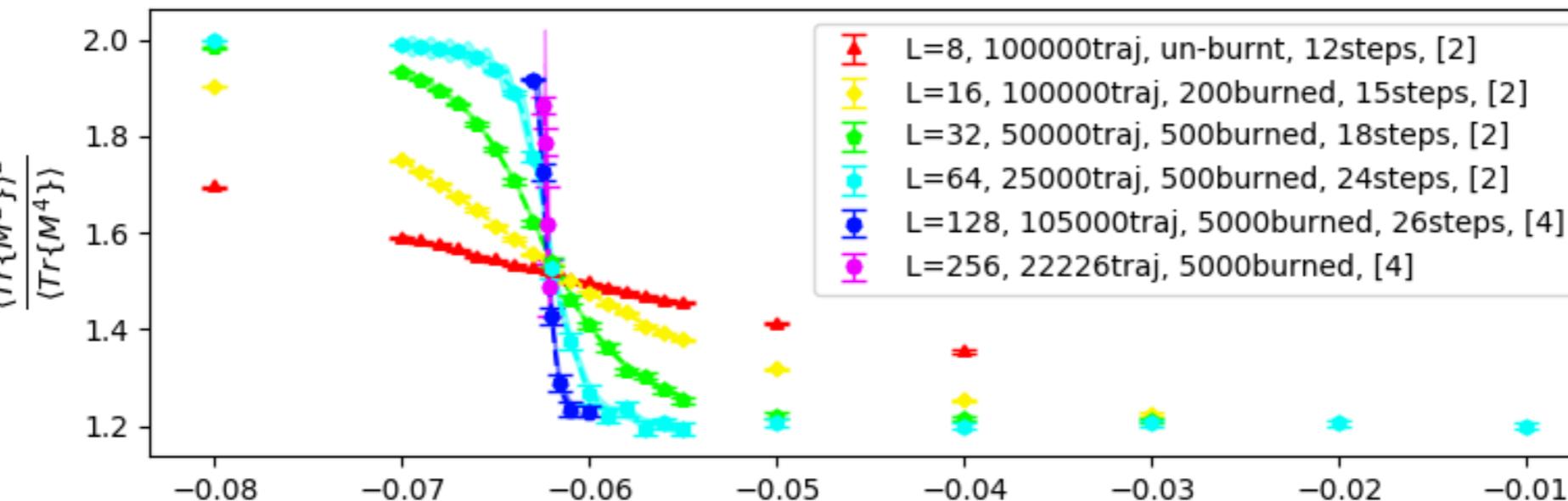
256^3 SU(2) lattice with $g = 0.2$, critical at $m^2 = -0.0625$

HMC near the critical point



256^3 SU(2) lattice with $g = 0.2$, critical at $m^2 = -0.0625$

HMC near the critical point



SU(2) lattice with $g = 0.2$, critical at $m^2 = -0.0625$

Renormalisation of the energy-momentum tensor

Continuum Ward identity

- Local translations

$$\phi(x) \xrightarrow{\alpha} \phi[x + \alpha(x)] = \exp[\alpha(x) \cdot \partial] \phi(x)$$

- Noether current in direction ν .

$$T_{\mu\nu} = \frac{N}{g} \text{tr} \left\{ 2(\partial_\mu \phi)(\partial_\nu \phi) + \delta_{\mu\nu} \left[(\partial_\rho \phi)(\partial_\rho \phi) + m^2 \phi^2 + \phi^4 \right] \right\}$$

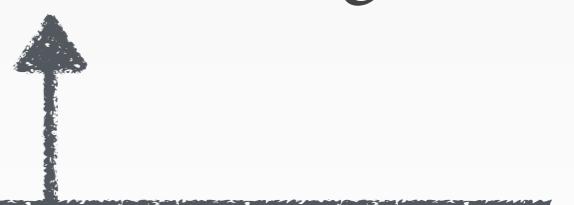
- Ward identity

$$\sum_\mu \partial_\mu \langle T_{\mu\nu}(x) P(x_1, \dots, x_n) \rangle = - \left\langle \frac{\delta P(x_1, \dots, x_n)}{\delta \alpha_\nu(x)} \right\rangle$$

- $T_{\mu\nu}$ does not renormalise.

Lattice Ward identity

- No infinitesimal translations,
use transformation $\phi(x) \xrightarrow{\alpha} \exp[\alpha(x) \cdot \bar{\delta}] \phi(x)$
- Central derivative $\bar{\delta}$: unitary transformation
- Same EMT with $\partial_\mu \mapsto \bar{\delta}_\mu$.
- Ward identity Contains cubic divergences

$$\bar{\delta}_\mu \langle T_{\mu\nu}(x) P(x_1, \dots, x_n) \rangle = - \left\langle \frac{\delta P(x_1, \dots, x_n)}{\delta \alpha_\nu(x)} \right\rangle + a^2 \boxed{\langle X_\nu(x) P(x_1, \dots, x_n) \rangle}$$


Renormalisation scheme

- On the lattice, the EMT mixes with

$$O_{\mu\nu}^{(1)} = \text{tr}[(\bar{\delta}_\mu \phi)(\bar{\delta}_\nu \phi)]$$

$$O_{\mu\nu}^{(2)} = \delta_{\mu\nu} \text{tr}[(\bar{\delta}_\rho \phi)(\bar{\delta}_\rho \phi)]$$

$$O_{\mu\nu}^{(3)} = \delta_{\mu\nu} \text{tr}(\phi^2)$$

$$O_{\mu\nu}^{(4)} = \delta_{\mu\nu} \text{tr}(\phi^4)$$

$$O_{\mu\nu}^{(5)} = \delta_{\mu\nu} \text{tr}[(\bar{\delta}_\nu \phi)(\bar{\delta}_\nu \phi)]$$

- Condition: the renormalised tensor

$$T_{\mu\nu}^{(R)} = T_{\mu\nu} + \sum_{j=1}^5 c_j(a) O_{\mu\nu}^{(j)}$$

has to verify the continuum Ward identity.

RI/MOM renormalisation

- RI/MOM condition for amputated vertex functions
- $$\Lambda_{T,\mu\nu}^{(2)}(p,p) + \sum_j c_j(a) \Lambda_{j,\mu\nu}^{(2)}(p,p) = -\delta_{\mu\nu} D(p)^{-1}$$
- [Caracciolo *et al.*, NPB 309(4), 1988]: identical at $p^2 = 0$.
 - Heavily relies on IR-finiteness, important to check.
 - Should be derived for 4 and 6 legs, 5 coefficients to determine.

Signal-to-noise issues

- Non-zero momentum data extremely noisy.
- Zero transfer momentum clean but requires huge disconnected subtractions: noisy again.
- Need to investigate variance reduction literature.
 - e.g. some symmetries are only true in average and can be enforced configuration per configuration.

Conclusion & perspectives

Conclusion

- CMB: real world data to constrain holographic conjectures. Exciting!
- Dual theory has a non-perturbative IR regime.
- Lattice simulations can provide a complete answer.
- IR/UV hierarchy challenge looks doable with current supercomputing technology.
- Additional lattice complication: broken EMT needs renormalisation.

Perspectives

- Achieving precise EMT renormalisation.
- Renormalise the EMT 2-pt function.
- Study finite-volume & continuum scaling.
- Confront to low-multipole Planck data.
- Think about introducing gauge fields.

Thank you!