Form factors for *b* hadron decays from lattice QCD

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Why search for new physics in b decays?

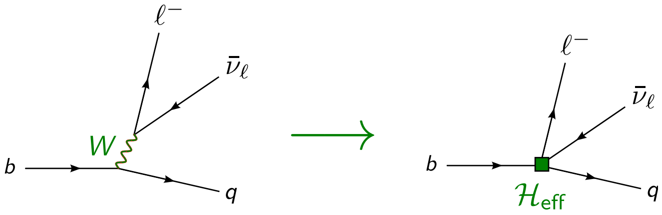
- The b is the heaviest quark that forms hadrons. Consequently there are many possible decay channels (also with τ leptons).

- The dominant decays are already CKM-suppressed:

$$|V_{cb}|^2 \approx 0.0017, \;\; |V_{ub}|^2 \approx 0.000014.$$

- CP-violating effects can be very large.

Effective weak Hamiltonian for $b \to q\ell^-\bar{\nu}_\ell$ decays



$$\mathcal{H}_\text{eff} = \frac{G_F}{\sqrt{2}} V_{qb}\ \bar{q}\gamma^\mu(1-\gamma_5)b\ \bar{\ell}\gamma_\mu(1-\gamma_5)\nu$$

+additional Dirac structures beyond the SM

Hadronic matrix elements for $b \to q\ell^- \bar{\nu}_\ell$ decays

Exclusive $H_b \to H_q \ell^- \bar{\nu}_\ell$:

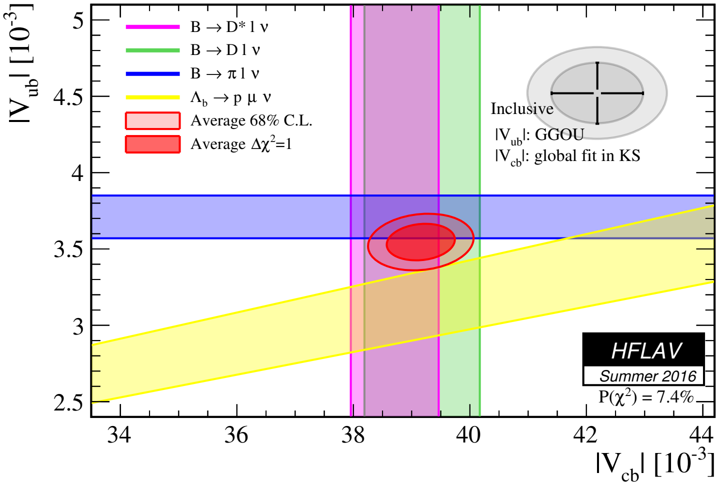
$$\langle H_q(p') | J_\mu | H_b(p) \rangle$$

Inclusive $H_b \to X_q \ell^- \bar{\nu}_\ell$:

$$\mathrm{Im}\left[-i \int \mathrm{d}^4 x \, e^{-iq\cdot x} \langle H_b(p) | \, \mathrm{T} J_\mu^\dagger(x) J_\nu(0) \, | H_b(p) \rangle \right]$$

where $J_\mu = \bar{q}\gamma_\mu(1-\gamma_5)b$

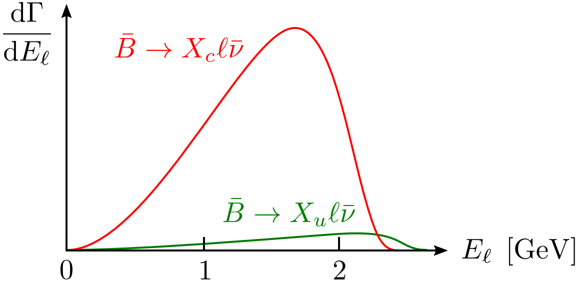
$|V_{ub}|$ and $|V_{cb}|$, 2016



[http://www.slac.stanford.edu/xorg/hflav/semi/summer16/html/ExclusiveVub/exclVubVcb.html]

Note: the $\bar{B} \to D^{(*)}\ell\bar{\nu}_\ell$ results shown here used extrapolation to zero recoil with the CLN form factor parametrization.

Inclusive B decay lepton energy spectra (schematic)

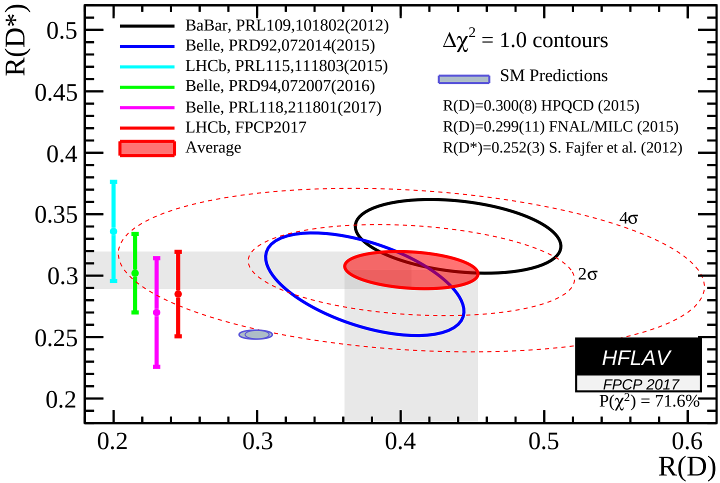


(Not to scale. $\bar{B} \to X_u \ell \bar{\nu}$ rate is actually even lower.)

Can lattice QCD predict the shapes?

[M. Hansen, H. Meyer, D. Robaina, arXiv:1704.08993/PRD 2017]

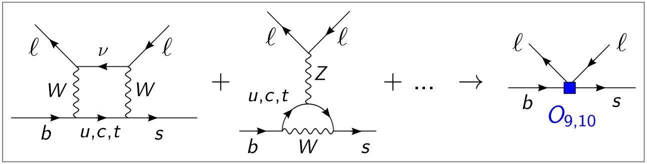
$R(D^{(*)}) = \Gamma(B \to D^{(*)}\tau\bar{\nu})/\Gamma(B \to D^{(*)}\ell\bar{\nu})$, 2017

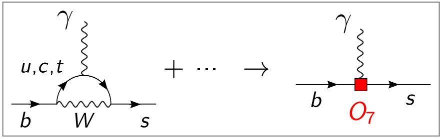


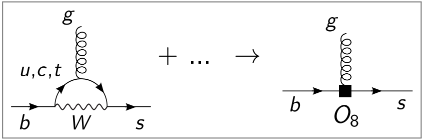
[http://www.slac.stanford.edu/xorg/hflav/semi/fpcp17/RDRDs.html]

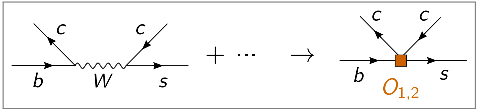
Note: the SM prediction for $R(D^*)$ used $\bar{B} \to D^{(*)}\ell\bar{\nu}_\ell$ experimental data and the CLN form factor parametrization.

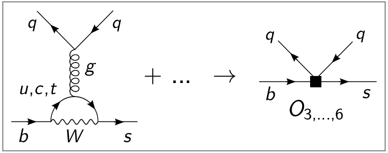
Effective weak Hamiltonian for $b \to s\ell^+\ell^-$ decays











Effective weak Hamiltonian for $b \to s\ell^+\ell^-$ decays

$$\mathcal{H}_{\text{eff}} = -\frac{4G_F}{\sqrt{2}} V_{tb} V_{ts}^* \sum_i C_i O_i$$

with

$$\begin{aligned}
O_1 &= \bar{c}^b \gamma^\mu b_L^a \;\; \bar{s}^a \gamma_\mu c_L^b, \\
O_2 &= \bar{c}^a \gamma^\mu b_L^a \;\; \bar{s}^b \gamma_\mu c_L^b, \\
O_7 &= \frac{e\, m_b}{16\pi^2} \;\bar{s}\sigma^{\mu\nu} b_R \;\; F_{\mu\nu}^{(\text{e.m.})}, \\
O_9 &= \frac{e^2}{16\pi^2} \;\bar{s}\gamma^\mu b_L \;\; \bar{\ell}\gamma_\mu \ell, \\
O_{10} &= \frac{e^2}{16\pi^2} \;\bar{s}\gamma^\mu b_L \;\; \bar{\ell}\gamma_\mu \gamma_5 \ell, \\
&\ldots
\end{aligned}$$

In the Standard Model, $\overline{\text{MS}}$ scheme, at $\mu = 4.2$ GeV,

C_1	C_2	C_7	C_9	C_{10}	...
-0.288	1.010	-0.336	4.275	-4.160	...

[Computed using EOS, `https://eos.github.io/`]

Hadronic matrix elements for exclusive $b \to s\ell^+\ell^-$ decays

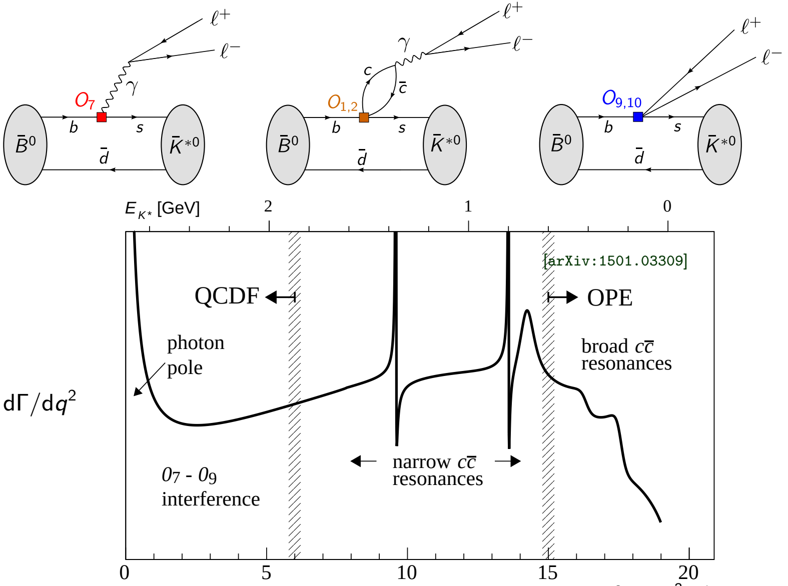
O_7, O_9, O_{10}:

$$\langle H_s(p')| \, \bar{s}\Gamma b \, |H_b(p)\rangle$$

$O_{1,\ldots,6}, O_8$:

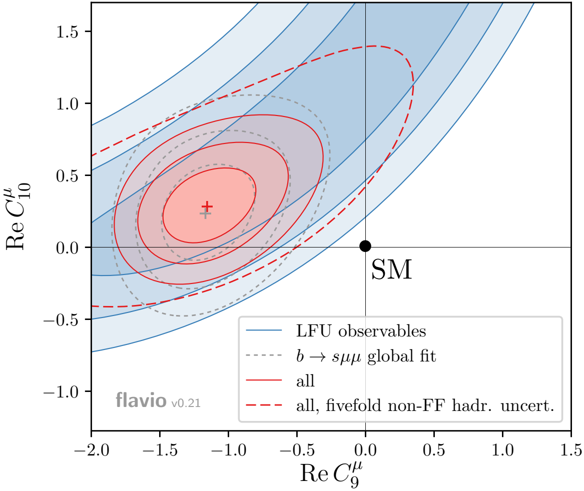
$$\int \mathrm{d}^4x \;\; e^{iq\cdot x} \, \langle H_s(p')| \, \mathrm{T} \; O_i(0) \; J^\mu_{\mathrm{e.m.}}(x) \, |H_b(p)\rangle$$

Hadronic matrix elements for exclusive $b \to s\ell^+\ell^-$ decays



(schematic only)

$b \to s\ell^+\ell^-$: Fit of C_9^μ and C_{10}^μ to experimental data (mesons only), 2017



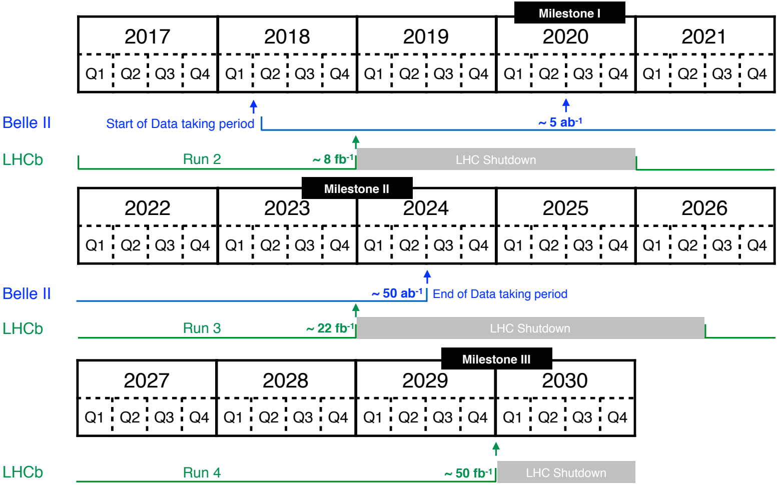
LFU observables:

$$R_{K^{(*)}} = \frac{\int_{q^2_{\text{start}}}^{q^2_{\text{stop}}} \frac{d\Gamma(B \to K^{(*)}\mu^+\mu^-)}{dq^2} dq^2}{\int_{q^2_{\text{start}}}^{q^2_{\text{stop}}} \frac{d\Gamma(B \to K^{(*)}e^+e^-)}{dq^2} dq^2}$$

[W. Altmannshofer, P. Stangl, D. M. Straub, arXiv:1704.05435/PRD 2017]

Note: in the lattice QCD calculation of $B \to K^*$ form factors, the K^* was treated as if it were stable.

Belle II and LHCb timeline



[J. Albrecht *et al.*, arXiv:1709.10308]

Lattice methods for b quarks: overview

Challenge: wide range of scales

$$m_\pi \approx 0.1 \text{ GeV}, \;\; m_b \approx 5 \text{ GeV}$$

Approaches:

Special heavy-quark lattice action for the b

- Lattice HQET
- Lattice NRQCD/mNRQCD
- Wilson-like actions with m_Q-dependent, anisotropic coefficients

Same action for b as for light quarks

- Use very fine lattice spacings and/or extrapolate/interpolate in m_b
- Main advantage: renormalization simplified or unnecessary $\rightarrow$ smaller systematic uncertainty

Large momenta of final-state light mesons are also challenging.

Lattice HQET

Leading-order HQET in rest frame:

$$S_\psi = \delta m \int \mathrm{d}^4 x \;\; \psi^\dagger \psi + \int \mathrm{d}^4 x \;\; \psi^\dagger D_0 \, \psi$$

Lattice discretization:

$$S_{\psi,\mathrm{lat.}} = \sum_x \psi^\dagger(x) \big[(1+\delta m)\psi(x) - U_0^\dagger(x-\hat{0})\psi(x-\hat{0}) \big]$$

[E. Eichten, B. Hill, PLB **240**, 193 (1990)]

Higher-order $1/m$ corrections, starting with

$$\mathcal{L}_\psi^{(1)} = \psi^\dagger \left[-\frac{\mathbf{D}^2}{2m} - g \frac{\boldsymbol{\sigma} \cdot \mathbf{B}}{2m} \right] \psi,$$

are treated as insertions in correlation functions, with nonperturbative renormalization. This means that the theory remains renormalizable, and one can go to the continuum limit.

[J. Heitger, R. Sommer, arXiv:hep-lat/0310035/JHEP 2004]

Lattice HQET can only be used for singly-heavy hadrons.

Lattice NRQCD

Continuum action (with tree-level matching coefficients):

$$\mathcal{S}_\psi = \int d^4x \; \psi^\dagger \Big[\underbrace{D_0 - \frac{\mathbf{D}^2}{2m}}_{\mathcal{O}(v^2)}$$

$$\underbrace{- \frac{g}{2m}\boldsymbol{\sigma}\cdot\mathbf{B} + \frac{g}{8m^2}\left(i\mathbf{D}^{\mathrm{ad}}\cdot\mathbf{E} - \boldsymbol{\sigma}\cdot(\mathbf{D}\times\mathbf{E} - \mathbf{E}\times\mathbf{D})\right) - \frac{\mathbf{D}^4}{8m^3}}_{\mathcal{O}(v^4)} + \ldots \Big]\psi$$

Here, the power counting indicated is based on v^2, the average heavy-quark velocity-squared inside heavy quarkonium. For bottomonium, $v^2 \approx 0.1$.

Lattice NRQCD is a discretization of this, where all terms are kept in the action [G. P. Lepage, L. Magnea, C. Nakhleh, U. Magnea, K. Hornbostel, arXiv:hep-lat/9205007/PRD 1992].
One must keep $am \gtrsim 1$ in the simulations.

Matching coefficients for the action and for currents have been computed using one-loop lattice perturbation theory.

[See for example C. Monahan, J. Shigemitsu, R. Horgan, arXiv:1211.6966/PRD 2013;

R. Dowdall, C. Davies, T. Hammant, R. Horgan, C. Hughes, arXiv:1309.5797/PRD 2014]

Lattice "moving NRQCD"

This is a Lorentz-boosted version of lattice NRQCD. The **v** below is the boost velocity, not the power-counting parameter discussed before. The continuum action of mNRQCD (with tree-level matching coefficients) is

$$\begin{aligned} S &= \int \mathrm{d}^4x \; \psi_v^\dagger \Bigg[D_0 - i\mathbf{v}\cdot\mathbf{D} - \frac{\mathbf{D}^2 - (\mathbf{v}\cdot\mathbf{D})^2}{2\gamma m} - \frac{g}{2\gamma m}\boldsymbol{\sigma}\cdot\mathbf{B}' \\ &\quad - \frac{i}{4\gamma^2 m^2}\left(\left\{\mathbf{v}\cdot\mathbf{D},\ \mathbf{D}^2\right\} - 2(\mathbf{v}\cdot\mathbf{D})^3\right) + \frac{g}{8m^2}\left(i\mathbf{D}^{\mathrm{ad}}\cdot\mathbf{E} + \mathbf{v}\cdot(\mathbf{D}^{\mathrm{ad}}\times\mathbf{B})\right) \\ &\quad - \frac{g}{8\gamma m^2}\,\boldsymbol{\sigma}\cdot\left(\mathbf{D}\times\mathbf{E}' - \mathbf{E}'\times\mathbf{D}\right) + \frac{g}{8(\gamma+1)m^2}\left\{\mathbf{v}\cdot\mathbf{D},\ \boldsymbol{\sigma}\cdot(\mathbf{v}\times\mathbf{E}')\right\} \\ &\quad - \frac{(2-\mathbf{v}^2)g}{16m^2}\left(D_0^{\mathrm{ad}} + i\mathbf{v}\cdot\mathbf{D}^{\mathrm{ad}}\right)(\mathbf{v}\cdot\mathbf{E}) - \frac{ig}{4\gamma^2 m^2}\left\{\mathbf{v}\cdot\mathbf{D},\ \boldsymbol{\sigma}\cdot\mathbf{B}'\right\} \\ &\quad - \frac{1}{8\gamma^3 m^3}\left(\mathbf{D}^4 - 3\left\{\mathbf{D}^2,\ (\mathbf{v}\cdot\mathbf{D})^2\right\} + 5(\mathbf{v}\cdot\mathbf{D})^4\right) + \ldots \Bigg]\,\psi_v. \end{aligned}$$

Lattice mNRQCD allows to give high momentum ($> a^{-1}$) to b-hadrons on the lattice while keeping discretization errors under control.

[R. R. Horgan, L. Khomskii, S. Meinel, M. Wingate, K. M. Foley, G. P. Lepage, G. M. von Hippel, A. Hart, E. H. Mller, C. T. H. Davies, A. Dougall, K. Y. Wong, arXiv:0906.0945/PRD 2009]

Wilson-like actions with m_Q-dependent, anisotropic coefficients

Wilson-like actions have a smooth heavy-quark limit. Cutoff effects can be understood and (partially) removed using HQET/NRQCD analysis.

[A. El-Khadra, A. Kronfeld, P. Mackenzie, arXiv:hep-lat/9604004/PRD 1997;

A. Kronfeld, arXiv:hep-lat/0002008/PRD 2000;

J. Harada, S. Hashimoto, K.-I. Ishikawa, A. Kronfeld, T. Onogi, N. Yamada, arXiv:hep-lat/0112044/PRD 2002;

J. Harada, S. Hashimoto, A. Kronfeld, T. Onogi, arXiv:hep-lat/0112045/PRD 2002]

- Fermilab approach: am tuned to yield correct heavy-light meson kinetic mass (the energy at zero momentum is irrelevant).
- Columbia approach: (also known as RHQ action): am, ν, and c_P tuned to yield correct heavy-light meson kinetic mass, rest mass, and hyperfine splitting [N. Christ, M. Li, H.-W. Lin, arXiv:hep-lat/0608006/PRD 2007]
- Oktay-Kronfeld action: adds dimension-6 and dimension-7 operators [M. Oktay, A. Kronfeld, arXiv:0803.0523/PRD 2008]
- Matching of currents is usually done with the "mostly nonperturbative method":

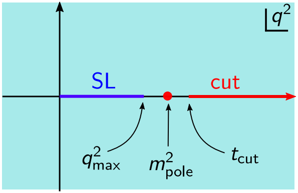
$$J_\Gamma = \underbrace{\sqrt{Z_V^{(qq)} Z_V^{(bb)}}}_{\text{nonperturbative}} \rho_\Gamma \left[\bar{q}\Gamma b + \mathcal{O}(a) \text{ improvement terms} \right]$$

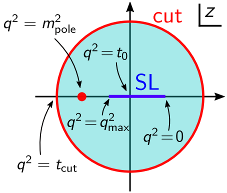
The z expansion

To fit the q^2 $[= (p - p')^2]$ dependence of form factors in a model-independent way, it is convenient to consider them as functions of a new dimensionless variable z, defined as

$$z = \frac{\sqrt{t_{\text{cut}} - q^2} - \sqrt{t_{\text{cut}} - t_0}}{\sqrt{t_{\text{cut}} - q^2} + \sqrt{t_{\text{cut}} - t_0}}$$

This maps the complex q^2 plane, cut along the real axis for $q^2 > t_{\text{cut}}$, to the interior of the unit disk:



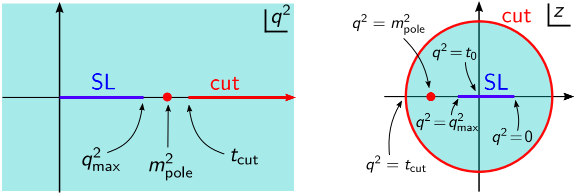


The BCL "simplified" z expansion for a form factor with a single pole below t_{cut} reads

$$f(q^2) = \frac{1}{1 - q^2/m^2_{\text{pole}}} \sum_{k=0}^{\infty} a_k\, z^k$$

[C. Bourrely, I. Caprini, L. Lellouch, arXiv:0807.2722/PRD 2009]

The z expansion



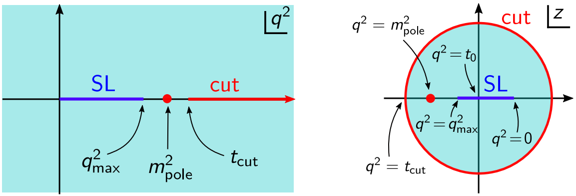
$$f(q^2) = \frac{1}{1 - q^2/m^2_{pole}} \sum_{k=0}^{\infty} a_k \, z^k$$

Analyticity guarantees convergence. Unitarity provides bounds on the sizes of the coefficients a_k. It is sufficient to keep only the first few terms of the series.

The unitarity bounds take a simple form in the original BGL variant of the z expansion, which however requires a complicated "outer function".

[C. Boyd, B. Grinstein, R. Lebed, arXiv:hep-ph/9412324/PRL 1995]

The z expansion



Example: the form factor $f_+(\Lambda_b \to p)$

- t_{cut} is set to the onset location of the two-particle branch cut created by the current $J^\mu = \bar{u}\gamma^\mu b$,

$$t_{\text{cut}} = (m_B + m_\pi)^2$$

- m_{pole} is set to the mass of the $J^P = 1^-$ bound state created by the current $J^\mu = \bar{u}\gamma^\mu b$,

$$m_{\text{pole}} = m_{B*}$$

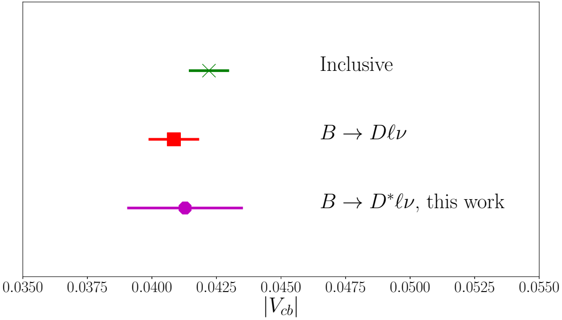
- t_0 determines which value of q^2 gets mapped to $z = 0$. I used

$$t_0 = q^2_{\text{max}} = (m_{\Lambda_b} - m_p)^2$$

News on $|V_{cb}|$

The 2016 HFLAV exclusive determinations of $|V_{cb}|$ used extrapolations of the experimental data for the $\bar{B} \to D^{(*)}\ell\bar{\nu}$ differential decay rates to zero recoil with the one-parameter CLN form factor parametrizations [I. Caprini, L. Lellouch, M. Neubert, arXiv:hep-ph/9712417/NPB 1998], where the shapes are fixed by HQET and dispersive bounds.

Using the less constrained BGL z expansion [C. Boyd, B. Grinstein, R. Lebed, arXiv:hep-ph/9412324/PRL 1995] (and new Belle data for the angular distribution of $\bar{B} \to D^{*}\ell\bar{\nu}$ [arXiv:1702.01521]) gives larger values for $|V_{cb}|$ that are closer to the inclusive value.

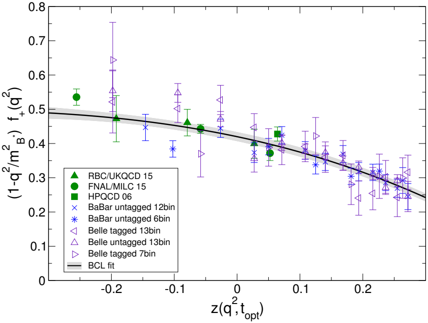


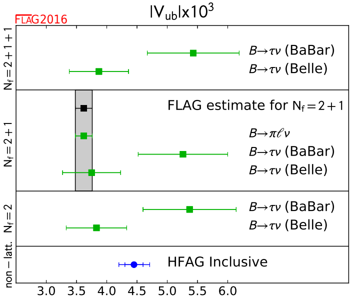
[J. Harrison, C. Davies, M. Wingate, arXiv:1711.11013/PRD 2018]

In the remainder of this section, I will only discuss heavy-to-light meson form factors (for lack of time).

$B \to \pi$ form factors

Reference	LQ action	HQ action	m_π [MeV]	a [fm]	Notes
RBC/UKQCD `arXiv:1501.05373`/PRD	DWF	RHQ	290 - 420	0.08, 0.11	
FNAL/MILC `arXiv:1503.07839`/PRD `arXiv:1507.01618`/PRL	AsqTad	Fermilab	175 - 420	0.045, 0.06, 0.09, 0.12	$B \to \pi \ell^+ \ell^-$
FNAL/MILC `arXiv:1710.09442`(proc.)	HISQ	Fermilab	**135** - 300	0.09, 0.12, 0.15	
HPQCD `arXiv:1510.07446`/PRD	HISQ	NRQCD	**135** - 300	0.09, 0.12, 0.15	zero recoil
HPQCD C. Bouchard, Beauty 2018	HISQ/AsqTad	NRQCD	280 - 520	0.09, 0.12	entire q^2 range
JLQCD `arXiv:1710.07094`(proc.)	DWF	DWF	300 - 500	0.044, 0.055, 0.08	

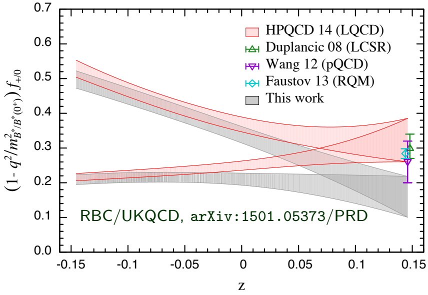




$B_s \to K$ form factors

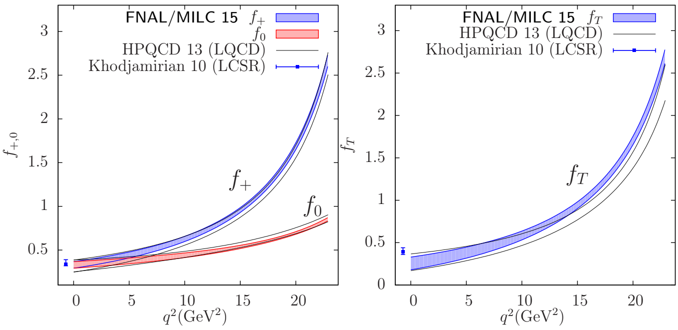
LHCb will measure a ratio of $B_s \to K\ell\bar{\nu}$ and $B_s \to D_s\ell\bar{\nu}$ decay rates, which will allow a new determination of $|V_{ub}/V_{cb}|$ [M. Calvi, Talk at Challenges in semileptonic B decays workshop, MITP, 2018].

Reference	LQ action	HQ action	m_π [MeV]	a [fm]	Notes
HPQCD arXiv:1406.2279/PRD	AsqTad	NRQCD	175 - 300	0.09, 0.12	
RBC/UKQCD arXiv:1501.05373/PRD	DWF	RHQ	290 - 420	0.08, 0.11	
ALPHA arXiv:1601.04277/PLB	Clover	HQET	175 - 420	0.05, 0.065, 0.075	leading order
arXiv:1711.01158(proc.)					order $1/m$
FNAL/MILC arXiv:1711.08085(proc.)	AsqTad	Fermilab	175 - 420	0.06, 0.09, 0.12	
FNAL/MILC arXiv:1710.09442(proc.)	HISQ	Fermilab	**135** - 300	0.09, 0.12, 0.15	



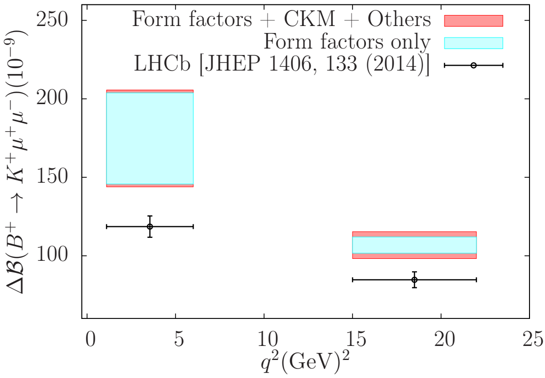
$B \to K$ form factors

Reference	LQ action	HQ action	m_π [MeV]	a [fm]
HPQCD `arXiv:1306.2384`/PRD	AsqTad	NRQCD	270 - 400	0.09, 0.12
FNAL/MILC `arXiv:1509.06235`/PRD	AsqTad	Fermilab	175 - 420	0.045, 0.06, 0.09, 0.12
FNAL/MILC `arXiv:1710.09442`(proc.)	HISQ	Fermilab	**135** - 300	0.09, 0.12, 0.15



$B^+ \to K^+\mu^+\mu^-$ differential branching fraction

Contributions from $O_{1\ldots6;8}$ treated with OPE and QCDF.



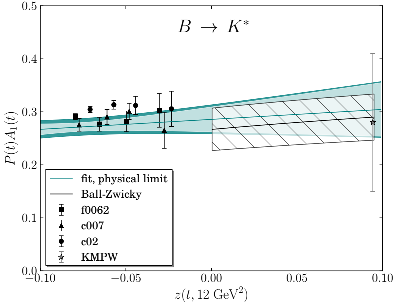
[D. Du *et al.* (FNAL/MILC), arXiv:1510.02349/PRD 2016]

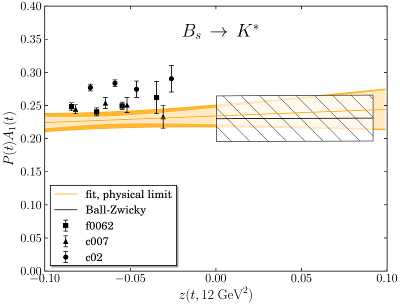
$B \to K^*$ and $B_s \to K^*$ form factors

Reference	LQ action	HQ action	m_π [MeV]	a [fm]	Notes
Cambridge group arXiv:1310.3722/PRD	AsqTad	NRQCD	310 - 520	0.09, 0.12	K^* treated as stable

Note: earlier, quenched calculations by other groups (see references in arXiv:1310.3722).

Seven form factors; only one example shown below.

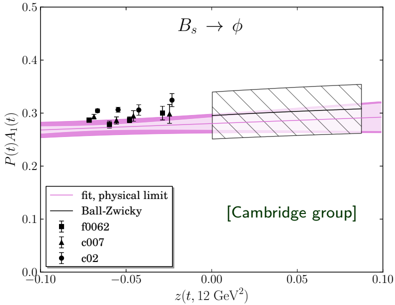


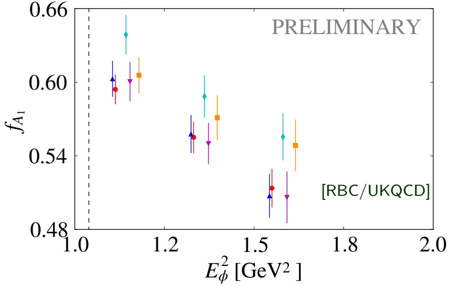


$B_s \to \phi$ form factors

Reference	LQ action	HQ action	m_π [MeV]	a [fm]	Notes
Cambridge group arXiv:1310.3722/PRD	AsqTad	NRQCD	310 - 520	0.09, 0.12	ϕ treated as stable
RBC/UKQCD arXiv:1612.05112(proc.)	DWF	RHQ	290 - 420	0.08, 0.11	ϕ treated as stable

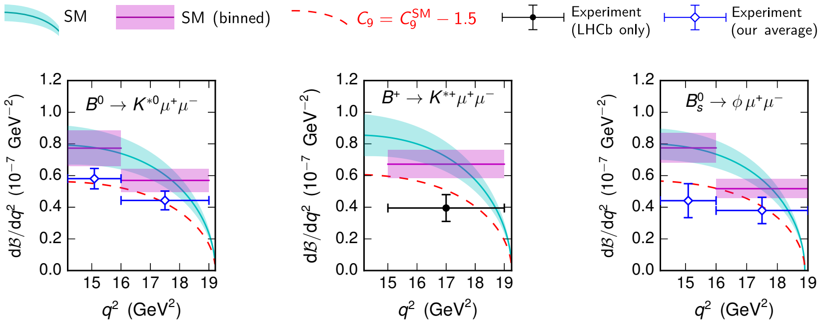
Seven form factors; only one example shown below.





$B \to K^*\mu^+\mu^-$ and $B_s \to \phi\mu^+\mu^-$ differential branching fractions at high q^2

Contributions from $O_{1\ldots6;8}$ treated with OPE.



[R. R. Horgan, Z. Liu, S. Meinel, M. Wingate, arXiv:1310.3887/PRL 2014]

b (and c) baryon decay form factors: overview

Early work on $\Lambda_b \to \Lambda_c$ (quenched, focused on Isgur-Wise function):

K. C. Boweler *et al.* (UKQCD Collaboration), arXiv:hep-lat/9709028/PRD 1998
S. Gottlieb and S. Tamhankar, arXiv:hep-lat/0301022/Lattice 2002

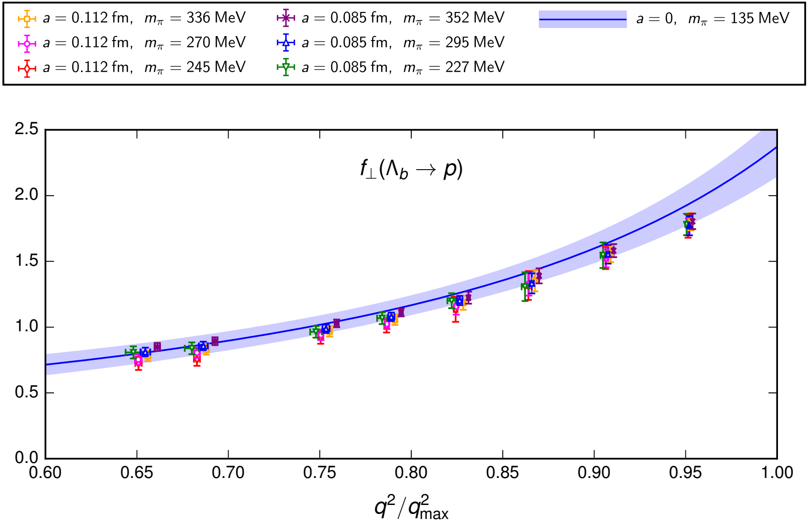
Our work, using RBC/UKQCD $2+1$ flavor DWF ensembles:

Transition	m_b	a [fm]	m_π [MeV]	Reference
$\Lambda_b \to \Lambda$	∞	0.11, 0.08	230–360	WD, DL, SM, MW, arXiv:1212.4827/PRD 2013
$\Lambda_b \to p$	∞	0.11, 0.08	230–360	WD, DL, SM, MW, arXiv:1306.0446/PRD 2013
$\Lambda_b \to p$	phys.	0.11, 0.08	230–360	WD, CL, SM, arXiv:1503.01421/PRD 2015
$\Lambda_b \to \Lambda_c$	phys.	0.11, 0.08	230–360	WD, CL, SM, arXiv:1503.01421/PRD 2015; AD, SK, SM, AR, arXiv:1702.02243/JHEP 2017
$\Lambda_b \to \Lambda$	phys.	0.11, 0.08	230–360	WD, SM, arXiv:1602.01399/PRD 2016
$\Lambda_b \to \Lambda^*$	phys.	0.11	340	SM, GR, arXiv:1608.08110/Lattice 2016
$\Lambda_b \to \Lambda_c^*$	phys.	0.11, 0.08	300–430	SM, GR, Later in this talk
$\Lambda_c \to \Lambda$		0.11, 0.08	**140**–360	SM, arXiv:1611.09696/PRL 2017
$\Lambda_c \to p$		0.11, 0.08	230–360	SM, arXiv:1712.05783/PRD 2018

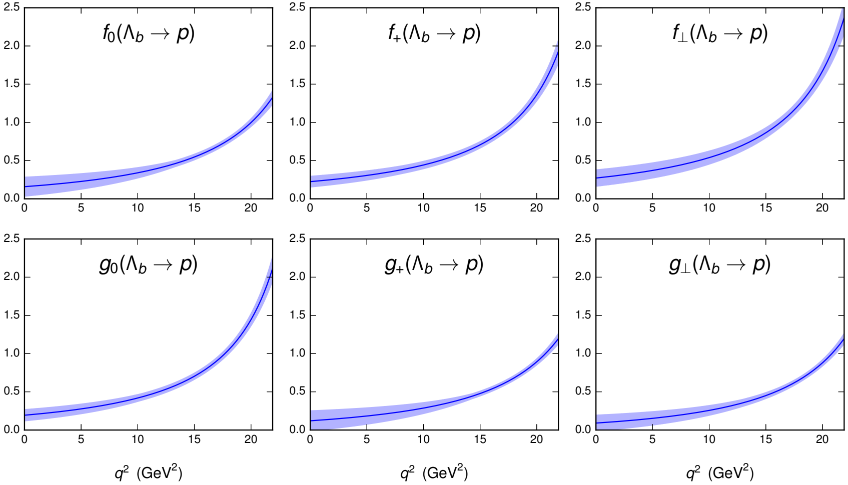
$m_b = \infty$ using Eichten-Hill action with HYP smearing. Current matching with 1-loop PT.
$m_b =$ phys. using RHQ action. Current matching with mostly NPR (perturbative coefficients to 1 loop).

WD = William Detmold
DL = C.-J. David Lin
SM = Stefan Meinel
MW = Matthew Wingate
CL = Christoph Lehner
AD = Alakabha Datta
SK = Saeed Kamali
AR = Ahmed Rashed
GR = Gumaro Rendon (graduate student at U of A)

$\Lambda_b \to p$ form factors

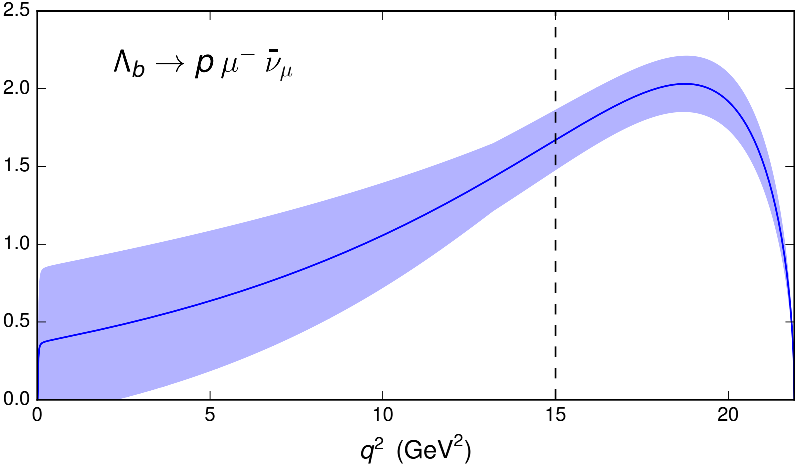


$\Lambda_b \to p$ form factors

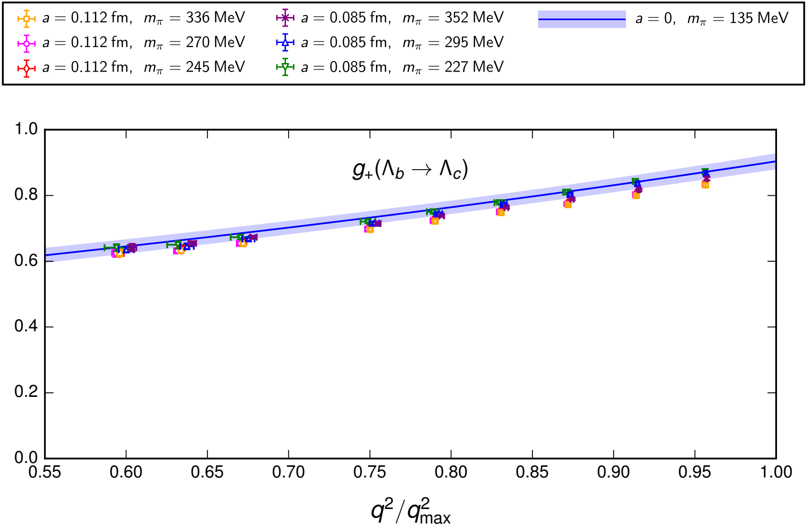


$\Lambda_b \to p\,\mu^-\bar{\nu}_\mu$ differential decay rate

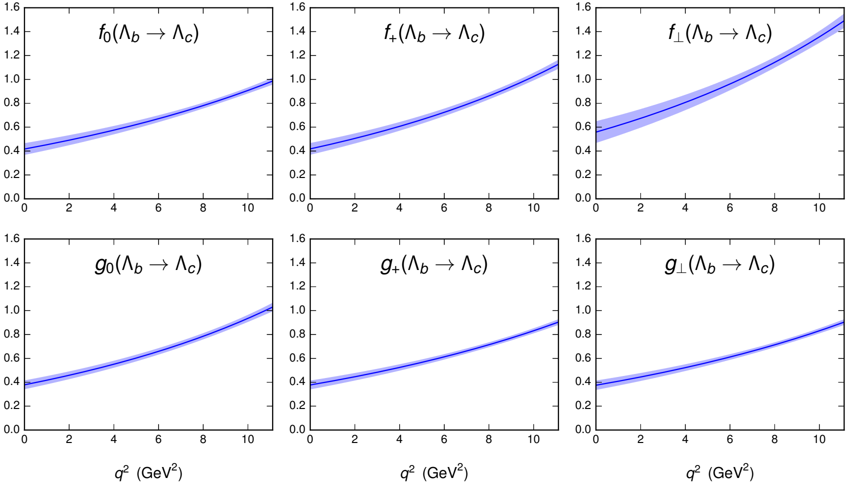
$\frac{d\Gamma/dq^2}{|V_{ub}|^2}$ (ps^{-1} GeV^{-2})



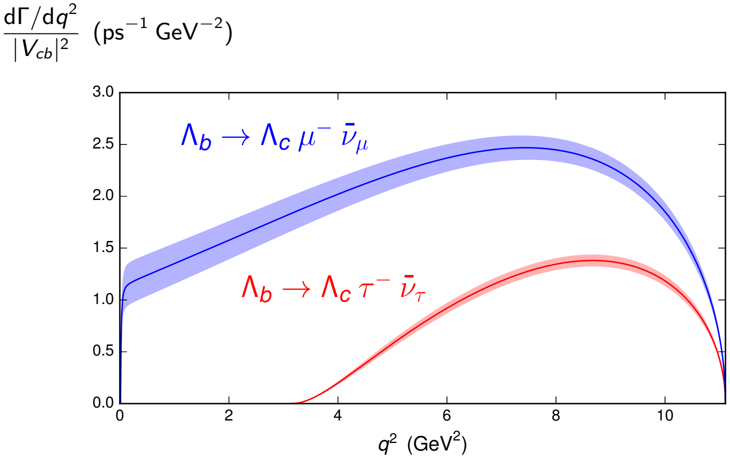
$\Lambda_b \to \Lambda_c$ form factors



$\Lambda_b \to \Lambda_c$ form factors



$\Lambda_b \to \Lambda_c\, \mu^- \bar{\nu}_\mu$ and $\Lambda_b \to \Lambda_c\, \tau^- \bar{\nu}_\tau$ differential decay rates



$$R(\Lambda_c) = \frac{\Gamma(\Lambda_b \to \Lambda_c\, \tau^- \bar{\nu}_\tau)}{\Gamma(\Lambda_b \to \Lambda_c\, \mu^- \bar{\nu}_\mu)} = 0.3328 \pm 0.0074_{\text{stat}} \pm 0.0070_{\text{syst}}$$

Ratio of $\Lambda_b \to p\,\mu^-\bar{\nu}_\mu$ and $\Lambda_b \to \Lambda_c\,\mu^-\bar{\nu}_\mu$ decay rates

Lattice QCD:

$$\frac{\dfrac{1}{|V_{ub}|^2}\displaystyle\int_{15\text{ GeV}^2}^{q^2_{\rm max}} \frac{d\Gamma(\Lambda_b \to p\,\mu^-\bar{\nu}_\mu)}{dq^2}dq^2}{\dfrac{1}{|V_{cb}|^2}\displaystyle\int_{7\text{ GeV}^2}^{q^2_{\rm max}} \frac{d\Gamma(\Lambda_b \to \Lambda_c\,\mu^-\bar{\nu}_\mu)}{dq^2}dq^2} = 1.471 \pm 0.095_{\,\rm stat.} \pm 0.109_{\,\rm syst.}$$

Experiment:

$$\frac{\displaystyle\int_{15\text{ GeV}^2}^{q^2_{\rm max}} \frac{d\Gamma(\Lambda_b \to p\,\mu^-\bar{\nu}_\mu)}{dq^2}dq^2}{\displaystyle\int_{7\text{ GeV}^2}^{q^2_{\rm max}} \frac{d\Gamma(\Lambda_b \to \Lambda_c\,\mu^-\bar{\nu}_\mu)}{dq^2}dq^2} = (1.00 \pm 0.04 \pm 0.08) \times 10^{-2}$$

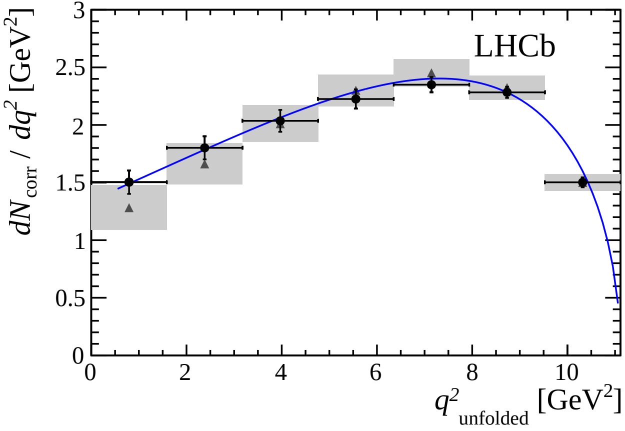
[LHCb Collaboration, arXiv:1504.01568/Nature Physics 2015]

Combine lattice QCD and experiment:

$$\frac{|V_{ub}|}{|V_{cb}|} = 0.083 \pm 0.004_{\,\rm expt} \pm 0.004_{\,\rm lat}$$

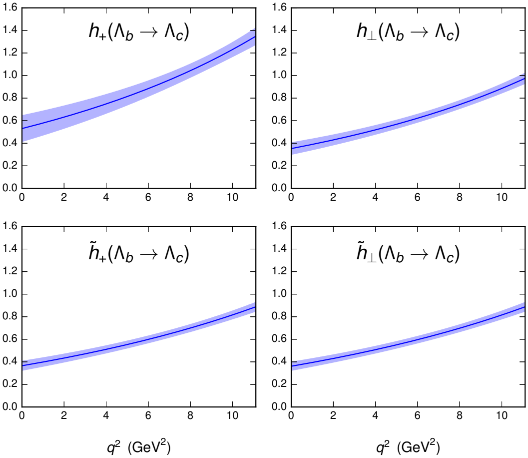
Shape of the $\Lambda_b \to \Lambda_c \mu^- \bar{\nu}_\mu$ decay rate from LHCb

Gray rectangles (triangles = central values): Lattice QCD prediction
Black circles: LHCb

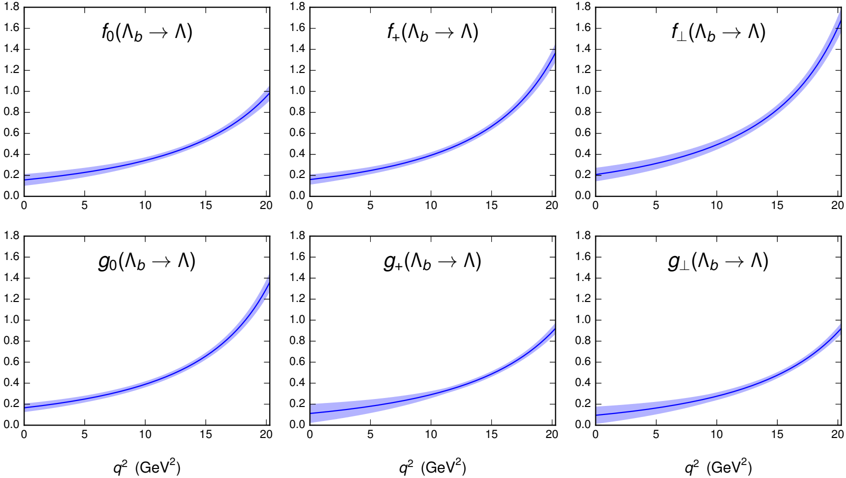


[LHCb Collaboration, arXiv:1709.01920/PRD 2017]

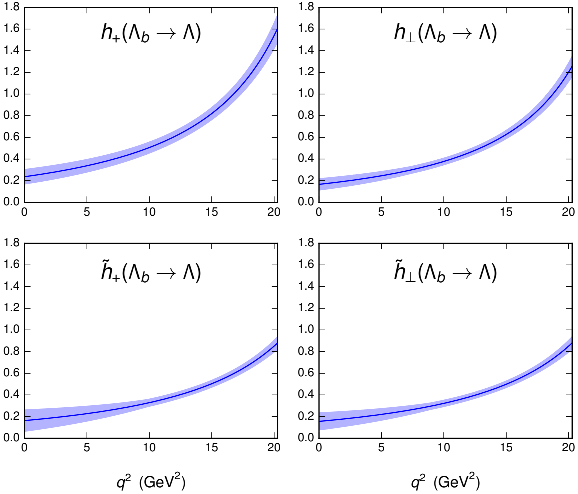
$\Lambda_b \to \Lambda_c$ tensor form factors (for BSM studies)



$\Lambda_b \to \Lambda$ vector and axial vector form factors

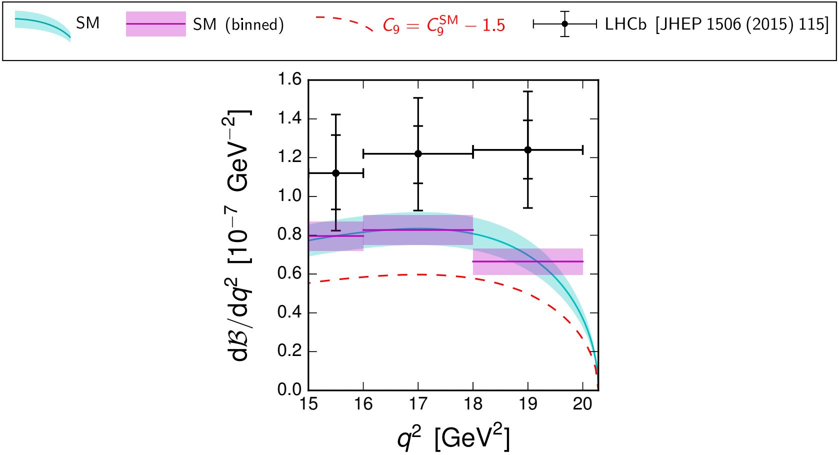


$\Lambda_b \to \Lambda$ tensor form factors



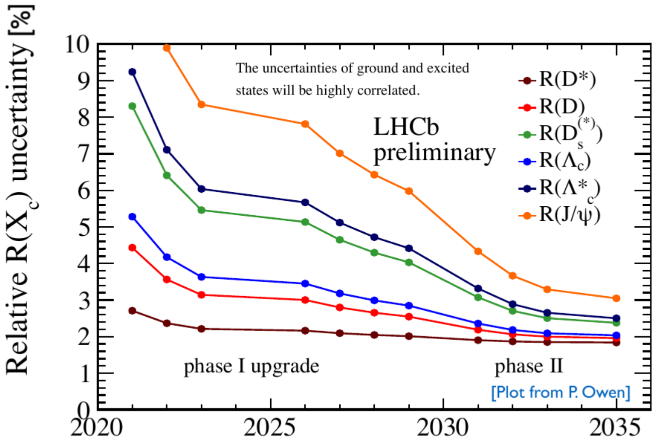
$\Lambda_b \to \Lambda\mu^+\mu^-$ differential branching fraction at high q^2

Contributions from $O_{1\ldots6;8}$ treated with OPE.



Deviation in opposite direction compared to mesonic decays?

Motivation



[G. Cohan, Talk at 2017 LHCb Implications Workshop]

The Λ_c^* baryons

Name	J^P	Mass [MeV]	Width [MeV]	Strong decay modes
$\Lambda_c^*(2595)$	$\frac{1}{2}^-$	2592.25(28)	2.6(6)	$\Lambda_c\,\pi^+\pi^-$
$\Lambda_c^*(2625)$	$\frac{3}{2}^-$	2628.11(19)	< 0.97	$\Lambda_c\,\pi^+\pi^-$

(decays proceed partly through $\Lambda_c^* \to \Sigma_c^{(*)}(\to \Lambda_c\pi)\pi$)

[2017 Review of Particle Physics]

In the following, we will treat the Λ_c^* baryons as if they were stable.

Some notation to define the form factors

$$\langle \Lambda^*_{c\,\frac{1}{2}^-}(\mathbf{p}', s')|\, \bar{c}\Gamma b\, |\Lambda_b(\mathbf{p}, s)\rangle = \bar{u}(m_{\Lambda^*_{c\,\frac{1}{2}^-}}, \mathbf{p}', s')\, \gamma_5\, \mathscr{G}^{(\frac{1}{2}^-)}[\Gamma]\, u(m_{\Lambda_b}, \mathbf{p}, s)$$

$$\langle \Lambda^*_{c\,\frac{3}{2}^-}(\mathbf{p}', s')|\, \bar{c}\Gamma b\, |\Lambda_b(\mathbf{p}, s)\rangle = \bar{u}_\lambda(m_{\Lambda^*_{c\,\frac{3}{2}^-}}, \mathbf{p}', s')\, \mathscr{G}^{\lambda(\frac{3}{2}^-)}[\Gamma]\, u(m_{\Lambda_b}, \mathbf{p}, s)$$

$$\sum_s u(m, \mathbf{p}, s)\bar{u}(m, \mathbf{p}, s) = m + \not{p}$$

$$\sum_{s'} u_\mu(m', \mathbf{p}', s')\bar{u}_\nu(m', \mathbf{p}', s') =$$
$$-(m' + \not{p}')\left(g_{\mu\nu} - \frac{1}{3}\gamma_\mu\gamma_\nu - \frac{2}{3m'^2}p'_\mu p'_\nu - \frac{1}{3m'}(\gamma_\mu p'_\nu - \gamma_\nu p'_\mu)\right)$$

$\frac{1}{2}^+ \to \frac{1}{2}^-$ vector and axial vector form factors

$$
\begin{aligned}
\mathscr{G}^{(\frac{1}{2}^-)}[\gamma^\mu] \quad = \quad & f_0^{(\frac{1}{2}^-)} (m_{\Lambda_b} + m_{\Lambda_c^*}) \frac{q^\mu}{q^2} \\
& + f_+^{(\frac{1}{2}^-)} \frac{m_{\Lambda_b} - m_{\Lambda_c^*}}{s_-} \left(p^\mu + p'^\mu - (m_{\Lambda_b}^2 - m_{\Lambda_c^*}^2) \frac{q^\mu}{q^2} \right) \\
& + f_\perp^{(\frac{1}{2}^-)} \left(\gamma^\mu + \frac{2 m_{\Lambda_c^*}}{s_-} p^\mu - \frac{2\, m_{\Lambda_b}}{s_-} p'^\mu \right),
\end{aligned}
$$

$$
\begin{aligned}
\mathscr{G}^{(\frac{1}{2}^-)}[\gamma^\mu \gamma_5] \quad = \quad & - g_0^{(\frac{1}{2}^-)} \gamma_5 \, (m_{\Lambda_b} - m_{\Lambda_c^*}) \frac{q^\mu}{q^2} \\
& - g_+^{(\frac{1}{2}^-)} \gamma_5 \frac{m_{\Lambda_b} + m_{\Lambda_c^*}}{s_+} \left(p^\mu + p'^\mu - (m_{\Lambda_b}^2 - m_{\Lambda_c^*}^2) \frac{q^\mu}{q^2} \right) \\
& - g_\perp^{(\frac{1}{2}^-)} \gamma_5 \left(\gamma^\mu - \frac{2 m_{\Lambda_c^*}}{s_+} p^\mu - \frac{2\, m_{\Lambda_b}}{s_+} p'^\mu \right),
\end{aligned}
$$

$$s_\pm = (m_{\Lambda_b} \pm m_{\Lambda_c^*})^2 - q^2$$

$\frac{1}{2}^{+} \to \frac{1}{2}^{-}$ tensor form factors

$$
\begin{aligned}
\mathscr{G}^{(\frac{1}{2}^{-})}[i\sigma^{\mu\nu} q_\nu] \quad = \quad & -h_{+}^{(\frac{1}{2}^{-})} \frac{q^2}{s_-} \left(p^\mu + p'^\mu - (m_{\Lambda_b}^2 - m_{\Lambda_c^*}^2) \frac{q^\mu}{q^2} \right) \\
& -h_{\perp}^{(\frac{1}{2}^{-})} (m_{\Lambda_b} - m_{\Lambda_c^*}) \left(\gamma^\mu + \frac{2\, m_{\Lambda_c^*}}{s_-}\, p^\mu - \frac{2\, m_{\Lambda_b}}{s_-}\, p'^\mu \right)
\end{aligned}
$$

$$
\begin{aligned}
\mathscr{G}^{(\frac{1}{2}^{-})}[i\sigma^{\mu\nu} \gamma_5 q_\nu] \quad = \quad & -\widetilde{h}_{+}^{(\frac{1}{2}^{-})} \gamma_5 \frac{q^2}{s_+} \left(p^\mu + p'^\mu - (m_{\Lambda_b}^2 - m_{\Lambda_c^*}^2) \frac{q^\mu}{q^2} \right) \\
& -\widetilde{h}_{\perp}^{(\frac{1}{2}^{-})} \gamma_5 \,(m_{\Lambda_b} + m_{\Lambda_c^*}) \left(\gamma^\mu - \frac{2m_{\Lambda_c^*}}{s_+}\, p^\mu - \frac{2m_{\Lambda_b}}{s_+}\, p'^\mu \right)
\end{aligned}
$$

$\frac{1}{2}^+ \to \frac{3}{2}^-$ vector and axial vector form factors

$$\mathscr{G}^{\lambda(\frac{3}{2}^-)}[\gamma^\mu] = f_0^{(\frac{3}{2}^-)} \frac{m_{\Lambda_c^*}}{s_+} \frac{(m_{\Lambda_b} - m_{\Lambda_c^*})\, p^\lambda q^\mu}{q^2}$$

$$+ f_+^{(\frac{3}{2}^-)} \frac{m_{\Lambda_c^*}}{s_-} \frac{(m_{\Lambda_b} + m_{\Lambda_c^*})\, p^\lambda (q^2(p^\mu + p'^\mu) - (m_{\Lambda_b}^2 - m_{\Lambda_c^*}^2) q^\mu)}{q^2\, s_+}$$

$$+ f_\perp^{(\frac{3}{2}^-)} \frac{m_{\Lambda_c^*}}{s_-} \left(p^\lambda \gamma^\mu - \frac{2\, p^\lambda (m_{\Lambda_b} p'^\mu + m_{\Lambda_c^*} p^\mu)}{s_+} \right)$$

$$+ f_{\perp'}^{(\frac{3}{2}^-)} \frac{m_{\Lambda_c^*}}{s_-} \left(p^\lambda \gamma^\mu - \frac{2\, p^\lambda p'^\mu}{m_{\Lambda_c^*}} + \frac{2\, p^\lambda (m_{\Lambda_b} p'^\mu + m_{\Lambda_c^*} p^\mu)}{s_+} + \frac{s_-\, g^{\lambda\mu}}{m_{\Lambda_c^*}} \right)$$

$$\mathscr{G}^{\lambda(\frac{3}{2}^-)}[\gamma^\mu \gamma_5] = -g_0^{(\frac{3}{2}^-)} \gamma_5 \frac{m_{\Lambda_c^*}}{s_-} \frac{(m_{\Lambda_b} + m_{\Lambda_c^*})\, p^\lambda q^\mu}{q^2}$$

$$- g_+^{(\frac{3}{2}^-)} \gamma_5 \frac{m_{\Lambda_c^*}}{s_+} \frac{(m_{\Lambda_b} - m_{\Lambda_c^*})\, p^\lambda (q^2(p^\mu + p'^\mu) - (m_{\Lambda_b}^2 - m_{\Lambda_c^*}^2) q^\mu)}{q^2\, s_-}$$

$$- g_\perp^{(\frac{3}{2}^-)} \gamma_5 \frac{m_{\Lambda_c^*}}{s_+} \left(p^\lambda \gamma^\mu - \frac{2\, p^\lambda (m_{\Lambda_b} p'^\mu - m_{\Lambda_c^*} p^\mu)}{s_-} \right)$$

$$- g_{\perp'}^{(\frac{3}{2}^-)} \gamma_5 \frac{m_{\Lambda_c^*}}{s_+} \left(p^\lambda \gamma^\mu + \frac{2\, p^\lambda p'^\mu}{m_{\Lambda_c^*}} + \frac{2\, p^\lambda (m_{\Lambda_b} p'^\mu - m_{\Lambda_c^*} p^\mu)}{s_-} - \frac{s_+\, g^{\lambda\mu}}{m_{\Lambda_c^*}} \right)$$

$\frac{1}{2}^+ \to \frac{3}{2}^-$ tensor form factors

$$
\begin{aligned}
\mathscr{G}^{\lambda(\frac{3}{2}^-)}[i\sigma^{\mu\nu} q_\nu] = & -h_+^{(\frac{3}{2}^-)} \frac{m_{\Lambda_b} m_{\Lambda_c^*}}{s_-} \frac{p^\lambda (q^2 (p^\mu + p'^\mu) - (m_{\Lambda_b}^2 - m_{\Lambda_c^*}^2) q^\mu)}{m_{\Lambda_b}\, s_+} \\
& -h_\perp^{(\frac{3}{2}^-)} \frac{m_{\Lambda_b} m_{\Lambda_c^*}}{s_-} \frac{m_{\Lambda_b} + m_{\Lambda_c^*}}{m_{\Lambda_b}} \left(p^\lambda \gamma^\mu - \frac{2\, p^\lambda (m_{\Lambda_b} p'^\mu + m_{\Lambda_c^*} p^\mu)}{s_+} \right) \\
& -h_{\perp'}^{(\frac{3}{2}^-)} \frac{m_{\Lambda_b} m_{\Lambda_c^*}}{s_-} \frac{m_{\Lambda_b} + m_{\Lambda_c^*}}{m_{\Lambda_b}} \left(p^\lambda \gamma^\mu - \frac{2\, p^\lambda p'^\mu}{m_{\Lambda_c^*}} + \frac{2\, p^\lambda (m_{\Lambda_b} p'^\mu + m_{\Lambda_c^*} p^\mu)}{s_+} + \frac{s_-\, g^{\lambda\mu}}{m_{\Lambda_c^*}} \right)
\end{aligned}
$$

$$
\begin{aligned}
\mathscr{G}^{\lambda(\frac{3}{2}^-)}[i\sigma^{\mu\nu} q_\nu \gamma_5] = & -\widetilde{h}_+^{(\frac{3}{2}^-)} \gamma_5 \frac{m_{\Lambda_b} m_{\Lambda_c^*}}{s_+} \frac{p^\lambda (q^2 (p^\mu + p'^\mu) - (m_{\Lambda_b}^2 - m_{\Lambda_c^*}^2) q^\mu)}{m_{\Lambda_b}\, s_-} \\
& -\widetilde{h}_\perp^{(\frac{3}{2}^-)} \gamma_5 \frac{m_{\Lambda_b} m_{\Lambda_c^*}}{s_+} \frac{m_{\Lambda_b} - m_{\Lambda_c^*}}{m_{\Lambda_b}} \left(p^\lambda \gamma^\mu - \frac{2\, p^\lambda (m_{\Lambda_b} p'^\mu - m_{\Lambda_c^*} p^\mu)}{s_-} \right) \\
& -\widetilde{h}_{\perp'}^{(\frac{3}{2}^-)} \gamma_5 \frac{m_{\Lambda_b} m_{\Lambda_c^*}}{s_+} \frac{m_{\Lambda_b} - m_{\Lambda_c^*}}{m_{\Lambda_b}} \left(p^\lambda \gamma^\mu + \frac{2\, p^\lambda p'^\mu}{m_{\Lambda_c^*}} + \frac{2\, p^\lambda (m_{\Lambda_b} p'^\mu - m_{\Lambda_c^*} p^\mu)}{s_-} - \frac{s_+\, g^{\lambda\mu}}{m_{\Lambda_c^*}} \right)
\end{aligned}
$$

Λ_c^* interpolating fields

We work in the Λ_c^* rest frame to allow exact spin-parity projection. We use

$$(\Lambda_c^*)_{j\gamma} = \epsilon^{abc}\,(C\gamma_5)_{\alpha\beta}\Big[\tilde{c}^a_\alpha\,\tilde{d}^b_\beta\,(\nabla_j\tilde{u})^c_\gamma - \tilde{c}^a_\alpha\,\tilde{u}^b_\beta\,(\nabla_j\tilde{d})^c_\gamma + \tilde{u}^a_\alpha\,(\nabla_j\tilde{d})^b_\beta\,\tilde{c}^c_\gamma - \tilde{d}^a_\alpha\,(\nabla_j\tilde{u})^b_\beta\,\tilde{c}^c_\gamma\Big]$$

(˜ denotes Gaussian smearing) [S. Meinel and G. Rendon, arXiv:1608.08110/Lattice2016]

This requires light-quark propagators with derivative sources.

We project to $J^P = \frac{1}{2}^-$ and $J^P = \frac{3}{2}^-$ using

$$P^{(\frac{1}{2}^-)}_{jk} = \frac{1}{3}\gamma_j\gamma_k\frac{1+\gamma_0}{2},$$

$$P^{(\frac{3}{2}^-)}_{jk} = \left(g_{jk} - \frac{1}{3}\gamma_j\gamma_k\right)\frac{1+\gamma_0}{2}.$$

Lattice methods

- Gauge field configurations generated by the RBC and UKQCD collaborations
  [Y. Aoki *et al.*, arXiv:1011.0892/PRD 2011]
- u, d, s quarks: domain-wall action
  [D. Kaplan, arXiv:hep-lat/9206013/PLB 1992; V. Furman and Y. Shamir, arXiv:hep-lat/9303005/NPB 1995]
- All-mode averaging with 1 exact and 32 sloppy propagators per configuration
  [E. Shintani *et al.*, arXiv:1402.0244/PRD 2015]
- c, b quarks: anisotropic clover with three parameters, re-tuned more accurately to $D_s^{(*)}$ and $B_s^{(*)}$ dispersion relation and HFS
- "Mostly nonperturbative" renormalization
  [A. El-Khadra *et al.*, hep-ph/0101023/PRD 2001]
- Three-point functions with 9 source-sink separations

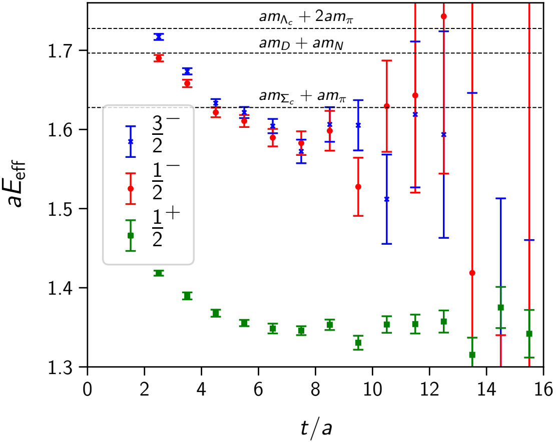
Lattice parameters

Name	$N_s^3 \times N_t$	β	$am_{u,d}$	am_s	a (fm)	m_π (MeV)	Run status
C01	$24^3 \times 64$	2.13	0.01	0.04	≈ 0.111	≈ 430	1/4 cfgs done
C005	$24^3 \times 64$	2.13	0.005	0.04	≈ 0.111	≈ 340	1/4 cfgs done
F004	$32^3 \times 64$	2.25	0.004	0.03	≈ 0.083	≈ 300	1/4 cfgs done

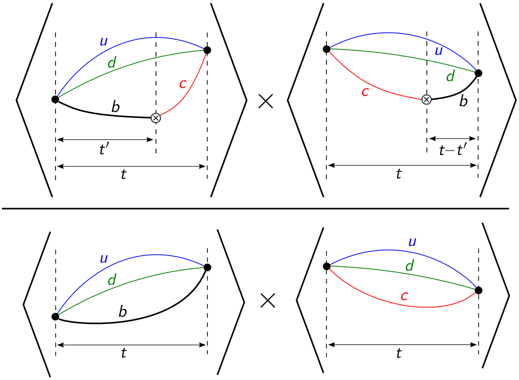
Λ_c^* two-point functions preliminary

Results from $24^3 \times 64$, $am_{u,d} = 0.005$ ensemble, 78 configs $\times$ 32 sources

$a^{-1} = 1.785(5)$ GeV



Extracting the form factors from ratios of 3pt and 2pt functions



t = source-sink separation
t' = current insertion time

We have data for two different Λ_b momenta: $\mathbf{p} = (0,0,2)\frac{2\pi}{L} \approx 0.9$ GeV and $\mathbf{p} = (0,0,3)\frac{2\pi}{L} \approx 1.4$ GeV

Extracting the form factors from ratios of 3pt and 2pt functions

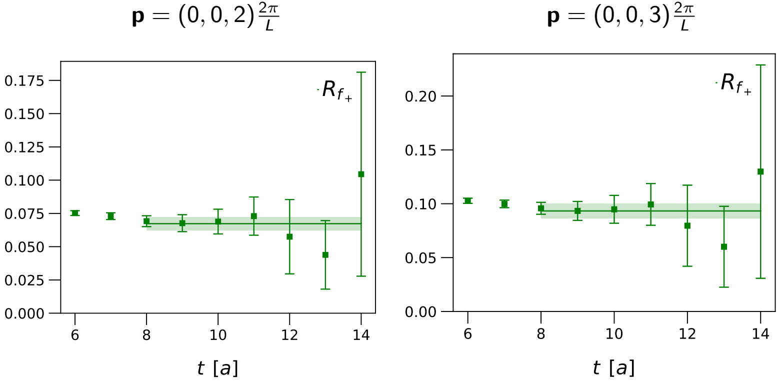
Schematically,

$$R_f(\mathbf{p},t) \quad = \quad \sqrt{(\text{kinematic factors}) \times (\text{polarization vectors}) \times (\text{ratio at } t' = t/2)}$$

$$\rightarrow \quad f(\mathbf{p}) \quad \text{for large } t$$

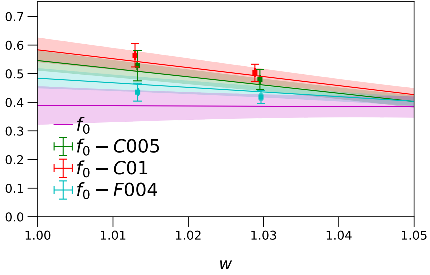
Example: R_{f_+} for $\Lambda_b \to \Lambda_c^* \left(\frac{3}{2}^-\right)$ preliminary

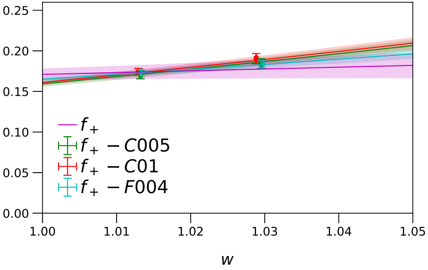
Results from $24^3 \times 64$, $am_{u,d} = 0.005$ ensemble, 78 configs $\times$ 32 sources

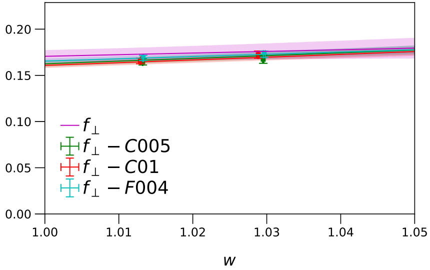


$\Lambda_b \to \Lambda_c^* \left(\frac{1}{2}^-\right)$ vector form factors

very preliminary



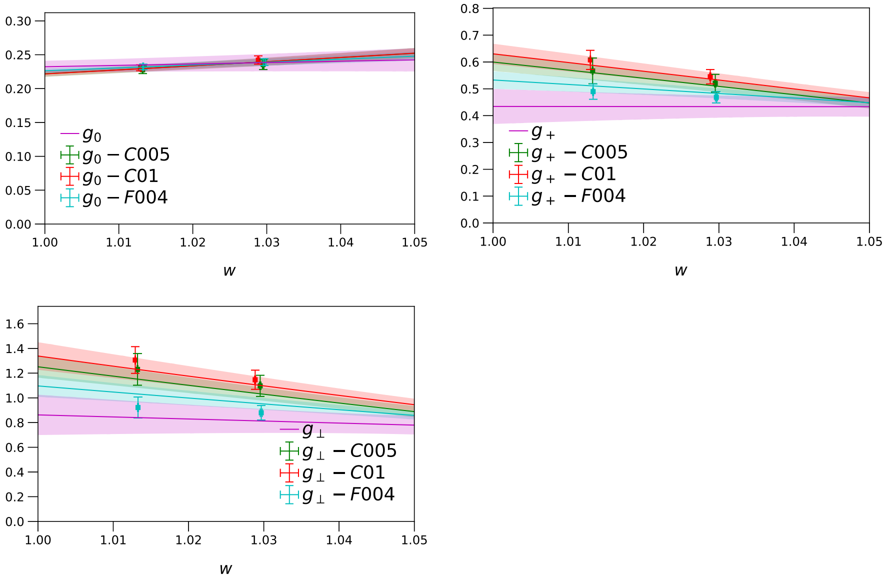




$$w = v \cdot v' = \frac{m_{\Lambda_b}^2 + m_{\Lambda_c^*}^2 - q^2}{2 m_{\Lambda_b} m_{\Lambda_c^*}}$$

Only the statistical uncertainties are shown.

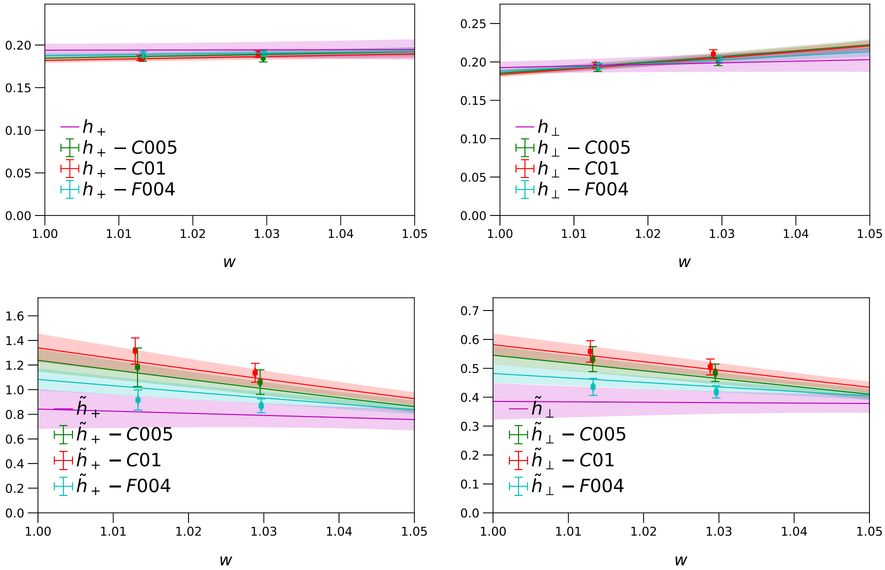
$\Lambda_b \to \Lambda_c^* \left(\frac{1}{2}^-\right)$ axial vector form factors very preliminary



Only the statistical uncertainties are shown.

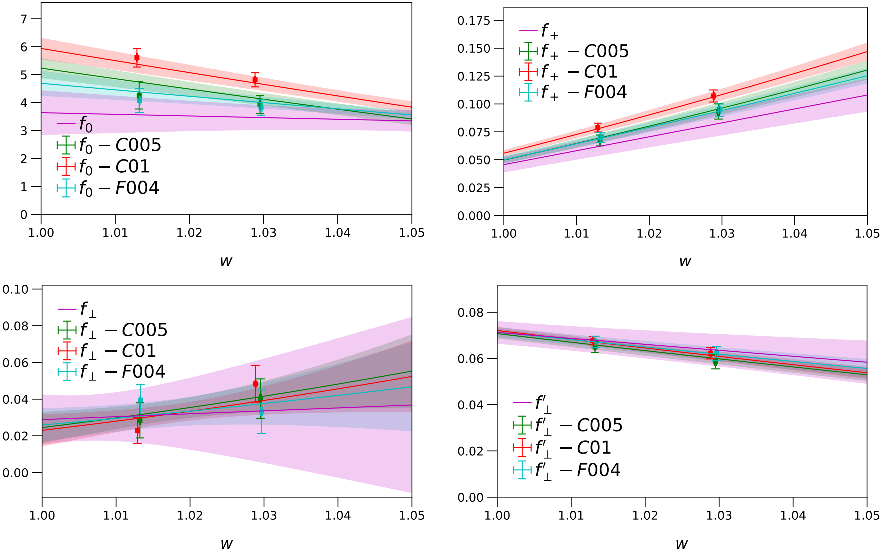
$\Lambda_b \to \Lambda_c^* \left(\frac{1}{2}^-\right)$ tensor form factors

very preliminary



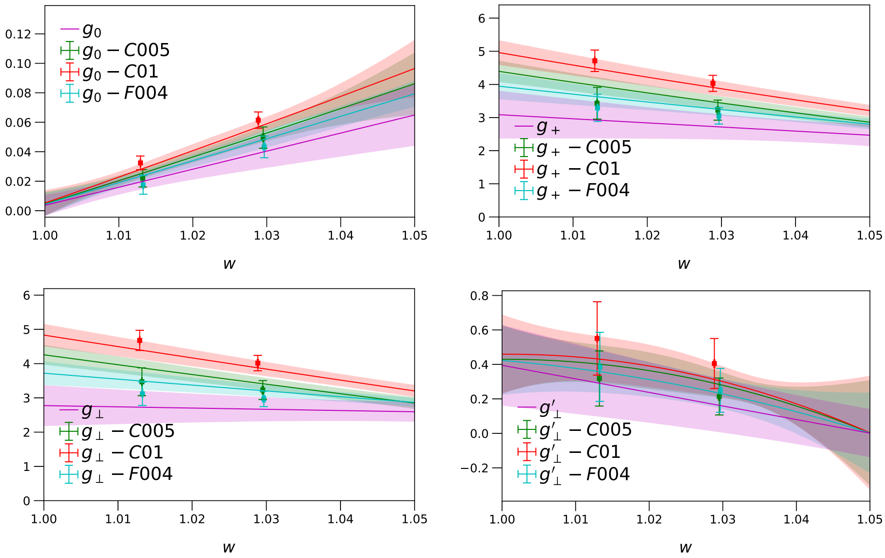
Only the statistical uncertainties are shown.

$\Lambda_b \to \Lambda_c^* \left(\frac{3}{2}^-\right)$ vector form factors   very preliminary



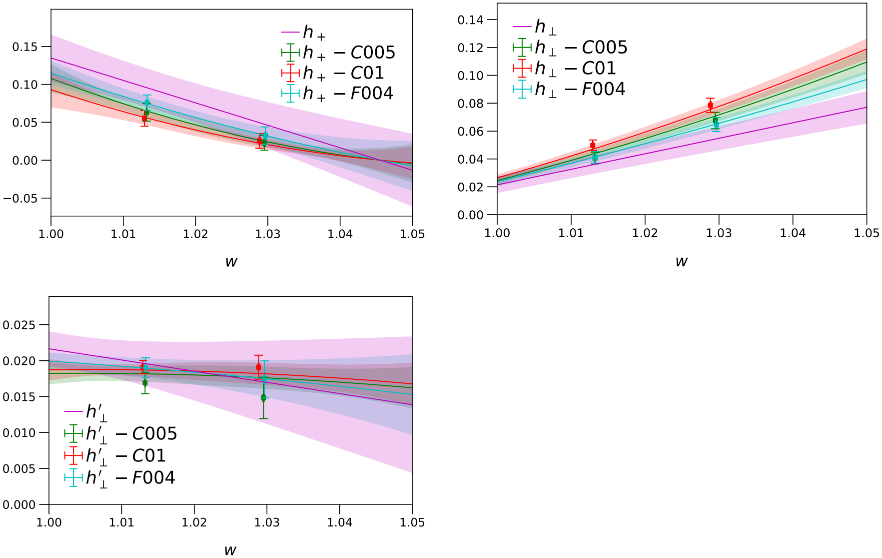
Only the statistical uncertainties are shown.

$\Lambda_b \to \Lambda_c^* \left(\frac{3}{2}^-\right)$ axial vector form factors very preliminary



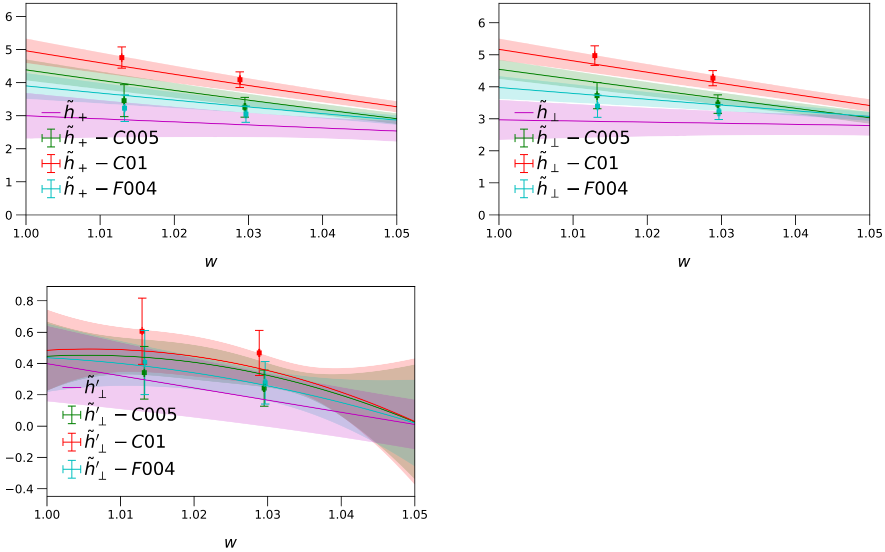
Only the statistical uncertainties are shown.

$\Lambda_b \to \Lambda_c^* \left(\frac{3}{2}^-\right)$ tensor form factors part 1 very preliminary



Only the statistical uncertainties are shown.

$\Lambda_b \to \Lambda_c^* \left(\frac{3}{2}^-\right)$ tensor form factors part 2 very preliminary

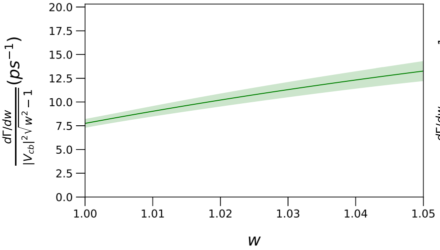


Only the statistical uncertainties are shown.

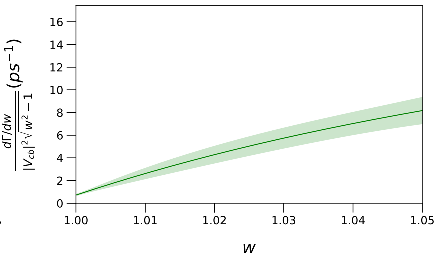
$\Lambda_b \to \Lambda_c^* \, \mu^- \bar{\nu}_\mu$ differential decay rates

very preliminary

$\Lambda_b \to \Lambda_c^*(2595) \, \mu^- \bar{\nu}_\mu$



$\Lambda_b \to \Lambda_c^*(2625) \, \mu^- \bar{\nu}_\mu$



Only the statistical uncertainties are shown.

To predict $R(\Lambda_c^*)$, we will combine the lattice QCD form factors (which are limited to low recoil) with experimental data for the shapes of the $\Lambda_b \to \Lambda_c^* \mu \bar{\nu}$ differential decay rates, making use of HQET.

[P. Boer, M. Bordone, E. Graverini, P. Owen, M. Rotondo, and D. Van Dyk, arXiv:1801.08367]